THE PITFALLS OF A PARTIALLY HONEST BUREAUCRACY: Bribery, Inefficiency, and Bureaucratic Delay

by

Chris Ahlin and Pinaki Bose

Working Paper No. 02-W24

December 2002

DEPARTMENT OF ECONOMICS
VANDERBILT UNIVERSITY
NASHVILLE, TN 37235

www.vanderbilt.edu/econ
THE PITFALLS OF A PARTIALLY HONEST BUREAUCRACY: BRIBERY, INEFFICIENCY, AND BUREAUCRATIC DELAY*

Christian Ahlin and Pinaki Bose

December 2002

Abstract

Bribery, it has been argued, allocates resources efficiently. We show that this conclusion need not hold in a dynamic extension of a simple static model in which it does. When permits are awarded over time and applicants can reapply, a partially honest bureaucracy results in inefficiency. This can take the form of both misallocation and bureaucratic delay, both of which are strategic maneuvers by dishonest bureaucrats to increase bribery income. Efficiency is a non-monotonic function of the fraction of bureaucrats that are honest. Consequently, small differences in monitoring costs may lead to very different optimal levels of corruption.

*Department of Economics, Vanderbilt University, and Department of Economics, University of Memphis, respectively. We are grateful for helpful comments to Kaushik Basu, James Foster, John Siegfried, Quan Wen, and seminar participants at Vanderbilt and at NEUDC 2002. Email: c.ahlin@Vanderbilt.edu, psbose@Memphis.edu.
1 Introduction

Corruption and delays in the performance of official tasks are commonly taken to be the two most prevalent vices of bureaucracy in developing economies. In such economies, business ventures may require numerous official sanctions and permits to operate (see De Soto [1989], Shleifer [1997], and Berkowitz and Li [2000], among many others, for examples). Government officials may also control the distribution of scarce resources, often at subsidized prices. Bureaucratic decisions, and the timeliness of such decisions, can, therefore, influence the profitability of many productive activities.

Consider the time-honored example (also modeled in this paper) of bureaucrats granting licenses that are essential for some surplus generating activity. If bureaucrats are not sufficiently monitored, their discretionary power in awarding licenses gives them the opportunity to extract rents by charging bribes. In this case, potential bribe income, rather than the official criteria for obtaining a license (merit or otherwise), decides the allocation of licenses, and inefficiency may result. This inefficiency may take the form of licenses being granted to the wrong recipients, or needless delay for the right recipients in obtaining the license. In particular, bureaucrats may create red tape and unnecessary delays if such tactics positively affect the magnitude of their bribes. Thus the possibility arises that bribery and bureaucratic delays may be strategically interrelated.

It has been argued, however, that allocation of goods or services by bribery contains certain efficiency properties. In the context of obtaining a contract or production license, for example, the most productive applicant is expected to be willing to pay the highest bribe. In the context of obtaining a bureaucratic service, the one with the highest value on time will be willing to pay most to move ahead in the queue. In these examples, allowing bribery guarantees efficient allocation, across people and across time.\footnote{For more sophisticated arguments along these lines, see Lien [1990] and Lui [1985]. (Clark and Riis [2000], however, show some fragility in the results of the former model.) Of course, in the context of credit market imperfections and wealth inequality, these efficiency properties may not hold either, an issue we examine in section 5.}

These arguments on the efficiency of bribery are commonly made in a static setting. In contrast, we examine a very simple dynamic model, in which the distribution of licenses takes place over two periods. In the baseline model, all applicants have unlimited ability, but not willingness, to pay bribes. Applicants who do not receive a license in the first period are allowed to re-apply in the second period to a new bureaucrat. We find that in this setting, the efficiency of bribery is far from a foregone conclusion, even though in the one-period version of our model it is. An applicant who is both more productive and who also has a higher waiting cost, may not secure a license in the first period. The reason is that once the ability to reapply in a subsequent period is introduced, productive candidates need not be the ones willing to pay most for the license; they may assess higher chances of a profitable reapplication.

Surprisingly, we find that social efficiency is a non-monotonic function of the fraction of bureaucrats who are opportunistic, i.e. maximize their returns from bribery. In particular, if all bureaucrats are opportunistic, or if all are honest, perfect efficiency results. In the former case, reapplication is pointless for all candidates, since there is no chance of finding an honest bureaucrat; hence, the productive candidates are willing to pay the highest bribes. In the latter case, all bureaucrats, by assumption, perform the duty of awarding licenses to the most deserving applicants. When there is a mix of honest and opportunistic bureaucrats, however, inefficiency can result.

Paradoxically, then, an increase in the fraction of honest bureaucrats can actually lower efficiency. It should be stressed that these results are different from the arguments of Leff [1964] and Huntington [1968], among others, who have argued that honest officials enforcing absurd regulations can lead to inefficiency. In our model, the honest officials award their licenses to the most productive applicants in a timely manner, by assumption.

The equilibrium inefficiency in our model can take the form of awarding licenses to the less productive candidates; it can also take the form of delaying licenses to the productive candidates. Thus we rationalize the equilibrium occurrence of inefficient bureaucratic delays.
We are not the first to do so. In Banerjee [1997] red tape (including bureaucratic delay) is an efficient mechanism designed to separate out the more productive applicants when the fact that potential applicants are credit-constrained makes monetary mechanisms, such as bribery, unable to do so. In our approach, bureaucrats have the ability to recognize the more productive applicants, so screening is not an issue. Delay arises because productive candidates may have increasing willingness to pay bribes over time as their re-application options are exhausted. This makes bureaucratic delay a potentially strategic choice that leads to higher bribe incomes for bureaucrats but contributes to overall inefficiency.

One surprising implication of the model is that inefficiency results from the availability of greater opportunities to entrepreneurs to choose among bureaucrats, and in particular, to leave a bureaucrat who has attempted to charge a bribe in search of an honest one. Elsewhere in the literature ability to choose is typically regarded as enhancing efficiency (see Shleifer and Vishny [1993] and Rose-Ackerman [1978], for example). In contrast, in the present paper, perfect efficiency results if entrepreneurs have only one choice.

Optimal monitoring for a principal that only cares about efficiency would be trivial in this setting, since complete dishonesty is perfectly efficient. However, if applicants differ not only in willingness to pay but also in ability to pay bribes, total surplus may remain non-monotonic but be uniquely maximized under a perfectly honest bureaucracy. This would lead to a sort of bimodality of optimal monitoring of bureaucracies, with small differences in monitoring costs leading to very different optimal corruption levels.

The basic model is outlined in section 2. The equilibrium is characterized and proved unique in section 3. Efficiency as a function of the level of honesty in the bureaucracy is solved for in section 4. We analyze a case of limited ability to pay bribes in section 5 and conclude in section 6.
2 The Model

There are $N$ 'officials' or 'bureaucrats’, each endowed with $L$ 'licenses' or 'permits’. These officials are delegated the power to grant the licenses to deserving applicants. Once granted, the permit enables a person possessing entrepreneurial skills to engage in some productive venture. There are $M$ potential 'applicants’, heterogeneous in terms of their productivity (or entrepreneurial ability). Specifically, we assume that an applicant, if granted a license, can generate a net present value $V_1$ from her venture if she is of the 'productive’ type (type one), and $V_2 \in (0, V_1)$ if she is of the 'unproductive’ type (type two). A fraction $\mu \in (0, 1)$ of the total population of applicants $M$, are productive. Applicants apply costlessly for licenses, and maximize expected net present value.

Officials are charged with the task of giving all their licenses to productive applicants, in as timely a manner as possible. A fraction $\nu \in [0, 1]$ of them are 'honest' and do just that. The rest, are 'dishonest’ or 'corrupt’, and thus maximize personal gain from bribery.

An official to whom an application is submitted can identify the type of the applicant costlessly. However, the process of awarding licenses is otherwise time-consuming (due to formalities and red-tape), so much so that any official can award at most $T$ licenses in a single period. We assume that all bureaucrats are given exactly two periods to complete their task of distributing the permits and that $T \in (L/2, L)$.\footnote{All results of this model do not appear to depend on this finite, two-period time horizon, but are robust to an infinite time horizon under the assumption $T \in (L/2, L)$. This is because the honest officials will finish license distribution in two periods, and the dishonest ones will therefore have no incentive to delay beyond two periods.} In words, one period is insufficient ($T < L$), but two periods are more than sufficient ($2T > L$), to award all licenses. Since two periods are more than enough, an official who gives away all $L$ licenses has some discretion over the timing of his license distribution. Applicants and bureaucrats discount payoffs in period 2 at rate $\delta \in (0, 1)$.

An official's type, honest or dishonest, is private information. An applicant can approach at most one official per period, whom she chooses at random. For simplicity, we assume that
a candidate who reapplies in the second period randomly chooses from all officials.\(^3\) We also assume that the act of soliciting a bribe is not observed by a third party. The result of these assumptions is that candidates do not know whether any given official is honest or corrupt, neither in the first nor second period; they only know the fraction of honest officials, \(\nu.\)\(^4\)

Applicants thus select randomly from among all officials in each period. Thinking of \(M\) as large, we assume that the same number of each type of applicants approaches each official in each period. Since the process of application is costless, all \(M\) applicants decide to attempt acquiring licenses in period 1. Therefore, each official receives \(M/N\) applications of which \(\mu M/N\) are of the productive type. We assume that

\[ \mu M/N \geq 2T, \quad (A1) \]

and that

\[ (1 - \mu) M/N \geq T. \quad (A2) \]

Assumptions A1 and A2 are made for simplicity because they eliminate key applicant-supply constraints from the bureaucrat’s decision. Assumption A1 ensures that at least \(T\) productive applicants approach every bureaucrat in each period, no matter what other bureaucrats do: at least \(2T\) apply in period 1, and even if all bureaucrats service them, there are at least \(T\) per bureaucrat remaining to apply in period 2.\(^5\) Similarly, assumption A2 guarantees at

\(^3\)In other words, the candidate samples with replacement. One might think of a bureaucracy with many clerks, with applicants randomly assigned to one of them each visit. The more prevalent case of sampling without replacement complicates the analysis without changing results, given that \(N\) is thought of as large.

\(^4\)We do not explore communication among candidates about first-period application experiences. It is not clear that communication would be effective, since it would be cheap talk and productive candidates have an incentive to divert other productive candidates to the dishonest officials, to alleviate excess demand for honest bureaucrats’ licenses. Further, it is realistic to rule out communication when entrepreneurial candidates make up a small percentage of the total population, and thus potentially never meet each other in daily interaction.

\(^5\)This assumption implies that there are more productive applicants than there are licenses, since \(2T > L\). Sufficient for our qualitative results is \(\mu M/N \geq L\), but we make this stronger assumption for ease of exposition.
least $T$ unproductive applicants apply to each bureaucrat in period 1. Since only $T$ licenses can be awarded in any period, lack of applicants of type 1, or of type 2 in period 1, will never create a binding constraint.

Under assumption A1, it is clear that an honest official will award all $T$ licenses in period 1, and $1 - T$ licenses in period 2, to productive applicants. The action of the dishonest official, however, is determined so as to maximize total payoffs he gets from take-it-or-leave-it offers to the applicants. He may find it profitable to award some number of permits less than $T$ in period 1, or award permits to less productive applicants, or both, depending on the willingness to pay of the various candidates in each period. Thus, both bureaucratic delay and allocative inefficiencies may arise from the strategic behavior of corrupt officials in our model. Let $G_{ij}^t$ denote the number of licenses given to type $i$ applicants by dishonest bureaucrat $j$ in period $t$, for $i, t \in \{1, 2\}$ and $j \in \mathcal{D}$, where $\mathcal{D}$ is the set of all dishonest bureaucrats.

For convenience of exposition, we define

$$1 + \kappa \equiv \frac{\mu M}{N L},$$

(1)

$$\tau \equiv \frac{T}{L},$$

(2)

$$\alpha \equiv \frac{V_2}{V_1},$$

(3)

and

$$g_{ij}^t \equiv \frac{G_{ij}^t}{L}.$$  

(4)

Since $T \in (L/2, L)$ we have that $\tau \in (1/2, 1)$. Assumption A1 implies that $1 + \kappa \geq 2\tau > 1$. 

7
Assumption A2 implies that $(1 + \kappa)(1 - \mu)/\mu \geq \tau$. Finally, $V_2 \in (0, V_1)$ implies that $\alpha \in (0, 1)$.

3 Equilibrium in the Two-Period Game

An equilibrium in this context involves honest officials awarding licenses to productive applicants in as timely a manner as possible; dishonest officials maximizing bribery revenue, given the actions of all other officials and applicants; and applicants maximizing expected revenue given the actions of officials and other applicants. We further restrict attention to subgame perfect equilibria.

We note here that in a static version of this model (or equivalently, a dynamic version with $\tau = 1$), perfect efficiency results. Both honest and dishonest officials would give all licenses to productive candidates without delay, the latter because productive candidates are willing to pay the most for the licenses. Alternatively, in a "no-competition" version of this model, in which each applicant has only one bureaucrat authorized to service her, perfect efficiency also results. The reasoning again is that productive candidates are willing to pay the most, since they have no outside options; and a dishonest bureaucrat will not needlessly delay because it only delays his own income.

In this model, however, the option of searching for an honest bureaucrat in the second period has value to a productive applicant and can thus reduce her willingness to pay below that of an unproductive applicant.

The behavior of an honest bureaucrat is straightforward: he gives a fraction $\tau$ of his licenses in period 1, and $1 - \tau$ in period 2, to productive applicants.\footnote{We have assumed that honest bureaucrats act efficiently with their own licenses. This strategy turns out to be optimal for a bureaucrat aiming to maximize efficiency of all license distribution, not just his own. One may imagine a case such an honest bureaucrat might sacrifice some efficiency in his own action if this gave other officials greater incentives toward the efficient action. This could never happen in our model, in part because the honest bureaucrat cannot credibly commit to acting dishonest in period 2.} The behavior of a dishonest bureaucrat in period 2 is also straightforward. Since this is the final date, a take-it-or-leave-it offer of a permit will be accepted as long as the price to a candidate of type $i$ is no greater than $V_i$. Thus the official can win $V_1$ from a productive candidate and $\alpha V_1 = V_2$
from an unproductive candidate in period 2. Turning to period 1 bribes, first note that an unproductive candidate will always get a payoff of zero in period 2. If she meets an honest official, she gets no license, since he gives all of his to productive candidates; if she meets a dishonest official, she loses all her surplus in the form of a bribe. Thus the waiting option of an unproductive candidate in period 1 is worth zero. It follows that even in period 1, a dishonest official will be able to extract the full amount \( \alpha V_1 \) from an unproductive candidate.

How much will a productive candidate be willing to pay for a license in period 1? If she reapplies for a license in the following period, she gets a positive payoff only if she meets an honest official, which happens with probability \( \nu \). Even if this occurs, the honest official may not have enough licenses for her. If \( \phi \) denotes her probability of being granted a license conditional on finding an honest official, then the expected payoff of waiting is \( \delta \nu \phi V_1 \).

We next derive \( \phi \). Note that it must be in \( (0,1) \), since in period 2 an honest official has \( L - T = L(1 - \tau) > 0 \) licenses remaining, while there are at least \( T = L \tau \) productive applicants. Specifically, let \( P_2 \) denote the total number of productive applicants who are without permits at the beginning of period 2. Then

\[
P_2 = \mu M - [\nu NT + \sum_{j \in D} G_{1j}] = NL[1 + \kappa - \nu \tau - \sum_{j \in D} \frac{g_{1j}}{N}],
\]

(5)

where the first expression subtracts from the total number of productive applicants \( \mu M \) the number serviced by honest bureaucrats \( \nu NT \) and by dishonest bureaucrats; and the second expression just rewrites the first using equations 1-4. Let \( p_2 \) denote the number of type 1 applicants in period 2 per official. Further, let

\[
\overline{g}_1 = \sum_{j \in D} \frac{g_{1j}}{[(1 - \nu)N]}
\]

(6)

be the average fraction of their licenses dishonest officials give to productive candidates in
period 1.\textsuperscript{7} Then \( p_2 \) can be expressed, using equations 5 and 6, as

\[
p_2 = \frac{P_2}{N} = L[1 + \kappa - \nu \tau - (1 - \nu)\overline{g}_1] = L[1 + \kappa - \tau + (1 - \nu)(\tau - \overline{g}_1)].
\]

In period 2, each honest bureaucrat has \( L(1 - \tau) \) licenses and \( p_2 \) productive applicants. Thus

\[
\phi(\overline{g}_1) = \frac{L(1 - \tau)}{p_2} = \frac{1 - \tau}{1 + \kappa - \tau + (1 - \nu)(\tau - \overline{g}_1)}.
\]

Note that as long as \( \nu < 1 \), \( \phi(\overline{g}_1) \) is strictly increasing in \( \overline{g}_1 \).

Given that the value to a productive candidate of waiting for period 2 is \( \delta \nu \phi(\overline{g}_1) \)\( V_1 \), the maximum bribe that can be extracted from her in period 1 is \( V_1 - \delta \nu \phi(\overline{g}_1) \)\( V_1 \). We define \( \gamma(\overline{g}_1) V_1 \) as

\[
\gamma(\overline{g}_1) = 1 - \delta \nu \phi(\overline{g}_1),
\]

so that the maximum bribe can also be written as \( \gamma(\overline{g}_1) \)\( V_1 \). Since \( \phi(\overline{g}_1) \) is strictly increasing in \( \overline{g}_1 \) (as long as \( \nu < 1 \)), \( \gamma(\overline{g}_1) \) is strictly decreasing in \( \overline{g}_1 \) for \( \nu \in (0, 1) \). The intuition is that the greater the number of licenses going to productive candidates in period 1, the smaller the number of productive candidates that honest bureaucrats will be unable to service in period 2; this raises productive candidates’ waiting option value and lowers their willingness to pay in period 1.

In summary, a dishonest bureaucrat in period 1 can earn \( \gamma(\overline{g}_1) \)\( V_1 \) from a productive candidate and \( \alpha V_1 = V_2 \) from an unproductive candidate. In period 2 he can earn \( \delta V_1 \) from a productive candidate and \( \delta \alpha V_1 \) from an unproductive candidate. Using equation 4, the dishonest bureaucrat’s payoff, call it \( \Pi \), can be written

\[
\Pi = LV_1[\gamma(\overline{g}_1) g_{ij}^1 + \alpha g_{2ij} + \delta g_{ij}^2 + \alpha \delta g_{2ij}^2],
\]

\textsuperscript{7}It is the sum of fractions of licenses granted by dishonest officials to productive candidates in period 1, divided by the total number of dishonest officials, \((1 - \nu)N\).
This is maximized, taking as given other bureaucrats’ decisions, subject to a total-license constraint, which can be written using equation 4 as \( g_{1j}^1 + g_{2j}^1 + g_{1j}^2 + g_{2j}^2 \leq 1 \); to the per-period license constraints, \( g_{1j}^1 + g_{2j}^1 \) and \( g_{1j}^2 + g_{2j}^2 \) each being less than \( \tau \); to non-negativity of \( g_{1j}^1 \); and to applicant-supply constraints.

We now prove a lemma that simplifies analysis of dishonest officials’ behavior.

**Lemma 1.** Under assumptions A1 and A2, dishonest official \( j \) will set \( g_{1j}^1 + g_{2j}^1 + g_{1j}^2 = 1 \).

**Proof.** To see that \( g_{2j}^2 \) must be zero at an optimum, consider feasible plan \( Z \) with \( g_{1j}^2 = x \) and \( g_{2j}^2 = y > 0 \). It gives a strictly lower payoff (see equation 10) than plan \( Z' \), identical to \( Z \) except that it sets \( g_{1j}^2 = x + y \) and \( g_{2j}^2 = 0 \). \( Z' \) is also feasible because it gives the same number of licenses in each period as \( Z \), and because assumption A1 guarantees that supply of productive applicants is never a binding constraint.

Now consider a feasible plan with \( g_{1j}^1 + g_{2j}^1 + g_{1j}^2 < 1 \). This implies that either \( g_{1j}^1 + g_{2j}^1 < \tau \) or \( g_{1j}^2 < 1 - \tau \), or both. In the latter case, payoff 10 can be increased by increasing \( g_{1j}^1 \). This can be done without violating any constraints, in part because assumption A1 guarantees the supply of productive applicants is not a binding constraint. In the former case, payoff 10 can be increased by increasing \( g_{2j}^1 \). This can be done without violating any constraints, in part because assumption A2 guarantees the supply of unproductive applicants in period one is not a binding constraint. Thus the constraint must bind at an optimum, that is, \( g_{1j}^1 + g_{2j}^1 + g_{1j}^2 = 1 \).

Lemma 1 establishes that no licenses will be awarded to unproductive candidates in period 2, since there are always enough candidates willing to buy them at a higher price. It also shows that no licenses are wasted, for largely the same reasons.

Utilizing lemma 1, we can eliminate the total-license constraint and write

\[
g_{1j}^2 = 1 - g_{1j}^1 - g_{2j}^1. \quad (11)
\]
As discussed earlier, assumptions A1 and A2 ensure that applicant-supply constraints never bind for productive applicants and for unproductive applicants in period one. The remaining per-period license constraints can then be written using equation 11

\[ 1 - \tau \leq g_{1j}^1 + g_{2j}^1 \leq \tau. \quad (12) \]

The dishonest bureaucrat’s program can then be written as

\[
\max_{g_{1j}^1 \geq 0, g_{2j}^1 \geq 0} \Pi = LV_1[\gamma(\bar{g}_i^1) g_{1j}^1 + \alpha g_{2j}^1 + \delta (1 - g_{1j}^1 - g_{2j}^1)]
\]

subject to (12).

A solution to this program clearly exists, since the maximand is continuous in both choice variables (see equations 6, 8, and 9) and the choice set is compact. The program would be completely linear were it not for the dependence of \( \gamma \) on \( \bar{g}_i^1 \), which in turn depends on \( g_{1j}^1 \). The remaining linearity, however, tends to push solutions to corners.

Take for example the case where \( \alpha < \delta \). It is clear that the solution takes the form

\[
\begin{align*}
g_{1j}^1 & = 0, \quad g_{2j}^1 = 1 - \tau, \quad \text{if } \gamma(\bar{g}_i^1) < \alpha < \delta \\
g_{1j}^1 & \in [0, 1 - \tau], \quad g_{2j}^1 = 1 - \tau - g_{1j}^1, \quad \text{if } \gamma(\bar{g}_i^1) = \alpha < \delta \\
g_{1j}^1 & = 1 - \tau, \quad g_{2j}^1 = 0, \quad \text{if } \alpha < \gamma(\bar{g}_i^1) < \delta \\
g_{1j}^1 & \in [1 - \tau, \tau], \quad g_{2j}^1 = 0, \quad \text{if } \alpha < \gamma(\bar{g}_i^1) = \delta \\
g_{1j}^1 & = \tau, \quad g_{2j}^1 = 0, \quad \text{if } \alpha < \delta < \gamma(\bar{g}_i^1).
\end{align*}
\]

The first three lines involve setting \( (g_{1j}^1 + g_{2j}^1) \) at their lower bound \( 1 - \tau \), reserving \( \tau \) for productive applicants in period 2, since \( \delta \) is the largest payoff. The \( 1 - \tau \) are divided in period 1 according to how \( \alpha \) compares to \( \gamma(\bar{g}_i^1) \); if they are equal, any division is optimal. The last two lines involve setting \( g_{2j}^1 \) to zero and awarding all licenses to productive applicants, with the timing depending on how \( \delta \) compares to \( \gamma(\bar{g}_i^1) \); if they are equal, any timing is optimal.

It is already evident how both strategic delay and allocative inefficiency may arise in
equilibrium. For example, assume for the moment that \( \gamma(0) < \alpha < \delta. \) In this case \( g_{ij}^1 = 0 \) (and since \( j \) is arbitrary, \( \mathbb{g}_i^1 = 0 \)), \( g_{ij}^3 = 1 - \tau \), and \( g_{ij}^2 = \tau \) give the optimal strategy. But this involves awarding some licenses to unproductive candidates \( (g_{ij}^1 > 0) \), as well as delaying licenses to productive candidates \( (g_{ij}^2 > 1 - \tau) \).

If \( \alpha > \delta \), no strategic delay would arise in equilibrium. An official would rather give out \( T \) licenses to unproductive candidates in period 1 than delay licenses to productive candidates in period 2. Consequently, we focus on the case in which \( \delta > \alpha \) in this paper.\(^9\)

Note that the equilibrium is characterized fully by the collection of solutions to the dishonest bureaucrat’s maximization problem, one for each \( j \in \mathcal{D} \), since the behavior of honest bureaucrats and optimizing behavior of applicants are incorporated into this program. Of course, these solutions are tied together by the relationship of equation 6, reproduced here:

\[
\bar{g}_i^1 = \sum_{j \in \mathcal{D}} g_{ij}^1 / [(1 - \nu) N].
\]

Thus the equilibrium is summed up by a set \( \{(g_{ij}^1, g_{ij}^3) : j \in \mathcal{D}\} \), such that for each \( j \in \mathcal{D} \), \( (g_{ij}^1, g_{ij}^3) \) solves the above maximization problem, taking as given all others’ actions, and such that \( \bar{g}_i^1 \) satisfies equation 6.

It can be shown that bureaucrat \( j \)'s solution to program 13 is unique, given the actions of other bureaucrats. However, in equilibrium solutions may be non-unique across bureaucrats.\(^{10}\) This non-uniqueness of the individual bureaucrats’ solutions is not problematic for our purposes, since the aggregate number of licenses given to each type in each period (equivalently, \( \bar{g}_i^1 \) and analogously-defined \( \bar{g}_i^2 \) and \( \bar{g}_i^3 \)) is unique, as we now show.

**Proposition 1.** Under assumptions A1 and A2 and for \( \alpha < \delta \), all equilibria involve the

---

8 Conditions under which this holds will be established later.

9 The other case is available from the authors upon request. The analysis is almost identical and adds no new insight to this case.

10 This can happen when in equilibrium \( \gamma(\bar{g}_i^1) = \alpha \), for example. Some bureaucrats may give more licenses to productive, others to unproductive, candidates, but all reap the same optimal payoff.
same values for $\overline{g}_1^1$, $\overline{g}_2^1$, and $\overline{g}_3^1$.

Proof. If $\nu = 1$, the equilibrium is trivial: $\overline{g}_1^1 = \tau$ and $\overline{g}_2^1 = 1 - \tau$. If $\nu = 0$, equation 9 gives that $\gamma(\overline{g}_1^1) = 1$, and the optimal solution 14 for each bureaucrat then ensures that $\overline{g}_1^1 = \tau$ and $\overline{g}_2^1 = 1 - \tau$.

Now consider $\nu \in (0, 1)$ and assume that $\overline{g}_1^1$ and $\overline{g}_3^1 > \overline{g}_1^1$ are both equilibrium outcomes for $\overline{g}_1^1$. We know then that $\gamma(\overline{g}_1^1) > \gamma(\overline{g}_3^1)$, since $\gamma$ is strictly decreasing in $\overline{g}_1^1$ for $\nu \in (0, 1)$. Call these $\gamma_a$ and $\gamma_b$, respectively.

Note from equations 14 that the optimal $g_{1j}^1$ is weakly increasing in $\gamma$, in the sense that if $\gamma_a > \gamma_b$, then the optimal $g_{1j}^1$ for $\gamma_a$ is at least as great as the optimal $g_{1j}^1$ for $\gamma_b$. Since this is true for all $j$, it must be true that $\overline{g}_1^1 \geq \overline{g}_3^1$. This is a contradiction, and thus there can only be one equilibrium value for $\overline{g}_1^1$.

Take the equilibrium value for $\overline{g}_1^1$ as given, and let $\gamma^* = \gamma(\overline{g}_1^1)$. Consider two cases. First, if $\alpha \neq \gamma^*$, then by equations 14, $g_{2j}^1$ is strictly pinned down for each bureaucrat $j$; clearly $\overline{g}_2^1$ must also be. Second, if $\alpha = \gamma^*$, then equations 14 give that the sum $(g_{1j}^1 + g_{2j}^1)$ is pinned down at $1 - \tau$ for every bureaucrat. Thus $\overline{g}_1^1 + \overline{g}_2^1 = 1 - \tau$, so $\overline{g}_2^1$ is pinned down at $1 - \tau - \overline{g}_1^1$. Finally, since equation 11 holds for each bureaucrat, $\overline{g}_1^2$ is pinned down at $1 - \overline{g}_1^1 - \overline{g}_2^1$. $\blacksquare$

Proposition 1 establishes that the equilibrium is unique if we restrict attention to aggregate variables. An immediate corollary is that there exists a unique symmetric equilibrium. It will therefore be without loss of generality (again, from the standpoint of aggregates) that we restrict attention to symmetric equilibria in following sections.

For clarity, we define the following equilibrium quantities. Let $M$ be the amount of misallocation, that is, the total number of licenses awarded to unproductive candidates. Then

$$M = (1 - \nu)NL\overline{g}_2^1;$$

(15)
this is the total number of licenses in the hands of dishonest bureaucrats, \((1 - \nu)NL\), times the average fraction each one gives to unproductive candidates (which can occur only in period 1). Let \(D\) be the total number of licenses delayed, that is, given late to productive candidates, above and beyond the amount necessary. In particular, even honest officials cannot service all productive candidates in period 1, and so give out a fraction \(1 - \tau\) of their licenses to them in period 2. So only the fraction in excess of \(1 - \tau\) counts in the quantity \(D\):

\[
D = (1 - \nu)NL[\gamma^2_1 - (1 - \tau)] = (1 - \nu)NL[\tau - \gamma^1_1 - \gamma^2_2],
\]

where the second equality uses equation 11. Quantities \(M\) and \(D\) represent the two types of efficiency losses in the model.

4 Effects of Honest Agents

In this section we trace the effects of \(\nu\), the fraction of honest officials, on the amount of license misallocation \(M\) and strategic delay \(D\). We continue to assume \(\alpha < \delta\). The basic result is a non-monotonic (in fact, W-shaped) relationship of efficiency with the percent of honest agents, \(\nu\). To see the non-monotonicity, consider the extreme cases. If \(\nu = 0\), every official is corrupt; a productive applicant derives no value from waiting, since he can never meet an honest official, and so will pay the full amount \(V_1\) even in period 1 (i.e. \(\gamma = 1\)); and thus all officials give all their licenses to productive candidates, a fraction \(\tau\) in period 1 and \(1 - \tau\) percent in period 2. If \(\nu = 1\), all officials are honest and the exact same behavior results. For intermediate values of \(\nu\), however, corrupt bureaucrats find it optimal to pass by productive candidates in period 1 since they are not willing to pay the full amount \(V_1\).

The problem for \(\nu \in (0, 1)\) is illustrated graphically in Figure 1. The increasing step function (technically, correspondence) represents the \(\gamma\) required to make a given \(g^1_{ij}\) optimal; it is merely the inverse of the solution of equations 14. Specifically, setting \(g^1_{ij} = 0\) is justified
\[ \alpha = 0.6, \delta = 0.9 \]
\[ \tau = 0.6, \kappa = 0.2 \]

\[ g_{ij}^1, g_1^1. \]

Figure 1: Determination of equilibrium.

as an optimal choice by any \( \gamma \leq \alpha \). Similarly, setting \( g_{ij}^1 = 1 - \tau \) is optimal when \( \alpha \leq \gamma \leq \delta \), and setting \( g_{ij}^1 = \tau \) is optimal when \( \delta \leq \gamma \). These cases make up the vertical pieces of the step function. For \( g_{ij}^1 \in (0, 1 - \tau) \), we know that a dishonest official must be indifferent between productive and unproductive applicants in period 1, and thus \( \gamma \) must equal \( \alpha \). Similarly, for \( g_{ij}^1 \in (1 - \tau, \tau) \), a dishonest official must be indifferent between productive applicants in periods 1 and 2, respectively, and thus \( \gamma \) must equal \( \delta \). These cases make up the horizontal pieces of the step function.

The decreasing dashed lines represent the function \( \gamma(\overline{y}_1) \) for four different values of the parameter \( \nu \), which are discussed below. As mentioned earlier, the function \( \gamma(\overline{y}_1^1) \) is strictly decreasing in \( \overline{y}_1^1 \) when \( \nu \in (0, 1) \). Thus there is exactly one intersection of a given dashed line with the step function, which gives the unique equilibrium value of \( \overline{y}_1^1 \).

It is evident from inspection of equations 8 and 9 that \( \gamma(\overline{y}_1^1) \) is strictly decreasing in \( \nu \) for a given \( \overline{y}_1^1 \). Thus Figure 1 corresponds to \( \nu_1 < \nu_2 < \nu_3 < \nu_4 \); the higher \( \nu \), the lower the \( \gamma(\overline{y}_1^1; \nu) \) curve. More specifically, the values for \( \nu \) are chosen to put the intersections at
the corners of the step function. In particular, \( \nu_1 \) is defined to set \( \gamma(\tau; \nu_1) \) equal to \( \delta \). Then using equations 8 and 9,

\[
\nu_1 = \frac{1 - \tau + \kappa}{1 - \tau} \frac{1 - \delta}{\delta}.
\]  

By construction, for \( \nu < \nu_1 \), \( \gamma(\tau; \nu) > \delta \) and the equilibrium involves \( \overline{y}_1 = \tau \) and \( \overline{y}_2 = 0 \) (see equation 14). Next, \( \nu_2 \) is defined to set \( \gamma(1 - \tau; \nu_2) = \delta \) and \( \nu_3 \) is defined to set \( \gamma(1 - \tau; \nu_3) = \alpha \). This gives that

\[
\nu_2 = \frac{\tau + \kappa}{\frac{\delta}{1 - \delta}(1 - \tau) + 2\tau - 1}
\]  

and

\[
\nu_3 = \frac{\tau + \kappa}{\frac{\delta}{1 - \alpha}(1 - \tau) + 2\tau - 1}.
\]  

Clearly for \( \nu \in (\nu_2, \nu_3) \), \( \alpha < \gamma(1 - \tau; \nu) < \delta \) and the equilibrium involves \( \overline{y}_1 = 1 - \tau \) and \( \overline{y}_2 = 0 \). Finally, \( \nu_4 \) is defined to set \( \gamma(0; \nu_4) = \alpha \):

\[
\nu_4 = \frac{1 + \kappa}{\frac{\delta}{1 - \alpha}(1 - \tau) + \tau}.
\]  

For \( \nu > \nu_4 \), \( \gamma(0; \nu) < \alpha \) and the equilibrium involves \( \overline{y}_1 = 0 \) and \( \overline{y}_2 = 1 - \tau \). In the current discussion, we are assuming that \( 0 < \nu_1 < \nu_2 < \nu_3 < \nu_4 < 1 \); this will be shown in proposition 2.

The remaining intervals, \([\nu_1, \nu_2]\) and \([\nu_3, \nu_4]\), involve intersections through the horizontal segments of the step function in Figure 1. For \( \nu \in [\nu_1, \nu_2] \), Figure 1 makes clear that in equilibrium \( \gamma(\overline{y}_1; \nu) = \delta \). Using this fact in equations 8 and 9, we can solve for \( \overline{y}_1 \) as a
function of $\nu$:

$$g_1^1(\nu) = \tau - \frac{\delta(1 - \tau)}{1 - \delta} \frac{\nu - \nu_1}{1 - \nu}, \quad \nu \in [\nu_1, \nu_2].$$  \hfill (21)

It can be checked that this expression ranges from $\tau$ to $1 - \tau$ as $\nu$ increases from $\nu_1$ to $\nu_2$. Since $\gamma = \delta > \alpha$, we know $g_2^1 = 0$. For $\nu \in [\nu_3, \nu_4]$, Figure 1 makes clear that in equilibrium $\gamma(g_1^1; \nu) = \alpha$. Again, we can use equations 8 and 9 to solve for $g_1^1$:

$$g_1^1(\nu) = 1 - \tau - \frac{\delta(1 - \tau)}{1 - \alpha} + 2\tau - 1 \left[ \frac{\nu - \nu_3}{1 - \nu} \right], \quad \nu \in [\nu_3, \nu_4].$$  \hfill (22)

It can be checked that this expression ranges from $1 - \tau$ to 0 as $\nu$ increases from $\nu_3$ to $\nu_4$. Since $\alpha = \gamma < \delta$, we know that each bureaucrat gives out exactly the fraction $1 - \tau$ of his licenses in period 1, and so $g_2^1 = 1 - \tau - g_1^1$.

We can summarize how $g_1^1$ and $g_2^1$ vary with $\nu$ as follows:

\begin{align*}
& g_1^1 = \tau, & g_2^1 = 0, & i f \ \nu \in [0, \nu_1] \\
& g_1^1 = \tau - \frac{\delta(1 - \tau)}{1 - \delta} \frac{\nu - \nu_1}{1 - \nu}, & g_2^1 = 0, & i f \ \nu \in [\nu_1, \nu_2] \\
& g_1^1 = 1 - \tau, & g_2^1 = 0, & i f \ \nu \in [\nu_2, \nu_3] \\
& g_1^1 = 1 - \tau - \frac{\delta(1 - \tau)}{1 - \alpha} + 2\tau - 1 \left[ \frac{\nu - \nu_3}{1 - \nu} \right], & g_2^1 = 1 - \tau - g_1^1, & i f \ \nu \in [\nu_3, \nu_4] \\
& g_1^1 = 0, & g_2^1 = 1 - \tau, & i f \ \nu \in [\nu_4, 1].
\end{align*}

(23)

These equilibrium choices are expressed graphically in Figure 2. It is clear that for moderate values of $\nu$, productive candidates may experience significant delay. For $\nu$ higher still, they experience outright denial as licenses are allocated to unproductive candidates.

We quantify the inefficiency by using the equilibrium values of 23 in equations 15 and 16.
Figure 2: The effect of $\nu$ on corrupt officials’ license allocation.

to calculate total misallocation of licenses $M$ and strategic delay $D$.

$$M = \begin{cases} 
0, & \text{if } \nu \in [0, \nu_3] \\
(\nu - \nu_3)NL\left[\frac{[\alpha - \delta(1-\tau)]}{1-\alpha}\right] + 2\tau - 1, & \text{if } \nu \in [\nu_3, \nu_4] \\
(1 - \nu)NL(1 - \tau), & \text{if } \nu \in [\nu_4, 1] 
\end{cases} \quad (24)$$

$$D = \begin{cases} 
0, & \text{if } \nu \in [0, \nu_1] \\
(\nu - \nu_1)NL\left[\frac{[\alpha - \delta(1-\tau)]}{1-\alpha}\right], & \text{if } \nu \in [\nu_1, \nu_2] \\
(1 - \nu)NL(2\tau - 1), & \text{if } \nu \in [\nu_2, 1] 
\end{cases} \quad (25)$$

Both $M$ and $D$ consist of three linear pieces: the first flat, the second and third forming an inverted V. Total inefficiency, call it $\Lambda$ for loss, is a weighted sum of $M$ and $D$.\textsuperscript{11} For each license delayed, the payoff in total surplus is reduced by $V_1$ and increased by $\delta V_1$, for a net

\textsuperscript{11}We will think of $\Lambda$ as negative if there is a loss in efficiency.
gain of $(\delta - 1)V_1 < 0$. Similarly, for each license misallocated, the net gain to total surplus is $(\alpha - 1)V_1 < 0$. Thus

$$
\Lambda = V_1[D(\delta - 1) + M(\alpha - 1)].
$$

(26)

Using this expression and equations 24 and 25, we see that

$$
\Lambda = \begin{cases} 
0, & \nu \in [0, \nu_1] \\
-V_1NL(\nu - \nu_1)\delta(1 - \tau), & \nu \in [\nu_1, \nu_2] \\
-V_1NL(1 - \nu)(1 - \delta)(2\tau - 1), & \nu \in [\nu_2, \nu_3] \\
-V_1NL\{(1 - \nu)(1 - \delta)(2\tau - 1) + (\nu - \nu_3)[\delta(1 - \tau) + (1 - \alpha)(2\tau - 1)]\}, & \nu \in [\nu_3, \nu_4] \\
-V_1NL(1 - \nu)[(1 - \delta)(2\tau - 1) + (1 - \alpha)(1 - \tau)], & \nu \in [\nu_4, 1]. 
\end{cases}
$$

(27)

By inspection, total surplus is piecewise linear and W-shaped as a function of $\nu$ when $\nu \in [V_1, 1]$.\footnote{For $\nu \in [\nu_3, \nu_4]$, one can calculate the slope on $\nu$ to be $-V_1NL[\delta(1 - \tau) + (\delta - \alpha)(2\tau - 1)]$. This is strictly negative since $\alpha < \delta$ and $\tau \in (1/2, 1)$.} Thus while efficiency per dishonest bureaucrat is monotonically declining in $\nu$, total efficiency is non-monotonic. Perfect efficiency results when there are few enough honest bureaucrats, or all are honest. Total surplus is pictured in Figure 3 as a percent of first-best surplus, which equals $V_1[NT + \delta N(L - T)] = V_1NL[\tau + \delta(1 - \tau)]$.

We have argued that total efficiency is a W-shaped function of $\nu$, the fraction of honest people. It remains to provide conditions ensuring that indeed $0 < \nu_1 < \nu_2 < \nu_3 < \nu_4 < 1$.

**Proposition 2.** Under assumptions A1 and A2 and for $\alpha < \delta$, the condition

$$
\delta > (1 - \tau + \kappa) \max\left\{\frac{1 - \alpha}{1 - \tau}, \frac{1}{2(1 - \tau) + \kappa}\right\}
$$

guarantees that

- For $\nu \in (\nu_1, 1)$, strategic delay occurs in equilibrium, that is, $\bar{y}_{1}^{2} > 1 - \tau$;
• For $\nu \in (\nu_3, 1)$, misallocation of licenses occurs in equilibrium, that is, $\bar{y}_{2} > 0$; and

• Total surplus is W-shaped in $\nu$, reaching local minima at $\nu = \nu_2$ and $\nu = \nu_4$, and attaining the first best only for $\nu \in [0, \nu_1]$ and $\nu = 1$.

Proof. The preceding text establishes each of these facts, under the assumption that $0 < \nu_1 < \nu_2 < \nu_3 < \nu_4 < 1$, which we here show. Recall that $\delta \in (0,1)$, $\tau \in (1/2,1)$, and $\alpha \in (0, \delta)$ by assumption. Further, assumption A1 implies that $\kappa \geq 2\tau - 1$ and is thus strictly positive.

Given these facts, inspection of equation 17 reveals that $\nu_1 > 0$. Comparison of equations 18 and 19 reveals that $\nu_2 < \nu_3$, since $\alpha < \delta$. Comparison of equations 19 and 20 reveals that $\nu_3 < \nu_4$ as long as $\nu_4 < 1$. To see this, let $\nu_4 \equiv x/y$, and note that $\nu_3 = (x - z)/(y - z)$, where $z = 1 - \tau > 0$. Subtracting a positive constant from both numerator and denominator of any fraction less than one reduces the amount, as long as both numerator and denominator remain positive, which is true here.
It remains only to show that $\nu_1 < \nu_2$ and $\nu_1 < 1$. Using equations 17 and 18, one can show that

$$\delta > \frac{1 - \tau + \kappa}{2(1 - \tau) + \kappa}$$

is necessary and sufficient for $\nu_1 < \nu_2$. (It is also necessary and sufficient for both $\nu_1 < 1$ and $\nu_2 < 1$.) Further, using equation 20, one can show that

$$\delta > \frac{(1 - \tau + \kappa)(1 - \alpha)}{1 - \tau}$$

is necessary and sufficient for $\nu_1 < 1$. (It is also necessary and sufficient for both $\nu_3 < 1$ and $\nu_3 < \nu_4$.)

Note that these two conditions can be satisfied. For example, if $\epsilon > 0$ is small enough, any combination of parameters satisfying $\delta > \alpha = \tau$ and $\kappa = 2\tau - 1 + \epsilon$, or residing in a sufficiently small neighborhood of such a combination, satisfies these conditions. ■

5 Limited Wealth and Optimal Monitoring

The previous section establishes an interesting non-monotonic relationship between efficiency and the honesty of the bureaucracy. Perfect efficiency only results from an extremely honest or dishonest bureaucracy. In the former situation, officials award all licenses to the most productive type as part of performance of their duty. In the latter case, bribery ensured allocation efficiency, as the type that generated the greatest surplus from each license paid the highest bribe.

If complete honesty among all bureaucrats is as efficient as total dishonesty, it may be argued that the government should not spend any resources on costly corruption-deterring activity, but should rather encourage bureaucrats to behave as revenue farmers who are un-
restricted in their attempt to maximize earnings from the sale of licenses. Such a conclusion, however, overlooks what frequently may be an important justification for awarding licenses at below their “market price” to the most productive applicants, rather than using price as a screening device for achieving allocation efficiency. In an economy where income is distributed unequally, and credit markets are imperfect, some applicants may be constrained in their ability to pay a bribe. Consequently, a less productive but wealthy agent may outbid a more productive but poor one, and revenue farming may lead to inefficient allocation of resources.

To illustrate, consider the following simple distribution of initial wealth: a proportion \( \omega \) of each type of agent (productive and unproductive) possesses wealth \( W > V_i \); \( 1 - \omega \) have zero wealth. Imperfections in the credit market are assumed to preclude agents from borrowing money for the purpose of bribing bureaucrats. We focus on the case where

\[
\tau < \omega(1 + \kappa) < \frac{(1 + \kappa)(1 - \tau)}{1 + \kappa - \nu_4 \tau} \equiv B, \tag{A3}
\]

where \( \nu_4 \) is defined in equation 20.\(^{13}\) Assumption A3 implies that the number of wealthy, productive agents approaching each bureaucrat in period one is at least \( T \). This eliminates the applicant-supply constraint involving wealthy, productive agents in period one. It also implies that the total number of wealthy, productive agents is less than \( L \), since \( B < 1 \). Now even if all bureaucrats are dishonest, inefficiency will result. There are not enough productive agents who can pay for a license, so some will be sold to unproductive agents. Note that assumption A3 also implies that

\[
\omega < \frac{1 - \tau}{1 + \kappa - \tau} \leq \frac{1 - \nu_4 \tau}{1 + \kappa - \nu_4 \tau}, \quad \nu \in [0, 1], \tag{28}
\]

a fact that will be useful.

\(^{13}\)We provide conditions in proposition 3 under which this assumption can be satisfied. Our goal here is not to characterize the equilibrium as a function of \( \omega \), but to provide an illustrative example. Conditions for \( \nu_4 < 1 \) are provided in proposition 2, which we now presume.
We make two further assumptions to simplify the analysis. One is that

$$\omega(1 + \kappa)(1 - \mu)/\mu > 2\tau.$$  \hspace{2cm} (A4)

This is a strengthening of assumption A2. It ensures there are at least $T$ wealthy, unproductive agents approaching each bureaucrat in each period. At least $2T$ per official apply in the first period, and even if the maximal amount are serviced, $T$ remain to apply in the second period. This eliminates the applicant-supply constraint involving wealthy, unproductive agents in both periods. The second assumption is that there are a continuum of officials and applicants. This implies that one dishonest bureaucrat cannot affect any aggregate $g_i^\tau$, since he is of measure zero.\(^{14}\)

The dishonest bureaucrat’s problem is as in section 3. He maximizes expression 10, reproduced here

$$\Pi = LV_1[\gamma(g_i^1)g_{i,j}^1 + \alpha g_{i,j}^2 + \delta g_{i,j}^1 + \alpha \delta g_{i,j}^2],$$  \hspace{2cm} (29)

subject to total-license constraint $g_{i,j}^1 + g_{i,j}^2 + g_{i,j}^2 + g_{i,j}^2 \leq 1$; to the per-period license constraints, $g_{i,j}^1 + g_{i,j}^1 \leq \tau$ and $g_{i,j}^2 + g_{i,j}^2 \leq \tau$; to non-negativity of $g_i^\tau$; and to wealthy applicant-supply constraints. Assumptions A3 and A4 eliminate all supply constraints except that of second-period wealthy, productive applicants. This constraint can be written

$$g_{i,j}^2 \leq \omega(1 + \kappa - \nu \tau) - (1 - \nu)g^1_i.$$  \hspace{2cm} (30)

The right-hand side is the fraction of licenses wealthy, productive applicants represent $(\omega(1 + \kappa))$ minus the measure serviced by honest bureaucrats in period one $(\omega \nu \tau)$ minus the measure serviced by dishonest bureaucrats in period one $((1 - \nu)g^1_i)$. This incorporates the fact that honest bureaucrats do not discriminate between wealthy and poor applicants, and thus give

\(^{14}\)This is for simplicity. The case of $N$ finite can be analyzed similarly.
a fraction of licenses $\omega \tau$ to wealthy productive applicants and $(1 - \omega)\tau$ to poor productive applicants. Dishonest bureaucrats, however, only give licenses to wealthy applicants, so $\bar{G}_1^i$ is not multiplied by $\omega$.

Again restricting attention to the case where $\alpha < \delta$, and focusing on symmetric equilibria, the solution is as follows:\footnote{It turns out efficiency would be improved if honest bureaucrats discriminated in favor of the poor applicants, since wealthy, productive agents have a chance of being serviced by dishonest bureaucrats. Here we explore the benchmark case of no wealth-discrimination by honest bureaucrats.}

\begin{align*}
g_{1j}^1 &= \tau, \quad g_{1j}^2 = \omega (1 + \kappa - \nu \tau) - (1 - \nu) \tau, \quad i f \quad \gamma(\bar{G}_1^i) > \alpha \\
g_{1j}^1 &\in [0, \tau], \quad g_{1j}^2 = \min\{\tau, \omega (1 + \kappa - \nu \tau) - (1 - \nu) \bar{G}_1^i\}, \quad i f \quad \gamma(\bar{G}_1^i) = \alpha \cdot (31) \\
g_{1j}^1 &= 0, \quad g_{1j}^2 = \min\{\tau, \omega (1 + \kappa - \nu \tau)\}, \quad i f \quad \gamma(\bar{G}_1^i) < \alpha
\end{align*}

Consider first the case where $\gamma(\bar{G}_1^i) > \alpha$. The above is an equilibrium because the maximal fractions of licenses are being given to wealthy, productive agents in each period: $\tau$ in period one, and the complete supply in period two, from constraint 30.\footnote{These solutions are not for arbitrary $\bar{G}_1^i$. They impose the symmetric equilibrium condition $g_{1j}^1 = \bar{G}_1^i$.} (Note that inequality 28 implies that here $g_{1j}^2 < 1 - \tau$.) Further, it is the only equilibrium. If $g_{1j}^1$ were less than $\tau$, it would have to be true at an optimum that $g_{1j}^1 + g_{1j}^2 = 1$; otherwise $g_{1j}^1$ could be increased at the expense of $g_{1j}^1$ or $g_{1j}^2$ without breaking any constraint, increasing payoff 29. But one can show that $g_{1j}^1 + g_{1j}^2 = 1$ is impossible in equilibrium.\footnote{Note that since each bureaucrat is of measure zero, he cannot affect the supply of wealthy, productive applicants in period two by withholding licenses from them in period one.}

Consider next the case where $\gamma(\bar{G}_1^i) < \alpha$. Clearly, any licenses given out in period one will be to unproductive agents, since the supply of both types is not a constraint. Thus $g_{1j}^1$
Figure 4: Determination of equilibrium under limited wealth.

must be zero. It follows that $g_{1j}^2$ will be set as high as possible below $\tau$ and constraint 30 at $\gamma_{ij} = 0$.

Finally, when $\gamma(\gamma_{ij}) = \alpha$, dishonest bureaucrats reserve as many as can be given to wealthy, productive applicants in period two. They then split the remaining licenses in period one, up to $\tau$, among productive and unproductive candidates, since any division of licenses is optimal in period one and there is ample supply of both. Any $g_{ij}^1 \in [0, \tau]$ can be a symmetric equilibrium. To see this, fix $g_{ij}^1 = a \in [0, \tau]$, and note that if every dishonest bureaucrat expects $\gamma_{ij}^1 = a$, they will optimally reserve $\min\{\tau, \omega(1 + \kappa - \nu \tau) - (1 - \nu)a\}$ fraction of licenses for wealthy, productive applicants in period two. They can then give $a$ to wealthy, productive applicants in period one as long as $a \leq 1 - g_{ij}^2$, which is easily verified using inequality 28.

The problem for $\nu \in (0, 1)$ is illustrated graphically in Figure 4, analogous to Figure 1. The increasing step function (technically, correspondence) represents the $\gamma$ required to make a given $g_{ij}^1$ optimal; it is merely the inverse of the solution of equations 31. The decreasing
dashed lines represent the function \( \gamma(\bar{\mathbf{f}}^1_1) \) for two different values of the parameter \( \nu \), which are discussed below. As mentioned earlier, the function \( \gamma(\bar{\mathbf{f}}^1_1) \) is strictly decreasing in \( \bar{\mathbf{f}}^1_1 \) when \( \nu \in (0, 1) \). Thus there is exactly one intersection of a given dashed line with the step function, which gives the unique symmetric equilibrium value of \( \bar{\mathbf{f}}^1_1 \).

As in section 4, \( \nu_4 \) is defined to set \( \gamma(0; \nu_4) = \alpha \); it satisfied equation 20. We define \( \nu_w \) to set \( \gamma(\tau; \nu_w) = \alpha \). Then using equations 8 and 9, \( \nu_w \) must satisfy

\[
\nu_w = \frac{1 - \tau + \kappa}{1 - \tau} \frac{1 - \alpha}{\delta}.
\]

By construction, for \( \nu < \nu_w \), \( \gamma(\tau; \nu) > \delta \) and the equilibrium involves \( \bar{\mathbf{f}}^1_1(\nu) = \tau \) and \( \bar{\mathbf{f}}^1_2(\nu) = 0 \) (see equation 31). For \( \nu > \nu_4 \), \( \gamma(0; \nu) < \alpha \) and the equilibrium involves \( \bar{\mathbf{f}}^1_1(\nu) = 0 \).

The remaining interval, \([\nu_w, \nu_4]\), involves intersection through the horizontal segment of the step function in Figure 4 at which \( \gamma(\bar{\mathbf{f}}^1_1; \nu) = \alpha \). Using this fact in equations 8 and 9, we can solve for \( \bar{\mathbf{f}}^1_1 \) as a function of \( \nu \):

\[
\bar{\mathbf{f}}^1_1(\nu) = \tau - \delta(1 - \tau) \frac{\nu - \nu_w}{1 - \nu}, \quad \nu \in [\nu_w, \nu_4].
\]

It can be checked that this expression ranges from \( \tau \) to 0 as \( \nu \) increases from \( \nu_w \) to \( \nu_4 \).

Note from equations 31 that the equilibrium value for \( \bar{\mathbf{f}}^2_1 \) always satisfies \( \min\{\tau, X\} \), where

\[
X \equiv \omega(1 + \kappa - \nu \tau) - (1 - \nu)\bar{\mathbf{f}}^1_1.
\]

Using the equilibrium values for \( \bar{\mathbf{f}}^1_1 \) from the previous paragraph, one can check that \( X \) increases in \( \nu \) up to \( \nu_4 \), then declines in \( \nu \). Thus it is maximized at \( \nu = \nu_4 \); assumption A3 ensures that even there, it is less than \( 1 - \tau \). Thus in equilibrium \( \bar{\mathbf{f}}^2_1 \) always stays less than
1 – \tau. This implies that in equilibrium \( \overline{g}_2^1 = 1 - \tau - \overline{g}_1^2, \overline{g}_2^2 = \tau - \overline{g}_1^3 \), and that

\[
\begin{align*}
\overline{g}_1^1 &= \tau, & \overline{g}_1^2 &= \omega (1 + \kappa - \nu \tau) - (1 - \nu) \tau, & \text{if } \nu \in [0, \nu_w] \\
\overline{g}_1^3 &= \tau - \frac{\delta (1 - \tau)}{1 - \alpha} \frac{\nu - \nu_w}{1 - \nu}, & \overline{g}_1^4 &= \omega (1 + \kappa - \nu \tau) - (1 - \nu) \overline{g}_1^3, & \text{if } \nu \in [\nu_w, \nu_4] \cdot \\
\overline{g}_1^0 &= 0, & \overline{g}_1^5 &= \omega (1 + \kappa - \nu \tau), & \text{if } \nu \in [\nu_4, 1] \quad (34)
\end{align*}
\]

These equilibrium choices are expressed graphically in Figure 5. Interestingly, all four types of applicants may receive licenses from the same bureaucrat, and misallocation always occurs when \( \nu < 1 \). However, delay is eliminated for these parameter values, in the sense that a fraction \( \tau \) of licenses is always given out in period one.\(^{19}\)

The inefficiency from the equilibrium values of 34 is slightly different from that of section 4. We continue to use \( M \) as defined in equation 15 to denote inefficiency from period

---

\(^{19}\)Delay can arise in equilibrium if assumption A3 is weakened, that is, if \( \omega \) is higher. We make the more restrictive assumption for simplicity of exposition.
one misallocation:

\[ M = (1 - \nu)NL\tilde{g}^2_2. \]

Now let \( MD \) be the total number of licenses misallocated in period two:

\[ MD = (1 - \nu)NL\tilde{g}^2_2. \quad (35) \]

Total inefficiency \( \Lambda \) is now a weighted sum of \( M \) and \( MD \). Note that for each license misallocated in period one, the net gain to total surplus is \((\alpha - 1)V_1 < 0\). For each license misallocated in period two, the net gain to total surplus is \( \delta(\alpha - 1)V_1 < 0 \). These two relationships hold because a dishonest bureaucrat gives the same number of licenses in each period here, so the comparison is between a productive and unproductive applicant within the same period. Thus

\[ \Lambda = V_1[M(\alpha - 1) + MD\delta(\alpha - 1)]. \quad (36) \]

Using these expressions, we calculate and graph total surplus as a percent of first-best surplus, which equals \( V_1[NT + \delta N(L - T)] = V_1NL[\tau + \delta(1 - \tau)] \) in Figure 6.

**Proposition 3.** Under assumptions A3, A4 and for \( \alpha < \delta \), the condition

\[ \frac{(1 - \tau + \kappa)(1 - \alpha)}{1 - \tau} < \delta < \frac{\tau(1 - \alpha)}{2\tau - 1} \]

guarantees that efficiency is uniquely maximized at \( \nu = 1 \).

**Proof.** The preceding text establishes that \( \tilde{f}_j^2 < 1 - \tau \) in any equilibrium with \( \nu < 1 \), under several presumptions. This guarantees inefficiency. The first presumption was that \( 0 < \nu_w < \nu_4 < 1 \). Inspection of equation 32 shows that \( 0 < \nu_w \). One can also check that the first inequality of the condition in the proof guarantees that \( \nu_4 < 1 \) and that \( \nu_w < \nu_4 \), (part
of which was shown in the proof of proposition 2).

The second presumption is necessary for assumption A3 to be possible:

\[ \tau < \frac{(1 + \kappa)(1 - \tau)}{1 + \kappa - \nu_4 \tau}. \]

One can check using equation 20 that the second inequality of the condition in the proof guarantees this.

Note that these conditions can be satisfied. For example, any combination of parameters satisfying \( \delta > \alpha = \tau, \tau < 2/3, \) and \( \kappa = 2\tau - 1, \) or residing in a sufficiently small neighborhood of such a combination, satisfies these conditions. ■

In this analysis, the door is opened for socially efficient investment in corruption-controlling activities. Consider, for example, a linear unit cost of indoctrinating officials, that is, making them honest. Examination of Figure 6 makes clear that for very high unit costs, no
investment is optimal; as the cost declines through a certain range, the optimal $\nu$ increases smoothly from 0 to $\nu_w$; as the cost declines through a further range, $\nu_w$ remains optimal; and finally, when the cost is low enough, the optimal $\nu$ jumps to 1.

Perhaps the most interesting conclusion here is that a sort of bimodality in implementation of optimal anti-corruption policies may exist. Countries with a wide range of unit costs above some cutoff will make little investment and allow or even encourage corruption as the norm, while those below the cutoff will expend resources to obtain a relatively, or completely, clean bureaucracy. The non-monotonicity of efficiency offers an explanation for corruption in bureaucracy persisting as the norm in some countries and be relatively infrequent in others.

6 Conclusion

Thus bribery is inefficient in a simple dynamic setting, and in fact creates a non-monotonic relationship between the degree of honesty of the bureaucracy and efficiency. The key mechanism is that the option of reapplying in hopes of finding an honest bureaucrat is worth most to productive candidates. This lowers their initial willingness to pay a bribe and thus can lead to misallocation of licenses toward less productive candidates and bureaucratic delay in awarding licenses to productive candidates. Thus our paper demonstrates that the twin features of misallocation of opportunities and bureaucratic delay are strategically interlinked, and their frequently observed co-existence is not one of pure coincidence.

References


