Rationalizing Seven Consumption-Saving Puzzles in a Unified Framework

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Abstract

Empirical evidence suggests that it may cost time, effort, and resources to properly implement a saving plan, though such cost may differ across individual consumers. We document seven facts on macroeconomic consumption and saving over the life cycle, and we enrich a simple life-cycle model by costly saving implementation to explain these facts. This friction is the sole and common mechanism in our model for rationalizing this series of facts, as the model abstracts from all existing mechanisms that are known to help explain some of them. The implementation costs in our model are small, yet they help resolve these macroeconomic consumption and saving puzzles in a simple and unified way.

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1. Introduction

This paper proposes a simple, unified model for a series of facts concerning macroeconomic consumption and saving over the life cycle. Our model enriches the standard life-cycle framework by an empirically relevant feature that it may cost time, effort, and resources to properly implement a consumption-saving plan, though such cost may differ across individual consumers. When making the decision, a rational, forward-looking consumer must weigh the expected welfare gains from the more efficient reallocation of resources that could be brought about by following an optimal consumption-saving plan against the costs of implementing the plan. This simple modification to the standard model offers a unified framework for thinking about the following life-cycle behaviors of macroeconomic saving and consumption:

1. Saving is usually postponed with heterogeneous implementation dates.
2. A fraction of the population never save.
3. There is heterogeneity in accumulated wealth at retirement holding lifetime earnings constant, yet consumption growth in the several years leading up to retirement is almost the same (and close to zero) across different wealth quartiles at retirement.
4. Aggregate consumption displays a hump during the work life.
5. Aggregate consumption experiences a discrete drop at retirement.
6. The discrete drop in aggregate consumption at retirement is largely anticipated.
7. Consumption tracks income over the work life for a small fraction of the population that has accumulated no or little savings at retirement.

A number of mechanisms have been proposed in the existing literature in the attempt to explain one or another of these facts, as we survey in Section 2. Yet, it appears to be difficult to jointly rationalize even a small subset of these facts with the existing theories, as some researchers have noted (e.g., Bernheim et al. 2001, Hurst 2006, Bullard and Feigenbaum 2007; and see Section 2 for details).
The contribution of this paper is to provide a unified theory to address the issue. The friction we model, that it is costly to implement an optimal consumption-saving plan, is the sole and common mechanism in our model framework for rationalizing the above series of facts, as our baseline model abstracts from those other mechanisms that are known to help explain some of them. Indeed, results produced from the degenerated version of our model that abstracts from this friction are at odds with every one of the facts: all consumers would implement a saving plan as soon as entering the workforce and accumulate the same amount of wealth at the time of retirement, and the aggregate consumption path would be flat over the life cycle and thus smooth across the retirement date, as the life-cycle consumption path for each individual consumer would behave this way, just like in the permanent-income paradigm. With the costs to implement saving plans introduced as suggested by the empirical evidence (see Section 2 for details), our model can account for the entire set of these facts.

We begin Section 2 by documenting these facts with more detail. We then review the existing theories in explaining some of these facts and motivate our approach. We present our model in Section 3, where we also characterize bounds on the implementation costs that can give rise to a non-degenerated equilibrium. We describe our model calibration, report the quantitative results, and provide some intuition in Section 4. Section 5 closes the paper.

2. Facts, existing theories, and our approach

In this section we first summarize the seven stylized facts about the behaviors of life-cycle saving and consumption. We next review the existing theories in explaining some of these facts and then motivate our approach.

2.1. Facts about life-cycle saving and consumption

There is a large body of literature that documents the behaviors of consumption and saving over the life cycle. The evidence obtained based on various sources of data suggests a number of stylized facts about life-cycle saving and consumption that we summarize here.

Fact 1: Saving is usually postponed with heterogeneity across the population in the date of implementing a saving plan. Micro data suggest that many people do not begin saving
in the early stage of their work life. The 1995 Survey of Consumer Finances (SCF)\(^1\) reveals that young households hold no or very little assets other than durable consumption goods (e.g., Fernández-Villaverde and Krueger 2001). Consistent evidence can be found in the 2001 SCF which shows that 48 percent of households with heads under the age of 35 do not yet have any retirement savings account, and in the 2001 Survey of Income and Program Participation (SIPP)\(^2\) which shows an even larger fraction of 58 percent (e.g., Purcell 2004). In actuality workers near retirement report dramatic heterogeneity in the date when they first started saving, which ranges from “during their twenties” to “never” (e.g., Loewenstein et al. 1999). For example, the 2001 SCF and SIPP both reveal that the share of middle-age people with a retirement saving account is significantly larger than in the under-35 group. Indeed, according to the 2006 Retirement Confidence Survey (RCS)\(^3\), American workers age 25 to 35 are less likely to have started saving for retirement than those age 35 and older (e.g., Helman et al. 2006) while not much saving occurs before the age of 50 (e.g., Lusardi 2003). The 2007 RCS shows a similar pattern. Corroborating evidence is also reported by Nyce (2005) based on a study of 300,000 employees, and Huberman et al. (2007) based on a sample of 800,000 employees with access to Vanguard defined contribution pension plans.

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\(^1\)The SCF is conducted every three years by the Board of Governors of the Federal Reserve System. The sample is random and representative, and the survey collects information on assets, liabilities, income, and other demographic characteristics for a cross section of U.S. households. The 1995 SCF interviews 4,299 households while the 2001 SCF panel consists of 4,449 families.

\(^2\)The SIPP is sponsored by the Census Bureau, and it collects detailed data on household assets and liabilities, along with income, labor force, program participation and eligibility, and demographic characteristics. The survey design is a continuous series of national panels, with sample size ranging from approximately 14,000 to 36,700 interviewed households and new multi-year panels of households introduced periodically. The 1996 redesign introduces a larger initial sample than in previous panels, with a target of 37,000 households, and the 1996 panel actually consists of 40,188 households.

\(^3\)The RCS sponsored by the Employee Benefit Research Institute, the American Savings Education Council, and Mathew Greenwald & Associates is a random and nationally representative survey with over 1,000 annual participants. It is a comprehensive study of the attitudes and behavior of American workers and retirees towards all aspects of saving, retirement planning, and long-term financial security.
Fact 2: A fraction of the population never save. The aforementioned work by Loewenstein et al. (1999) is one of many studies that find that some people never save. The 1996 Health and Retirement Survey (HRS) reveals that 33 percent of retirees have no private savings, and the Social Security Administration (SSA) 2005 Fact Sheet shows that 22 percent of social security beneficiaries have no other source of income during retirement. The 2005 RCS indicates that more than 20 percent of American workers do not intend to ever save for retirement, and the 2007 RCS confirms that 24 percent of retired Americans have indeed no private savings. Similar estimates are obtained by Hurst (2006) based on a cross-section of pre-retired households from the Panel Study of Income Dynamics (PSID), and Scholz et al. (2006) based on a sample of 10,523 respondents in 6,322 households from the HRS and social security earnings records.

Fact 3: There is heterogeneity in accumulated savings at retirement holding lifetime earnings constant, yet consumption growth in the several years leading up to retirement is almost the same (and close to zero) across different wealth quartiles at retirement. Many empirical studies have documented the heterogeneity in accumulated savings at retirement conditional on lifetime earnings. These include Venti and Wise (1998) based on the HRS data and earnings histories provided by the SSA for 4,000 households, Engen et al. (2000) for 2,500 households from the HRS and 1,500 households from the SCF, and Hurst (2006) for about 800 households from the PSID (see also the references therein). As shown by Bernheim et al.

4The HRS co-sponsored by the SSA and the National Institute on Aging at the National Institute of Health is conducted at the University of Michigan. The biennial national panel study surveys more than 22,000 Americans over the age of 50 about their financial status, retirement planning, insurance coverage, labor market status, family support systems, and physical and mental health.

5The PSID is a longitudinal study of a representative sample of U.S. individuals and the family units in which they reside. The study is conducted at the Survey Research Center, Institute for Social Research, and University of Michigan, and sponsored primarily by the National Science Foundation. Its sample size grew from 4,800 families in 1968 when it first began to more than 7,000 families in 2001 (the 2001 data was the most recent PSID data available when the analysis in Hurst (2006) was conducted).
(2001), based on 3,500 observations of 430 households from the PSID with additional consumption information from the Consumer Expenditure Survey (CEX)\textsuperscript{6} between the years of 1978 and 1990, what makes this fact particularly puzzling is another fact that consumption growth in the several years immediately before retirement is almost the same (and close to zero) across different wealth quartiles at retirement.

Fact 4: Aggregate consumption displays a hump during the work life. Many empirical studies based on the CEX and other data find that the work-life aggregate consumption profile is hump-shaped (e.g., Carroll and Summers 1991, Attanasio 1999, Gourinchas and Parker 2002, Fernández-Villaverde and Krueger 2007). The location of the hump is generally between 45 and 55 years of age, and the size of the peak (measured as the ratio of the peak consumption level to the level of consumption when first entering the work force) is generally above 1.1. This finding is robust after controlling for changes in family-size, occupation type, and education level, and it holds for not only the U.S., but other countries as well.

Fact 5: Aggregate consumption experiences a discrete drop at retirement. While a discrete drop in aggregate consumption at retirement in the range of 5 to 43 percent is reported in the literature, most studies find that the size of the drop is between 10 and 20 percent. The 1997 RCS suggests that such a drop is primarily fueled by a small subset of the population with very little savings, and recent studies confirm this. The empirical studies in this area include Hamermesh (1984) based on a sample of 500 households from the Retirement History Survey (RHS),\textsuperscript{7} Mariger (1987) on 800 households from the Surveys of Financial Characteristics of households.

\textsuperscript{6}The CEX survey conducted by the Bureau of Labor Statistics since 1980 contains a representative sample of the universe of U.S. households. It is a rotating panel with 5,000 randomly selected households in each quarter reporting their expenditures on a variety of goods and services in the previous three months. The survey accounts for about 95\% of all quarterly household expenditures in each consumption category from a highly disaggregated list of consumption goods and services. This renders the CEX data a main advantage over the PSID, which reports consumption expenditures for food only.

\textsuperscript{7}The RHS was designed by the Office of Research and Statistics at the SSA to be a longitudinal study of U.S. households nearing retirement age. It was conducted by the Census Bureau on a biennial basis and six surveys were conducted over the ten-year period from 1969 to 1979.
Consumers (SFCC) and Changes in Financial Characteristics of Consumers (SCFCC), and Haider and Stephens (2007) on 5,000 households from the five biennial surveys of the RHS over the period of 1969 to 1977 and 2,600 households from the first five biennial waves of the HRS covering the period of 1992 to 2000. Recent contributions also include Bernheim et al. (2001), Hurd and Rohwedder (2003, 2006), Scholz et al. (2006), Hurst (2006), and Ameriks et al. (2007), based on data from the PSID, CEX, HRS, and a survey of TIAA-CREF participants. Countries besides the U.S. also experience such a drop. For example, this is also the case for Canada according to Robb and Burbidge (1989) based on data spanning the six-year period from 1979 to 1985 from the Canadian Family Expenditure Survey (FAMES), and the U.K. according to Banks et al. (1998) based on an annual cross-sectional survey of 7,000 randomly selected British households from 1968 to 1992 under the Family Expenditure Survey.

**Fact 6: The discrete drop in aggregate consumption at retirement is anticipated.** While earlier work examining many existing mechanisms nearly always deduced that the observed drop in consumption at retirement must be something that takes households by surprise, recent survey evidence and empirical studies designed to directly test the issue of expectations all lead to the conclusion that the drop is in fact anticipated. For example, the 2007 Retirement Confidence Survey asks workers if they expect their spending to change at the date of retirement, and about 20 percent of the workers say that they expect a sharp decrease in spending at the date of retirement. In addition, the survey also asks retirees if they actually

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8The SFCC and SCFCC were the precursors to the SCF conducted by the Board of Governors of the Fed. Like the SCF both of the two earlier surveys were designed to be representative of all consumers and they interviewed the same set of 2,164 households and are meant to obtain information on the levels of the household’s assets and liabilities.

9The FAMES consisted of 14,000 Canadian households for the national survey and 7,000 for the urban survey, and it provided information on expenditures for detailed categories of consumer goods and services, as well as the income and other characteristics of families and individuals living in private households in Canada. It was replaced by the Survey of Household Spending in 1997.

10The FES is conducted by the Office for National Statistics, which is the government agency in the United Kingdom that collects statistics on the economy.
experienced a drop in spending at the date of retirement, and 20 percent of the retirees say that they did experience a sharp drop at retirement. Hence the expectations line up with the outcomes. Similar results are obtained by Hurd and Rohwedder (2003, 2006) based on a sample of 1,000 individuals from the HRS, and Ameriks et al. (2007) on a survey of 2,000 TIAA-CREF participants, both of which conclude that the drop is anticipated. Hurst (2007) provides a survey of the related evidence.

**Fact 7:** Consumption tracks income over the work life for a small group of people who have accumulated no or little savings at retirement. The fact that consumption tracks income has long been documented, and Browning and Crossley (2001) contains a survey of this literature. Hurst (2006) finds that such tracking exists for a small group of people who have accumulated no or little savings at retirement, which is about 20 percent of the population, but not for the larger group of people who are financially prepared to maintain their living standards during retirement.

2.2. Existing theories

In the literature various theories have been proposed in the attempt to explain one or another fact outlined above. Of all these facts, Fact 4 has probably received the most attention. There are several behavior-based stories. A “rule-of-thumb” story is offered by Campbell and Mankiw (1989) in which an agent simply consumes a constant fraction of his wage income that is hump-shaped over the work life. Short-term planning and overconfidence are shown to be also able to produce a work-life consumption hump (e.g., Caliendo and Aadland 2007, Caliendo and Huang 2007). Related to Fact 1, O’Donoghue and Rabin (1999) and Rabin and O’Donoghue (1999) argue that self-control problems due to time-inconsistent discounting combined with a lack of awareness of these self-control problems may play a role in generating procrastination in savings. The absent-minded consumer in Ameriks et al. (2004) is assumed to exert more effort in monitoring his income and expenditure when young and old than at the middle age. This hump-shaped absent-mindedness coupled with a hump-shaped wage profile point to some potential avenues for explaining Facts 1 and 4, as well as 5 and 6.
Models based on rational choice have also been developed in explaining Fact 4. One important mechanism has to do with demographic changes. The idea that family size dynamics may generate a life-cycle consumption hump takes its original root in Tobin (1967), and it has been confirmed by Browning et al. (1985), Attanasio and Browning (1995), Attanasio et al. (1999), Browning and Ejrnaes (2000), and Büttler (2001).

Another rational story for explaining Fact 4 concerns labor-income uncertainty. The idea can be traced back at least to Nagatani (1972), and it has been emphasized by Hubbard et al. (1994), and Carroll (1994, 1997). In Gourinchas and Parker (2002), the precautionary motive for saving due to income uncertainty in the early stage of one’s work life is offset by the urge to borrow due to high discounting relative to the interest rate, leading to a delay in saving (Fact 1) along with a hump in consumption.

Mortality risk is another factor that can generate a hump-shaped consumption profile during a rational agent’s work life. The idea goes back at least to Yaari (1965), and it has been revisited and confirmed by Büttler (2001), Hansen and İmrohoroğlu (2005), and Feigenbaum (2007). The role of consumption-leisure tradeoffs in generating a work-life consumption hump has been examined at least since Heckman (1974), and it has been verified by Büttler (2001), and Bullard and Feigenbaum (2007).

The idea that durable goods may help explain Facts 1 and 4 has been developed by Fernández-Villaverde and Krueger (2001). They find that the accumulation of durable goods early in the work life can lead people to rationally choose to wait until later in the working period to accumulate financial wealth, while this mechanism can also produce a hump in the consumption of durable and non-durable goods.

Concerning Fact 5, many studies conjecture that the story for a discrete drop in aggregate consumption at retirement must have to do with some sophisticated stochastic mechanism and the systematic arrival of unexpected adverse information. Deduction along this line nearly always leads to the inference that the drop must be unplanned, unexpected, and unintended (e.g., Banks et al. 1998, Loewenstein et al. 1999, Browning and Crossley 2001, Bernheim et al. 2001). But this is inconsistent with Fact 6. Some ideas for explaining
Facts 5 and 6 alternative to the hump-shaped absent-mindedness have been discussed in the recent literature. These include less work-related expenses, greater search effort, and more home production during the retirement phase, together with non-separability between consumption and leisure (e.g., Haider and Stephens 2007, Ameriks et al. 2007, and the references therein).

Some lessons about Fact 3 have also been learned. As Bernheim et al. (2001) illustrate, the fact that consumption growth during the several years immediately before retirement is almost the same (and close to zero) across different wealth quartiles at retirement implies that many well-known mechanisms must be ruled out as a possible explanation for the observed heterogeneity in accumulated savings at retirement. These mechanisms include variations in the discount rate, mortality risk, income uncertainty, and tastes for precaution. Their analysis also leads these authors to exclude other mechanisms, such as variations in unanticipated events that affect the timing of retirement, relative tastes for work and leisure at advanced ages, home production, and work-related expenses as a potential cause for the heterogeneity in wealth at retirement. They conclude that “...a broad range of standard life-cycle considerations are collectively incapable of accounting for the observed variation in wealth, holding lifetime earnings constant.” Some studies contend that a sophisticated stochastic life-cycle model that accounts for idiosyncratic earnings histories may generate cross-sectional variation in wealth (e.g., Engen et al. 2000, Scholz et al. 2006).

In sum, the literature has been most successful in explaining Fact 4, while various existing models also shed some important light on Facts 1, 3, 5, and 6. Yet there does not appear to be any unified theory in the existing literature that can explain all the facts summarized in Section 2.1. Hurst (2006) notes the difficulty in explaining even a small subset of these facts with the existing theories. For example, he shows that Facts 3, 5, and 7 cannot be jointly rationalized by the popular life-cycle considerations such as precautionary saving, habit persistence, variation in time preferences, substitutability between consumption and leisure, and liquidity constraints. In a recent study Bullard and Feigenbaum (2007) also note that “...there is little reason to believe that the [existing] mechanisms that are most pertinent
to resolving [puzzles during the retirement phase] are the mechanisms most pertinent to account for [puzzles during the working phase].”

2.3. Our approach

We provide a unified theory that can account for all the facts outlined in Section 2.1. Our approach enriches the standard life-cycle model by an empirically relevant feature that many households encounter nontrivial costs to properly implement a consumption-saving plan, and our rational-choice based model abstracts from the other mechanisms that are known to help explain some of these facts as surveyed in Section 2.2.

The idea that it can be costly to implement an optimal consumption-saving plan has been reflected in a vast empirical literature on financial literacy, retirement saving, default choice, and other related topics, based on various data sets (e.g., Madrains and Shea 2001, Benartzi and Thaler 2001, Lusardi 2003, Iyengar et al. 2004, Cronqvist and Thaler 2004, Ameriks et al. 2004, Beshears et al. 2005, Helman et al. 2006, Choi et al. forthcoming). It has also been manifested in the popular press and policy discussion (e.g., Bernanke 2006). More direct evidence can be found in various surveys and fee schedules for financial planning services. The cost may reflect time, effort, and resources spent in learning financial knowledge and consulting financial planning professionals, relatives, friends, or co-workers, as well as in budgeting and monitoring expenditures and flows of funds, portfolio choice and evaluation, asset allocation and re-balancing, and executing account transactions that are necessary for properly implementing a saving plan.

To give just a few examples, the 2006 RCS finds that 23 percent of Americans spend up to 1 hour per month reading about money and financial matters, 19 percent spend 1 to 2 hours, 24 percent spend 3 hours or more, and 34 percent spend no time.11 The heterogeneity in such cost across people is also revealed in the literature referred to in the above paragraph (see, also, Hogarth et al. 2003). The 2007 RCS shows that the average American spends 19 hours

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11There are several possibilities for this last cohort of Americans. For example, they may have already learned all the necessary knowledge, or already delegated the task to their financial advisors, or perhaps it is just too costly for them to do so anyway.
per year planning for retirement. A 2000 survey by the Canadian Association of Financial Planners reports that the average Canadian spends 2 to 3 hours per week managing their finances. On the other hand, fees for very basic financial advisory services (e.g., excluding annual checkup and account management) provided by professional financial advisors can easily run $1,000 a year.¹²

Envisioning that such implementation costs may lead to procrastination of saving for retirement, Lusardi (2006) puts the following item on the research agenda:

“...consumers facing high planning costs may optimally choose not to plan for retirement and not save optimally because those costs are much larger than the welfare gains of making these decisions. While these are plausible explanations, we need to explore these topics more formally.”

In what follows, we present a formal life-cycle model with costly implementation of an optimal consumption-saving plan and with forward-looking rational consumers, and we show that it can account for not only Fact 1, as Lusardi (2006) has envisioned, but also Facts 2-7.

3. A life-cycle model with costly saving implementation

Our basic framework augments the standard life-cycle consumption model with a cost to implement a saving plan. We consider a continuous time setting, with the starting and terminating dates of 0 and $T$, respectively.

We describe first the problem of an individual consumer. The consumer enters the workforce at 0, retires at $T$, and passes away at $\bar{T}$. When entering the workforce, the consumer has no savings or debt. At any date $t$ during the working period, he earns wage income $w(t)$ while facing a social security tax rate $\tau(t)$. At any date $t$ during his retirement phase, he receives a social security benefit $B(t)$.

There is a single perishable good that serves as the unit of account in the economy. At each date $t$, the consumer derives utility $u(c(t))$ from his consumption of the good, $c(t)$. His lifetime utility is

\[ \int_0^T e^{-\rho t} u(c(t)) dt, \]  

(1)

where \( \rho \) is a subjective discount rate.

There is an asset with a rate of return \( r \) that the consumer can use to transfer income across time. The defining feature of the model is that it is costly to implement a saving plan. The consumer’s problem is to choose between living hand-to-mouth all the time and implementing a saving plan at some date \( s \in [0, T] \) to maximize (1) subject to

\[ c(t) = y(t), \quad \text{for } t \in [0, s); \]  

(2)

\[ \dot{W}(t) = rW(t) - c(t), \quad \text{for } t \in [s, \bar{T}], \]

\[ W(s) = \int_s^T e^{-r(t-s)} y(t) dt - \kappa(s), \]  

(3)

\[ W(\bar{T}) = 0; \]

where \( y(t) \) denotes the consumer’s disposable income at date \( t \) given by

\[ y(t) = \begin{cases} [1 - \tau(t)] w(t), & \text{for } t \in [0, T), \\ B(t), & \text{for } t \in [T, \bar{T}], \end{cases} \]  

(4)

\( W(t) \) is the consumer’s wealth at date \( t \), and \( \kappa(s) \) is the cost to implement a saving plan at time \( s \).\(^{13}\) Before a saving plan is in place, the consumer lives hand-to-mouth by simply consuming his disposable income, but after implementing the saving plan he would live in the permanent-income paradigm.

Let \( V(\kappa) \) be the value function of the consumer facing the implementation cost \( \kappa \). The lifetime utility maximization problem is then described as follows:

\[ V(\kappa) = \max \{ \int_0^T e^{-\rho t} u(y(t)) dt, \max_{s \in [0, T]} [\int_0^s e^{-\rho t} u(y(t)) dt \]

\[ + (\max_{(c(t), W(t)) \in \mathbb{X}} \int_s^\bar{T} e^{-\rho t} u(c(t)) dt \quad \text{s.t.} \quad (3)) \}. \]  

(5)

\(^{13}\)We model this as a resource cost. Similar results are obtained for the case with an effort (utility) cost, and for both fixed and proportional costs. These additional results are not reported in the paper due to the space limit, but are available upon request from the authors.
To make progress, we need to take on some specific form of the utility function. Properties of the utility function that are essential for our results are that the function is strictly monotone and strictly concave. Thus, without loss of generality, we assume that the consumer’s instantaneous utility function is of the logarithm form. It can be shown with some algebra that the optimal consumption path conditional on an implementation date of \( s \in [0, \bar{T}) \) is characterized by

\[
c(t) = \begin{cases} y(t), & \text{for } t \in [0, s), \\ \frac{\int_{t}^{s} e^{-r(t-s)} y(t) dt - \kappa(s)}{\rho - 1} e^{(r-\rho)(t-s)}, & \text{for } t \in [s, \bar{T}]. \end{cases}
\]

Using the solution in (6), the lifetime utility maximization problem (5) reduces to

\[
V(\kappa) = \max \left\{ \int_{0}^{T} e^{-pt} \ln y(t) dt, \max_{s \in [0, T]} \left[ \int_{0}^{s} e^{-pt} \ln y(t) dt + V(\kappa|s) \right] \right\},
\]

where \( V(\kappa|s) \) denotes the value function on an implementation date \( s \), which we have derived as

\[
V(\kappa|s) = \left\{ \ln \frac{\int_{s}^{T} e^{-r(t-s)} y(t) dt - \kappa(s)}{\rho - 1} + \frac{r - \rho}{\rho} \right\} \frac{e^{-\rho s} - e^{-\rho \bar{T}}}{\rho} - \frac{(r - \rho)(\bar{T} - s)}{\rho} e^{-\rho \bar{T}}.
\]

Inspecting (7) and (8) reveals that whether the consumer would ever implement a saving plan and, if so, the choice of the implementation date, depends on how the benefit from being able to use the plan to reallocate his income more efficiently across time trades against the cost of implementing the plan.

In the special case with costless implementation of a saving plan (i.e., with \( \kappa = 0 \)), as in the standard life-cycle consumption model, it would always be optimal to implement a saving plan at the very beginning of the work life, that is, at date 0.\(^{14}\) This can be shown by

\(^{14}\)This does not exclude the possibility that, in some circumstances (depending on the dynamics of the disposable income \( y(t) \) and the relationship between the rate of return to saving \( r \) and the discount rate \( \rho \)),...
applying Jensen’s inequality to (7) and (8), and using the concavity property of the utility function.

On the other hand, if the implementation cost is too high, the consumer may choose to never implement a saving plan. To be specific, define a series \( \bar{\kappa} = \{ \bar{\kappa}(s)_{s \in [0, \bar{T}]} \} \) by

\[
\bar{\kappa}(s) = \int_s^{\bar{T}} e^{-r(t-s)} y(t) \, dt - \left[ \frac{1 - e^{-\rho(\bar{T}-s)}}{\rho} \right] e^{\rho \int_s^{\bar{T}} e^{-\rho(t-s)} \ln[y(t)] \, dt + (r - \rho)(\bar{T}-s) e^{-\rho(\bar{T}-s)}} + 1 - \frac{r}{\rho}, \tag{9}
\]

for all \( s \in [0, \bar{T}) \). It is clear that this series is determined completely by the fundamental characteristics of the economy. Working with (7) and (8) we can show that, if

\[
\kappa \geq \bar{\kappa}, \tag{10}
\]

then the consumer would never implement a saving plan, but live hand-to-mouth throughout the entire life cycle.

The more interesting case is when the implementation cost is nonzero, but small or moderate. Specifically, if there exists some \( s \in [0, \bar{T}) \) such that

\[
\kappa(s) < \bar{\kappa}(s), \tag{11}
\]

then the consumer would choose to implement a saving plan at some point during his life cycle. If this implementation date differs from 0, then a nontrivial delay in implementing a saving plan would arise.

To summarize our above discussion, let’s denote by \( S \) the collection of all time points that satisfy (11), that is,

\[
S = \{ s \in [0, \bar{T}) : \kappa(s) < \bar{\kappa}(s) \}, \tag{12}
\]

it might be equally optimal to implement a saving plan at a date later than 0, or never. For instance, if \( y(t) \) is flat across the life cycle (which, however, has little empirical relevance, especially given how income usually changes at retirement) and \( r = \rho \), then no saving plan can deliver a higher level of lifetime utility than living hand-to-mouth all the time. In this case, even a tiny cost for implementing a saving plan would make living hand-to-mouth strictly optimal.
If $S = \emptyset$, the consumer would choose to never implement a saving plan, while if $S \neq \emptyset$, he would choose to implement a saving plan at some point during his life cycle and the optimal implementation date would be an element in $S$.

With this progress, the lifetime value function further simplifies to

$$V(\kappa) = \begin{cases} \int_0^T e^{-\rho t} \ln y(t) dt, & \text{if } S = \emptyset, \\ \max_{s \in S} \left[ \int_0^s e^{-\rho t} \ln y(t) dt + V(\kappa|s) \right], & \text{if } S \neq \emptyset, \end{cases}$$

and the optimal date for implementing a saving plan is

$$s^*(\kappa) = \begin{cases} \text{nonexistent}, & \text{if } S = \emptyset, \\ \arg\max_{s \in S} \left[ \int_0^s e^{-\rho t} \ln y(t) dt + V(\kappa|s) \right], & \text{if } S \neq \emptyset. \end{cases}$$

This completes our description of an individual consumer’s problem.

There is a continuum of many such consumers indexed on the unit interval $[0,1]$. These consumers are identical in every respect, except for the costs of implementing a saving plan. A cost profile,

$$\mathcal{K} : [0,1] \to \mathbb{R}_{\geq 0}^T,$$

assigns to each consumer $i \in [0,1]$ a nonnegative-valued function from $[0, \bar{T})$ to $\mathbb{R}$, where we allow $\mathcal{K}(i) \neq \mathcal{K}(j)$, for $i, j \in [0,1]$ and $i \neq j$, to capture such heterogeneity revealed by the empirical evidence discussed in Section 2.3.

4. Quantitative results and some intuition

In this section we solve the model developed in Section 3 and discuss the results in light of the 7 facts documented in Section 2.1. To obtain a solution to the optimization problem for each individual consumer, we rely on the five equations, (8), (9), (12), (13), and (14) derived in Section 3. Recall that these consumers are identical in every respect except for the implementation costs that they face in implementing an optimal consumption-saving plan.

We begin by assigning values to the model’s parameters. We choose $T = 40$ and $\bar{T} = 55$, so all individuals start work at age 25, retire at age 65, and pass away at age 80. We set
the discount rate $\rho$ to 3.5%. We also set the real rate of return $r$ at 3.5%, as in the recent life-cycle literature (e.g., see McGrattan and Prescott 2000, Gourinchas and Parker 2002, Bullard and Feigenbaum 2007, Feigenbaum 2007). We set the two rates equal to ensure that, were the costly saving mechanism shut off, consumption would degenerate into a perfectly flat path across the entire life cycle, so as to isolate the effect of the costly saving mechanism on life-cycle saving and consumption.

We set $\tau(t) = 15.3\%$ (for all $t$) to match the payroll tax in the U.S.–employers and employees each legally pay half of the tax, but following the convention and since labor is supplied inelastically, the workers pay the full tax (e.g., Kotlikoff 1979, Feldstein 1985). The social security system is of the pay-as-you-go variety, so total taxes collected at any date equals total benefits paid out at that date; thus, social security benefits per retiree at time $t$ are $\tau(t)w(t)R(t)$ where $R(t)$ is the ratio of workers to retirees at time $t$. We set $R(t) = 3.3$ (for all $t$) in light of the current demographics in the U.S., and also to reflect the average value of this ratio over the past few decades.\(^{15}\)

We choose $w(0) = \$40,000$ for consistency with the current average annual wage income per worker in the U.S.;\(^{16}\) and, for consistency with the long-run trend growth in real wages, we set $\dot{w}(t)/w(t) = 1\%$.\(^{17}\) To start with, we intentionally abstract from age-based productivity effects over the working period (such as Feigenbaum’s quartic polynomial estimate) in order to distance our theory from other theories which invoke this assumption (some other theories can explain certain facts based on the assumption that household wage income over the work life follows a hump shape). After demonstrating that the costly saving mechanism can explain all the 7 facts under the simple, transparent calibration with exponential wage growth, we show that the results are robust and can even be strengthened when the age-based productivity effects are taken into account.


\(^{16}\)http://www.ssa.gov/OACT/cola/awidevelop.html

\(^{17}\)Bullard and Feigenbaum (2007) and Feigenbaum (2007) postulate a higher wage growth rate of 1.56\%. Our results can be strengthened by assuming a higher rate of growth in wages.
Recent survey evidence and data on fees charged by money managers provide a guide for the calibration of implementation costs. The finding from the 2007 RCS that the average American worker spends 19 hours a year planning for retirement suggests an annual cost of about $500, or 1% of the annual earnings for the average American worker, using the average hourly wage over the working period ($25) as a measure of the value of time and assuming 2,000 hours worked a year. In light of the evidence on heterogeneity in the costs as surveyed in Section 2.3, we assume that individuals draw an implementation cost from a uniform distribution with a zero lower bound and an upper bound equal to 2% of the average annual earnings (thus the mean cost is 1%). In dollar terms, this upper bound corresponds to the $1,000 annual fee for the very basic financial advisory services typically charged by professional financial advisors. This choice of the upper bound ensures that no individual in our model economy will draw an implementation cost that exceeds the cost of paying a financial advisor to do the job. This calibration can be conservative as the $1,000 fee is for advice only, whereas fees for managing portfolios and executing account transactions can be at least 4 times higher, and as other sources of data may suggest a greater cost of time for implementing a saving plan (see Section 2.3).

Table 1 contrasts the 7 life-cycle saving and consumption facts from the data against the predictions of the model. Concerning Fact 1, the data show that more than 50% of the population do not implement a saving plan when first entering the workforce, while the model predicts that 86% of the population procrastinate and only 14% implement immediately. The marginal individual who is indifferent between procrastination and immediate saving has an implementation cost equal to 0.28% of annual earnings (or $140). This cost is just large

\footnote{We assume a one-time implementation cost to keep the model parsimonious, which also allows to bypass a reversibility issue that would otherwise be encountered in a period-cost case. In an unreported study, we examine a model in which costs for managing a plan may be incurred in periods after it is put in place and an individual can jump on and off as many plans as he wants over the course of the life cycle. We find that, under some mild conditions, the results under this alternative modeling approach are the same as those under the current one. Hence, our calibration of the costs will ensure consistency between the two settings.}

\footnote{https://personal.vanguard.com/VGApp/hnw/accounttypes/advice/ATSArticle/AMS FeesContent.jsp}
enough to cancel the benefit from smoothed consumption over the life cycle that could be achieved through an optimal saving plan. The 14% of the population who choose to save immediately upon entering the workforce all have costs lower than 0.28% of annual earnings and these costs are more than compensated by the benefits from smoothed consumption over the life cycle. The 86% of the population who choose to procrastinate all have costs higher than 0.28% of annual earnings and these costs more than offset the potential benefits from smoothed consumption over the life cycle.

Among this latter group of individuals who procrastinate, some actually choose to never implement a saving plan. These are the ones who face implementation costs greater than 1.66% of annual earnings (or $830). The costs are so high that they always exceed the potential benefits from smoothed consumption regardless of when to implement a saving plan. These people constitute 17% of the population, and they live hand-to-mouth during the working period and then rely exclusively on the social security benefits during the retirement phase. This number comes close to accounting for Fact 2 where the data show that about 25% of the population live their retirement lives without any private savings.

Concerning Fact 3, the data show that there is heterogeneity in the level of accumulated savings at retirement holding lifetime earnings constant. This is also the case in the model. Whereas the 17% of the population who never save at all never accumulate any wealth, the remaining 83% of the population choose to implement saving plans at various points in time during the work life and accumulate various levels of wealth at retirement. Moreover, the data show that there is little variation in the preretirement consumption growth across the different wealth quartiles at retirement. The model also accounts for this feature of the data well. The largest difference in the average consumption growth rate over the 6 years leading up to retirement across the four equal-size quartiles based on the levels of accumulated savings at retirement is 1.4 percentage points in the data, while it is 0.7 percentage points in the model. Since the saving plans are generally implemented before the age of 59, even the highest and the lowest wealth quartiles, let alone the two middle ones, consist of a large portion of these savers who all share the same consumption growth rate in the 6 years prior
to retirement. This is why the model produces the heterogeneity in accumulated savings at retirement but little variation in the preretirement consumption growth rate across the wealth quartiles.

As Table 1 demonstrates, the model can account for not only the facts on saving, but also the facts on consumption. Concerning Fact 4, the data show that aggregate consumption displays a hump during the work life, with the size of the peak consumption generally above 1.1, and with the location of the peak usually between ages 45 and 55. In the model, the work-life aggregate consumption profile is indeed hump-shaped, with the size of the peak consumption equal to 1.26, and with the peak situated at age 53, both in line with the data (see also Figure 1). The consumption hump produced in the model is the consequence of aggregation over the individuals’ heterogeneous consumptions profiles. The 14% of the population who save immediately upon entering the workforce have a flat consumption profile across the life cycle. The 17% of the population who never save at all have their consumption expenditures follow one-for-one with the exponentially growing disposable wages during the working period, and then with the social security benefits in the retirement phase. The remaining 69% of the population choose to implement saving plans at different points in time during the work life. Before putting in place the saving plans, these individuals live hand-to-mouth, consuming the monotone rising disposable wages. On the dates the saving plans are adopted, these individuals scale back on their consumption expenditures as they begin to save for retirement. The sizes of the cutbacks in consumption expenditures are directly related to the implementation dates: the longer the individuals wait to implement the plans, the more aggressively they must save in order to provide for their retirement needs. This together with the variation in the implementation dates across these individuals work to produce a smooth hump in aggregate consumption over the work life.

The data also show that aggregate consumption displays a discrete drop of about 15% at the date of retirement (Fact 5) and that the drop is anticipated (Fact 6). In the model, the 83% of the population with implementation costs below 1.66% of annual earnings (or $830) all choose to implement saving plans at various dates during the work life, and thus their
consumption expenditures do not experience any drop at retirement. In contrast, the 17% of the population with implementation costs above 1.66% of annual earnings choose to never save, and thus at the date of retirement they must adjust their consumption down to a level not exceeding the social security benefits. This gives rise to a discrete drop in aggregate consumption of 8%, which accounts for more than 50% of the drop seen in the data (see also Figure 1). The drop is clearly anticipated in the model, as in the data, as these non-savers rationally choose to never save because of the high implementation costs.

Finally, concerning Fact 7, the data show that consumption tracks income over the entire working period for a group of people who have saved little or nothing at retirement and that these people account for 20% of the population. The prediction of the model conforms to this feature of the data. In the model, individuals with implementation costs above 1.66% of annual earnings (or $830) choose to never save at all, and thus they consume all of their disposable income during the work life and arrive at the retirement date without any private savings. These individuals displaying consumption-income tracking over the entire working period constitute 17% of the population, much in line with the data.

We have thus far abstracted from age-based productivity effects over the working period so as to isolate the role of the costly saving mechanism in shaping the 7 features of the data. As we can see from the results reported above, this mechanism by itself already does a fairly good job in accounting for all of the 7 facts on life-cycle saving and consumption.

In what follows we take into account age-dependent productivity effects by incorporating Feigenbaum’s (2007) quartic polynomial estimate of real wages over the working period. To fix from the above calibration the average productivity and implementation costs relative to the average earnings over the work life, we fine tune the initial wage from $w(0) = 40,000 in the above case to $w(0) = 39,157 for the current one while keeping unchanged the rest.

The magnitude of the drop in consumption at the date of retirement $T$ for each of these non-savers is

$$(1 - \tau)w(T) - \tau Rw(T) = 0.342 \ w(T),$$

where we have used the calibrated values $\tau = 15.3\%$ and $R = 3.3$ in deriving the above size.
of the calibration. We find that the model with this hump-shaped wage profile incorporated continues to fit the data well.

Table 2 reports the results for this case against the 7 facts from the data. The fit to Fact 1 is virtually the same as before: the fraction of the population who decide to procrastinate edges up only marginally, from 86% in the previous case to 88% in the current one, both of which are in line with the data. This similarity arises because the threshold cost that makes a marginal individual indifferent between procrastinating and implementing a saving plan immediately after entering the workforce edges down only slightly, from 0.28% of annual earnings before to 0.24% of annual earnings now (or from $140 before to $120 now).

Concerning Fact 2, the fraction of the population who choose to never save is much greater in this case (65%) than in the previous one (17%). This difference arises because the hand-to-mouth behavior over the life cycle gives rise to a relatively flatter consumption path when the wage profile is hump-shaped than when it is exponential. In consequence, the minimal cost required to prevent an individual from ever implementing a saving plan is much smaller, being 0.7% of annual earnings now compared to 1.66% of annual earnings before (or $350 now versus $830 before), resulting in a much larger fraction of the population who live their retirement lives without any private savings. While the previous case undershoots on this feature of the data (as Table 2 shows, this share of the population is 25% in the data), the current case clearly overshoots on it.

As for Fact 3, the model continues to produce heterogeneity in accumulated savings at retirement, as seen from the data. Whereas the 65% of the population who never save at all never accumulate any wealth, the remaining 35% of the population choose to implement saving plans at various dates over the working period and accumulate various levels of wealth at retirement. Because the fraction of the non-savers is much higher in this case than in the previous one, the lowest wealth quartile is now composed entirely with hand-to-mouth individuals whose consumption during the working period all changes at the rate at which the wage changes. On the other hand, the individuals in the highest wealth quartile implement saving plans before the age of 59, and thus they all have a zero consumption growth rate.
in the 6 years leading up to retirement. As a consequence, the largest difference across the wealth quartiles at retirement in the average consumption growth rate over the 6 years prior to retirement increases, from 0.7 percentage points before to 1.8 percentage points now, which is only slightly above what is seen in the data (1.4 percentage points).

In addition to the facts on saving, the model continues to fit the facts on consumption. A hump continues to emerge in the aggregate consumption profile over the work life, with the amplitude of the consumption peak slightly larger (1.30) than before (1.26), and with the peak situated at a slightly earlier age (44) than previously (53), both of which are still in line with the data (see also Figure 2).21 While continuing to account for Fact 4, the model now delivers a discrete drop in aggregate consumption at retirement with a size (13%) much closer to what is seen in the data (15%) than before (8%)–see also Figure 2. This improvement in the model’s ability in fitting Fact 5 comes from the greater fraction of non-savers in this case than in the previous one, as it is these non-savers that experience a discrete drop in consumption at retirement. Although the magnitude of the drop for each of the non-savers is now smaller (as the wage slips to the lowest level at retirement along the back end of the hump-shaped profile) than before (as the wage climbs to the highest level at retirement along the monotone increasing profile), the aggregate size of the drop is greater as a result of the larger share of the non-savers. The drop is of course still anticipated (Fact 6). Finally, concerning Fact 7, the model continues to predict consumption-income tracking for non-savers, as seen in the data. Because of the larger fraction of non-savers than before, the fraction of the population who display consumption-income tracking increases, from 17% before to 65% now, which is considerably larger than what is seen in the data (20%).

As a further robustness check under this hump-shaped wage profile, we also examine a case with smaller implementation costs. Specifically, we reduce the upper bound of the cost support from 2% of wage compensation down to 1%, while preserving the zero lower bound, as well as the rest of the calibration.

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21 While most studies report the location of the peak consumption between ages 45 and 55, one prominent study puts the location at a slightly younger age, at about 44 (e.g., Gourinchas and Parker 2002).
Again, the model continues to fit well all of the 7 facts, as we can see from Table 3. The threshold cost that makes a marginal individual indifferent between procrastinating and implementing a saving plan right after entering the workforce is the same as in the previous case, equal to 0.24% of annual earnings. But, with the upper bound of the cost support cut by half, the amount of individuals facing costs lower than this threshold value doubles, and thus the fraction of the population who implement immediately upon entering the workforce increases, from 12% in the previous case to 24% in the current one. Nevertheless, the fraction of the population who procrastinate is still above 50%, at 76%, much in line with the data.

The minimal cost required to prevent an individual from ever implementing a saving plan also remains the same as in the previous case, equal to 0.7% of annual earnings. Yet, the compression of the cost support toward the zero lower bound reduces the percentage of people having costs above this critical value, from 65% before to 30% now. As a consequence, the fraction of the population who never save goes down, from 65% in the previous case to 30% in the current one, and so does the fraction of the population who display consumption-income tracking over the working period. This improves the fit to Facts 2 and 7 that exhibit 25% non-savers and 20% consumption-income trackers, respectively.

Concerning Fact 3, the model continues to produce heterogeneity in accumulated savings at retirement, as implementation still may occur immediately (24%), with a delay (46%), or never at all (30%). Given that the lowest wealth quartile is still composed entirely with hand-to-mouth individuals whose consumption during the working period all changes at the rate at which the wage changes, and still the individuals in the highest wealth quartile implement saving plans before the age of 59 and thus all have a zero consumption growth rate in the 6 years leading up to retirement, the largest difference in the average consumption growth rate over the 6 years prior to retirement across wealth quartiles at retirement remains at 1.8 percentage points, which is broadly consistent with the data (1.4 percentage points).

As for Fact 4, aggregate consumption still displays a hump over the working period, with the size of the peak consumption (1.26) and the location of the peak (age 43) continuing to be in line with the data (see also Figure 3). Concerning Fact 5, the model continues to
produce a discrete drop in aggregate consumption at retirement (again, see also Figure 3). The size of the drop is reduced, from 13% in the previous case to 6% in the current one, due to the reduction in the share of hand-to-mouth individuals (from 65% before to 30% now). Nevertheless, it still accounts for nearly half of the size that is seen from the data. And, again, the drop is anticipated (Fact 6).

In sum, our costly saving model is quite successful in helping resolve the 7 puzzles on macroeconomic consumption and saving over the life cycle. This success stands regardless of whether age-based productivity effects are abstracted. It is important to note that, although taking into account these effects may help improve the fit of our model to some features of the data, the costs for properly implementing a saving plan are essential for the model to account for every single one of the facts. If we shut off the costs, the predictions of the model would be completely at odds with each one of the facts, regardless of the shape of the wage profile. By contrast, in the presence of the costs as suggested by the empirical evidence, the model can account for all of these facts successfully. Therefore, the implementation costs are an indispensable feature of the model that helps resolve these macroeconomic consumption and saving puzzles in a simple and unified way.

5. Concluding Remarks

We have documented 7 puzzles concerning macroeconomic saving and consumption over the life cycle and we have proposed a simple model with one small twist to resolve them. This twist is motivated by an empirically relevant feature that it may cost time, effort, and resources to properly implement a consumption-saving plan, though such cost may differ across individuals. Whereas the literature has already identified a number of mechanisms that may help explain one or another of these puzzles, it has proven to be difficult to jointly

\footnote{Note that this size (6%) is even slightly smaller than the one under the exponential wage profile (8%), although the share of hand-to-mouth individuals is bigger here (30%) than there (17%). This is so since the magnitude of the drop for each of the hand-to-mouth individuals is smaller here (as the wage slips to the lowest level at retirement along the back end of the hump-shaped profile) than there (as the wage climbs to the highest level at retirement along the monotone increasing profile), and this effect dominates.}
rationalize even a small subset of these facts within the existing theories. In contrast, the costly saving mechanism is the sole and common friction in the current model for rationalizing the entire set of these 7 facts. The implementation costs in the model are small, yet they help resolve these consumption and saving puzzles in a simple and unified way.
References


Hogarth, Jeanne M., Chris E. Anguelov, and Jinkook Lee (2003), Why Households Don’t Have Checking Accounts. Economic Development Quarterly 17(1), 75-94.


Table 1. Data versus Model:
The Case with Exponential Growth in Wages

<table>
<thead>
<tr>
<th>(Fact 1) % Who do not save immediately</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;50%</td>
<td></td>
<td>86%</td>
</tr>
<tr>
<td>(Fact 2) % Who never save at all</td>
<td>~25%</td>
<td>17%</td>
</tr>
<tr>
<td>(Fact 3) Heterogeneity in wealth on the date of retirement (holding income fixed)?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Maximal quartile variation in the rate of preretirement consumption growth*</td>
<td>1.4 points</td>
<td>0.7 points</td>
</tr>
<tr>
<td>(Fact 4) Consumption hump:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size of consumption peak**</td>
<td>&gt;1.1</td>
<td>1.26</td>
</tr>
<tr>
<td>Age when consumption peaks</td>
<td>45-55</td>
<td>53</td>
</tr>
<tr>
<td>(Fact 5) Discrete drop at retirement</td>
<td>~15%</td>
<td>8%</td>
</tr>
<tr>
<td>(Fact 6) Is the drop Anticipated?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>(Fact 7) Consumption-income tracking:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Who display tracking?</td>
<td>Low/Non savers</td>
<td>Non savers</td>
</tr>
<tr>
<td>% Who display tracking</td>
<td>20%</td>
<td>17%</td>
</tr>
</tbody>
</table>

*Consumption growth rates are for the six years immediately before retirement.
Quartiles are defined by wealth accumulation at retirement, and “maximal quartile variation” is defined as the largest difference in preretirement consumption growth rates across the wealth quartiles.

**Peak size is defined as the ratio of peak consumption to consumption when entering the workforce.

Note: Implementation costs are assigned to individual consumers from the uniform distribution with a zero lower bound and an upper bound of 2 percent of annual wages.
Table 2. Data versus Model:
Wages set to Feigenbaum’s Quartic Polynomial

<table>
<thead>
<tr>
<th>(Fact 1) % Who do not save immediately</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Fact 2) % Who never save at all</td>
<td>~25%</td>
<td>65%</td>
</tr>
<tr>
<td>(Fact 3) Heterogeneity in wealth on the date of retirement (holding income fixed)?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
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<td>Maximal quartile variation in the rate of preretirement consumption growth*</td>
<td>1.4 points</td>
<td>1.8 points</td>
</tr>
<tr>
<td>(Fact 4) Consumption hump:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size of consumption peak**</td>
<td>&gt;1.1</td>
<td>1.30</td>
</tr>
<tr>
<td>Age when consumption peaks</td>
<td>45-55</td>
<td>44</td>
</tr>
<tr>
<td>(Fact 5) Discrete drop at retirement</td>
<td>~15%</td>
<td>13%</td>
</tr>
<tr>
<td>(Fact 6) Is the drop Anticipated?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>(Fact 7) Consumption-income tracking:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Who display tracking?</td>
<td>Low/Non savers</td>
<td>Non savers</td>
</tr>
<tr>
<td>% Who display tracking</td>
<td>20%</td>
<td>65%</td>
</tr>
</tbody>
</table>

*Consumption growth rates are for the six years immediately before retirement.
Quartiles are defined by wealth accumulation at retirement, and “maximal quartile variation” is defined as the largest difference in preretirement consumption growth rates across the wealth quartiles.

**Peak size is defined as the ratio of peak consumption to consumption when entering the workforce.

Note: Implementation costs are assigned to individual consumers from the uniform distribution with a zero lower bound and an upper bound of 2 percent of annual wages.
Table 3. Data versus Model:

Wages set to Feigenbaum’s Quartic Polynomial, and Costs set to Low Values

<table>
<thead>
<tr>
<th>Fact</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Fact 1) % Who do not save immediately</td>
<td>&gt;50%</td>
<td>76%</td>
</tr>
<tr>
<td>(Fact 2) % Who never save at all</td>
<td>~25%</td>
<td>30%</td>
</tr>
<tr>
<td>(Fact 3) Heterogeneity in wealth on the date of retirement (holding income fixed)?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
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<td>1.8 points</td>
</tr>
<tr>
<td>(Fact 4) Consumption hump:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size of consumption peak**</td>
<td>&gt;1.1</td>
<td>1.26</td>
</tr>
<tr>
<td>Age when consumption peaks</td>
<td>45-55</td>
<td>43</td>
</tr>
<tr>
<td>(Fact 5) Discrete drop at retirement</td>
<td>~15%</td>
<td>6%</td>
</tr>
<tr>
<td>(Fact 6) Is the drop Anticipated?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>(Fact 7) Consumption-income tracking:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Who display tracking?</td>
<td>Low/Non savers</td>
<td>Non savers</td>
</tr>
<tr>
<td>% Who display tracking</td>
<td>20%</td>
<td>30%</td>
</tr>
</tbody>
</table>

*Consumption growth rates are for the six years immediately before retirement.

Quartiles are defined by wealth accumulation at retirement, and “maximal quartile variation” is defined as the largest difference in preretirement consumption growth rates across the wealth quartiles.

**Peak size is defined as the ratio of peak consumption to consumption when entering the workforce.

Note: Implementation costs are assigned to individual consumers from the uniform distribution with a zero lower bound and an upper bound of 1 percent of annual wages.
Figure 1. Aggregate consumption over the life cycle with the initial level normalized to 1. Implementation costs are assigned to individual consumers from the uniform distribution with a zero lower bound and an upper bound of 2 percent of annual wages.
Figure 2. Aggregate consumption over the life cycle with the initial level normalized to 1. Implementation costs are assigned to individual consumers from the uniform distribution with a zero lower bound and an upper bound of 2 percent of annual wages.
Figure 3. Aggregate consumption over the life cycle with the initial level normalized to 1. Implementation costs are assigned to individual consumers from the uniform distribution with a zero lower bound and an upper bound of 1 percent of annual wages.