Skill Differentiation and Income Disparity
in a Decentralized Matching Model of North-South Trade*†

Abstract
This paper develops a North-South trade model in which the South produces food and the North produces both food and a high-tech good. Food production is undertaken by unskilled workers while the high-tech product is made only by horizontally differentiated skilled workers. Due to the possibility of a peer-group effect, we allow the unskilled workers in the North to be equally or more productive than in the South. Horizontal matching of skilled workers is generally imperfect and the skilled wages are determined by a symmetric Nash bargain. We characterize two different types of equilibrium: a closed-economy equilibrium without trade and a free trade equilibrium without labor mobility. We then extend the benchmark framework to consider the presence of transport costs. In all cases with trade, the equilibrium properties of goods pricing, the volume of trade and wage disparities are examined.

Keywords: Skill Heterogeneity and Matching, North-South Trade, Wage Inequality

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1 Introduction

It is well-documented that since 1980, the demand for less-skilled workers has fallen significantly in advanced countries. In the U.S. this trend has occured in conjunction with a dramatic decline in real wages for the less-educated. For example, Chinhui, Murphy, and Topel (1991) find that the real wage for the lowest 20% of the American workforce in the 1990s was 25% below the 1973 level, while Juhn, Murphy and Pierce (1993) indicate that the real wage for the least skilled workers decreased by 5% but the skilled real wage rose by 40% over the period from 1963 to 1989. During the same period, there has been clear-cut evidence, such as in Sachs and Shatz (1994), that imports of low skill-intensive goods from less-developed economies have increased sharply. It should come as little surprise, therefore, that there is a lively debate on whether trade between developed and less-developed nations has detrimental effects on the welfare of unskilled workers in advanced societies. Specifically, Leamer (1993) and Wood (1994) suggest that the deteriorating situation of unskilled workers in developed countries is primarily caused by the expansion of trade with less-developed countries. Lawrence and Slaughter (1993) counter this view by arguing that trade is not a major factor driving the wage disparity in developed countries.¹

To gain a deeper understanding of this debate, it is necessary that we examine how the labor market in a developed country - which we can call North - is affected when North opens to trade with a less-developed country, South and what implications such trade carries for the determinants of the wage gap between the skilled and unskilled in developed countries. In other words, there is a need for a general-equilibrium model of North-South trade with completely specified skilled and unskilled labor markets.

This paper develops and analyzes such a model. If we were to appeal to a conventional two-sector Heckscher-Ohlin model, we might regard North as having a comparative advantage in producing a skill-intensive good. The Stolper-Samuelson theorem would then apply: a reduction in the relative

¹ Richardson (1995) provides an excellent survey of recent econometric estimates of the impact of trade on U.S. wage inequalities.
price of the less skill-intensive good as a result of free trade would result in a decline in the return to the unskilled labor and a rise in the skilled wage in North.\textsuperscript{2} However, it has been shown by Bond, Trask and Wang (2000) that various versions of the Heckscher-Ohlin theorem may fail to hold in a dynamic setting where both physical and human capital are endogenous accumulated. This paper further examines the consequences of a bilateral free trade agreement on economic welfare and wage disparity in a two-sector non-Walrasian framework of North-South trade in which skill-matching between workers and firms is imperfect and skilled wages are determined via a cooperative Nash bargain.

In particular, in our world of two countries, there is a highly-advanced North and a less-developed South. The two-sector structure features a modern high-tech sector in North that depends crucially on skill matching, and a traditional food-processing sector that can be operative in both countries. North is populated with a continuum of horizontally differentiated skilled workers and a continuum of unskilled workers, whereas South is populated only with unskilled workers. Due to the possibility of a peer-group effect, we allow the unskilled workers in North to be equally or more productive than those in South. Horizontal matching of skilled workers and firms in the high-tech industry is generally imperfect and the skilled wages are determined by a symmetric Nash bargain. Since the unskilled wage in North can be regarded as an outside alternative to skilled workers, it may or may not affect the determination of equilibrium wages of the skilled. It is important to note that the consideration of horizontal differentiation in skilled workers is particularly relevant. Should skilled workers be vertically different in quality, the consequent wage disparity would be fully justified. Since skilled workers in our model are on equal ground ex ante (as they are identical vertically in the quality aspect), any wage inequality would be a concern to a benevolent social planner.

We characterize two different types of equilibrium: an autarchy equilibrium without trade and a free trade equilibrium without labor mobility. In doing so, we focus on the effects of a bilateral

\textsuperscript{2} See a comprehensive survey by Jones and Neary (1984).
free trade agreement on wages, income inequalities and social welfare in each country. We then extend the benchmark model to consider the presence of transportation costs with trade.

The main findings of our paper are as follows. First, under free trade, an increase in the skilled productivity in North or the population of the unskilled in South raises the unskilled wage in North unambiguously. Second, whether the unskilled wage in North affects the outcome of skilled wage bargaining or not, free trade with no labor mobility always reduces the unskilled wage and the food price but raises the welfare of the unskilled in North. Third, free trade always narrows between-North-and-South inequality of the unskilled, whether the unskilled wage in North affects the skilled wage or not. Finally, when the unskilled wage does not affect the skilled wage in North, free trade widens the between-skilled-and-unskilled inequality in North without affecting the within-the-skilled-group wage inequality. By contrast, when the unskilled wage does affect the skilled wage in North, free trade reduces the within-the-skilled-group wage inequality but its effect on the between-skilled-and-unskilled inequality in North is ambiguous. Some skilled workers unambiguously benefit from trade but others - those with the least attractive outside options - unambiguously lose.

It may be worth mentioning that the recent literature on North-South trade focuses on trade with horizontally or vertically differentiated products. For example, Krugman (1979) constructs a static model of technology innovation in which North-South trade is motivated by a preference for variety in goods, whereas Stokey (1991) characterizes North-South trade in a model of vertical product differentiation where the composition of trade shifts over time toward high quality. In Grossman and Helpman (1992), dynamic comparative advantage is thoroughly analyzed for both horizontally and vertically differentiated products in trade between North and South. While these papers have provided valuable insights toward understanding the efficiency and equity consequences of North-South trade, the implications of trade for a horizontally differentiated labor market in a non-Walrasian setting remain unexplored.3 Our paper attempts to fill this gap.

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Moreover, it is interesting to adopt the matching framework to study the issues of North-South trade and income disparity for at least the following reasons. First, the framework captures the nature of spatial separation in trade. Second, it highlights the role of a threat-point effect in influencing the patterns of trade and the dispersion of wages, which is absent in the existing literature. Third, it enables clear-cut differentiation with respect to income inequalities within the skilled group, between the skilled and unskilled groups, as well as between the unskilled groups in North and South.

The remainder of the paper is organized as follows. In Section 2, we delineate a North-South trade model in which the South produces food and the North produces both food and a high-tech good. Section 3 characterizes two different types of equilibrium: an autarchy equilibrium without trade or factor mobility and a free trade equilibrium with no labor mobility in the unskilled labor. In section 4, we extend the basic framework to allow for the presence of transport costs in the case of trade with immobile labor. Finally, we conclude the research with general policy implications and plausible avenues for future work.

2 The Model

Consider two countries, North and South (indexed by $N$ and $S$, respectively). North is populated with a continuum of skilled labor of mass $M$ and a continuum of unskilled workers of mass $L_N$. South is populated with a continuum of unskilled workers of mass $L_S$. While unskilled workers are homogeneous (both within and across countries), skilled workers in North are of differentiated types (or skill characteristics). Let workers’ skill types be represented by $T$ defined over a unit circumference $[0, 1]$ and assume that the skilled workers are uniformly distributed in skill characteristics space, i.e., $M(T) = M$ for all $T \in [0, 1]$.

There are two final goods (labeled by $x$ and $z$) produced and traded at no transportation cost in the two-country world (to be relaxed later). A “food-processing” product, $x$, is made by skill matching in a very different context, namely, location theory.
unskilled workers under a simple constant-returns-to-scale technology. Production of the “high-tech” product is undertaken by high-tech firms that are differentiated by the type of skilled labor that they prefer.

Upon paying an exogenous entry (capital) cost of $K > 0$, a high-tech firm of type $J \in [0, 1]$ can employ a continuum of skilled workers of characteristics $T \in [J - D, J + D]$ to manufacture a homogenous high-tech product $z$, where $D$ measures the endogenously determined employment distance between high-tech firms. The closer the skilled worker’s type $T$ is to $J$, which can be regarded as the particular firm’s ideal skill requirement, the more productive the match is. In general, imperfect skill matching can be represented by the distance $\delta = T - J$ away from the ideal job requirement - we will refer to $\delta$ as the skill distance in the remainder of the paper.

While skilled workers are capable of producing the food product, skill requirements are such that the high-tech commodity cannot be manufactured by the unskilled so that all high-tech firms are located in North. Since South is populated with only unskilled workers, we assume at the pre-trade stage that only food-processing firms locate in South, hiring the unskilled to produce food. Thus, the pre-trade world features complete segmentation of unskilled labor and the competitively determined wage for the unskilled in North need not be the same as that in South. Finally, skilled workers’ wages are determined via a Nash bargain based upon their matching with high-tech firms under a one-to-one matching setting specified below.

2.1 Households

Each consumer/worker is endowed with one unit of labor. All consumers are identical in every respect except for their skill types, each having a utility function specified as:

$$U = \ln(x) + \ln(1 + z)$$

(1)

An immediate consequence of this utility function specification is that while food $x$ is a necessity, the high-tech good is not (i.e., $z = 0$ is permitted). Notably, this preference specification simplifies greatly the analysis, though most of our results remain unchanged under a more general utility
functional form.\footnote{We will discuss some the main deviations of the results from a general preference specification in Section 3 below.}

For notational convenience, let the high-tech good be the numéraire.\footnote{Since determination of the skilled wage is more complex, it is simpler to take good \( z \) as the numéraire.} Then the budget constraint facing a household is written as:

\[ p_N x + z = W_M (\delta) \]  \hspace{1cm} (2)

for skilled workers in North, and

\[ p_i x + z = W_i \]  \hspace{1cm} (3)

for unskilled workers in North and South, where \( p_i \) is the relative price of good \( x \) (in units of \( z \)) in country \( i \in \{N, S\} \), \( W_i \) is the corresponding unskilled wage rate and \( W_M (\delta) \) is the wage of a skilled worker with skill distance \( \delta \) in North.

By maximizing utility specified as in (1) subject to either (2) or (3), we obtain the demand for \( x \) and \( z \) by a skilled worker with skill distance \( \delta \) and an unskilled worker in country \( i \in \{N, S\} \):

\[ x_M (\delta) = \frac{W_M (\delta) + 1}{2p_N} \]  \hspace{1cm} (4)

\[ x_i = \frac{W_i + 1}{2p_i} \]  \hspace{1cm} (5)

\[ z_M (\delta) = \frac{W_M (\delta) - 1}{2} \]  \hspace{1cm} (6)

\[ z_i = \frac{W_i - 1}{2} \]  \hspace{1cm} (7)

\subsection*{2.2 Food Production}

The food product, \( x \), is made by unskilled workers using a simple constant-returns-to-scale technology:

\[ X_i = \gamma_i L_i \quad i \in \{N, S\} \]  \hspace{1cm} (8)

where \( X_i \) denotes the aggregate supply of good \( x \) in country \( i \) and \( \gamma_i \) measures the productivity of unskilled labor in country \( i \) with \( \gamma_N \geq \gamma_S > 0 \). Since there is no intrinsic difference in unskilled
workers between the two countries, the possibility of $\gamma_N > \gamma_S$ is due purely to the presence of a peer group effect examined in Lucas (1988): unskilled workers in the North enjoy positive knowledge spillovers from the skilled workers in that country.\footnote{An alternative approach would be to adopt the Romer (1986) convention and assume that this unskilled productivity differential can be written as an increasing function of the population of the skilled in the North: $\gamma_N - \gamma_S = \Gamma (L_N)$ with $\partial \Gamma (L_N) / \partial L_N > 0$.} In the extreme case, we will consider globally homogeneous unskilled workers where $\gamma_N = \gamma_S$.

Production efficiency implies that food producers hire unskilled workers at the competitive wage rate:

$$W_i = \gamma_i p_i \quad (9)$$

### 2.3 High-Tech Good Production

High-tech good production depends crucially on how well the skilled workers match with the skill requirements imposed by high-tech firms. Consider an individual firm of type $J \in [0,1]$ in the $z$-industry employing skilled labor of type $T \in [0,1]$. The highest productivity is reached when $T = J$ and we assume that this perfect match generates a maximal productivity of $a > 0$ per worker. In general, skill matching is imperfect. If a high-tech firm of type $J$ employs a skilled worker of type $T$ with skill distance of $\delta = T - J$ the productivity of this particular employee is assumed to be $a - c\delta$, where $c > 0$ measures the productivity loss from job (or skill) mismatch.\footnote{A similar skill matching structure is constructed in Abdel-Rahman and Wang (1995) in the context of a core-periphery urban economic framework.}

Figure 1 provides a graphical description of this productivity profile. The productivity schedule $a - c\delta$ is associated with skilled labor with skill distance $\delta$ from a type-0 firm and the productivity schedule $a - 2cD + c\delta$ is the productivity schedule for the same labor with skill distance $2D - \delta$ away from a type-2D firm. These two productivity schedules intersect at point $E$ where output per worker is $a - cD$. Rationality implies that a firm of type $J$ hires only skilled workers of type $T \in [J - D, J + D]$.

(Figure 1 near here)
Thus, by symmetry, the output of a high-tech firm of type $J$ is:

$$Z(J) = 2M \int_0^D (a - c\delta) d\delta$$

(10)

where $D$ can now be thought of as the, endogenously determined, maximal distance of skill mismatch acceptable to the representative firm. Since each firm employs skilled workers over an interval of $2D$, the number of firms in the $z$-industry is $N = 1/2D$. Given the wage schedule $W_M(\delta)$, the profit of a type $J$ firm is:

$$\Pi(J) = 2M \int_0^D [(a - c\delta) - W_M(\delta)] d\delta - K$$

(11)

2.4 Nash Wage Bargain in the Skilled Labor Market

Consider the bargaining situation facing a skilled worker with skill distance $\delta$ from the nearest high-tech firm. First we need to identify a skilled worker’s best alternative or threat point. If the unskilled wage is sufficiently low the best alternative facing a skilled worker is employment by the second nearest high-tech firm. When the unskilled wage is sufficiently high, however, a skilled worker’s best alternative becomes potential employment in the food industry and we assume the skilled to be capable of producing food with the same productivity as the unskilled in North. In order to be compatible with conventional studies, we do not allow any individual to influence the value of outside alternatives (i.e., all individuals are market-value takers). Throughout the analysis we impose:

**Assumption 1:** (Skilled Productivity) $a - cD > W_N$.

Assumption 1 ensures that all skilled workers will be employed in high-tech firms.

Now consider the nontrivial case where the minimum productivity from being employed by the second nearest high-tech firm is strictly less than the unskilled wage in the North but strictly greater than the unskilled wage in the South:$^8$

**Condition W:** (Unskilled Wage in the North) $W_N > a - 2cD > W_S$.

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$^8$ Once the equilibrium solution is obtained Assumption 1 and Condition W can be rewritten in primitives.
Provided that Condition W holds there is a unique critical point $D_c$ at which the potential output for a skilled labor working with the second nearest high-tech firm is equal to the unskilled wage (or the potential output per worker in units of goods $z$), that is, $W_N = a - 2cD + cD_c$. Under Condition W, we have:

$$D_c = \frac{W_N - (a - 2cD)}{c} \in [0, D] \quad (12)$$

For $\delta < D_c$ the best alternative facing a skilled worker with skill distance $\delta$ is $W_N$ whereas for $\delta > D_c$ it is $a - 2cD + c\delta$.

To obtain the symmetric Nash wage bargaining solution, we follow the bilateral bargaining literature to specify the total potential gains accrued from matching:\footnote{Our approach follows the symmetric Nash bargain in Diamond (1982). One may allow for an asymmetric split of surplus as in Burdett and Mortensen (1981) and Pissarides (1984) without altering the main results. Alternatively, one may use Rubinstein’s (1982) repeated alternating offer game, whose limiting case would generate our expression.}

$$G(\delta) = \begin{cases} 
a - c\delta - W_N & \delta \leq D_c \\
2c(D - \delta) & \delta > D_c 
\end{cases} \quad (13)$$

Using this, we can derive the skilled wage for a type-$\delta$ worker, which is the sum of the relevant best alternative and half of the total potential gain:

$$W_M(\delta) = \begin{cases} 
\frac{(a - c\delta + W_N)}{2} & \delta \leq D_c \\
\frac{a - cD}{2} & \delta > D_c 
\end{cases} \quad (14)$$

The use of the symmetric sharing rule is purely for the sake of simplicity. In general, we could allow for asymmetric sharing, yet we must assume that skilled workers would have at least half of the total potential gain (otherwise, skilled wages will be decreasing in the perfectness of the matches - which is obviously counter-intuition).\footnote{We will discuss the implications when skilled workers obtain a larger share than high-tech firms at the end of Section 3.} By exactly the same argument, it follows that if $W_N < a - 2cD$, so that Condition W does not hold, the skilled wage in North is

$$W_M(\delta) = a - cD \quad (15)$$

Figure 2 illustrates the determination of the skilled wages.
(Figure 2 near here)

Equations (14) and (15) indicate that when Condition W holds there will be wage disparity among skilled workers. Skilled workers with skill distances less that \( D_c \) are able to strike a better wage bargain because their best alternative is employment in a relatively high-wage agricultural sector rather than in a type-2D high-tech firm in which they would be relatively unproductive.

3 Equilibrium

We focus upon two types of equilibrium: (i) an equilibrium without trade (EA), and (ii) an equilibrium with free trade in the absence of unskilled labor mobility (ET). For the moment, we assume that free trade is associated with no trade barriers. We show later that our results in the case of free trade can be extended to the case of trade with frictions or transport costs, provided that such frictions or costs are sufficiently small compared to the gap between the minimum productivity from being employed by the second most adjacent firm in the North and the unskilled wage in the South.

3.1 Autarchy Equilibrium

We first examine the autarchy equilibrium (EA). The equilibrium in South is straightforward. From (5) aggregate demand for x is \( X_d^S = L_S (W_S + 1) / 2p_S \) while aggregate supply is \( X_s^S = \gamma_S L_S \). These equations and (9) give the equilibrium unskilled wage and the food price in South, \( W_S = 1 \) and \( p_S = 1/\gamma_S \).

Now consider North. In equilibrium we must have that the x market clears, the z market clears and, by free entry, each firm earns zero profit in equilibrium. By Walras Law we need only consider the latter two conditions. We need also to distinguish between the cases in which Condition W does or does not hold, that is, in which the unskilled wage does or does not impose a binding constraint upon the skilled wage.
3.1.1 Unskilled Wage Non-Binding

When the unskilled wage in North does not bind the skilled wage (i.e., Condition W does not hold), the zero-profit condition can be written, from equations (11) and (15), as:

$$0 = \Pi(J) = 2M \int_0^D [cD - c\delta] \, d\delta - K$$

which gives the condition:

$$\bar{D}^a = \sqrt{\frac{K}{cM}}$$

Aggregate demand for the high-tech product by skilled workers in each high-tech firm is $z(J) = 2M \int_0^D z_M(\delta) \, d\delta$. There are $1/2D$ such firms, so that aggregate demand for the high-tech product by skilled workers is, from equation (15), $Z_M(J) = M(a - 1 - cD)/2$. Aggregate demand for $z$ is this demand plus demand for $z$ by unskilled workers in North, $L_N(W_N - 1)/2$ plus demand for $z$ to cover the fixed costs of the high-tech firms, $K/2D$. Substituting from (17) gives aggregate demand for $z$ in North:

$$Z_N^d = ((a - 1)M + (W_N - 1)L_N)/2$$

(18)

Aggregate supply of $z$ in North is, from (10) and (17):

$$Z_N^s = aM - \sqrt{cKM}/2$$

(19)

It follows from (18) and (19) that the equilibrium unskilled wage when Condition W does not hold is:

$$\tilde{W}_N = 1 + \frac{(a + 1)M - \sqrt{cKM}}{L_N}$$

(20)

Substituting this into (9) then gives the relative price of $x$ in North:

$$\tilde{p}_N^B = \tilde{W}_N / \gamma_N$$

(21)

We need to check the consistency of this solution. Define

$$\alpha = \frac{L_N + M + c\bar{D}^a(2L_N - M)}{L_N - M}$$

(22)

which is strictly positive if $L_N > M$. We then have:
**Theorem 1** There exists a unique autarchy equilibrium \( \{\bar{D}^a, \bar{W}_N^a, \bar{p}_N^a\} \) given by (17), (20) and (9) in which the unskilled wage does not bind the skilled wage in North, provided that \( a > \bar{\pi} \) and \( L_N > M \).

**Proof.** For Condition W not to hold requires \( W_N < a - 2c\bar{D}^a \). Substituting from (20) and simplifying gives the inequality \( a > \bar{\pi} \) where \( \bar{\pi} > 0 \) provided that \( L_N > M \). For demand and supply to be consistent requires \( Z^a_N|_{W_N=1} < Z^a_N \), which is true provided that \( a > c\bar{D}^a - 1 \), a condition that is automatically satisfied under \( a > \bar{\pi} \). 

The intuition underlying Theorem 1 is simple enough. \( L_N > M \) is necessary to ensure that Assumption 1 holds, so that all skilled workers are employed in the high-tech sector. This of itself, however, is not sufficient to ensure that Condition W does not hold. For this to be the case it is necessary that the maximum skilled labor productivity be “sufficiently high” that the equilibrium unskilled wage in North does not impose an effective constraint on skilled wage bargaining. Note further that \( \partial\bar{\pi}/\partial c > 0, \partial\bar{\pi}/\partial K > 0, \partial\bar{\pi}/\partial L_N < 0 \) and \( \partial\bar{\pi}/\partial M > 0 \) for \( M \) “large enough”. In other words, wage bargaining in the skilled sector is less likely to be constrained by the unskilled wage if skilled labor productivity declines slowly with skill distance, if fixed costs are low (so that the maximum skill distance is small), and if unskilled (skilled) labor supply is relatively plentiful (scarce).

#### 3.1.2 Unskilled Wage Binding

When the unskilled wage in North binds the skilled wage (i.e., Condition W holds), we use equations (11) and (14) to obtain the zero-profit condition as:

\[
0 = \Pi(J) = 2M \int_0^{D_v} \left( a - c\delta \right) - \frac{1}{2} (a - c\delta + W_N) d\delta + 2M \int_{D_v}^{D} \left( (a - c\delta) - (a - cD) \right) d\delta - K
\]

(23)

Evaluating the integrals, substituting from (23) and simplifying gives the condition under autarchy (ZP^a):

\[
(a - W_N)(4cD - (a - W_N)) - 2c^2D^2 = 2cK/M
\]

(24)
which can be solved for \( W_N \) to give:

\[
W_N (D) = a - 2cD + \sqrt{\frac{2c(cD^2M - K)}{M}}
\]  
(25)

It is easy to confirm that, when Condition W holds,

\[
\frac{dW_N}{dD} \bigg|_{ZP_a} > 0
\]  
(26)

The zero-profit condition (24) describes an upward-sloping locus in \([D, W_N]\) space. In other words, the equilibrium employment range of high-tech firms in a closed economy is strictly increasing in the outside alternative measured by the unskilled wage in North. When the unskilled wage increases, high tech firms’ bargaining power reduces and their profits decrease. As a result, some high-tech firms exit and the equilibrium employment range for the surviving high-tech firms increases.

The zero-profit condition also requires further parameter restrictions. The upper limit on \( D \) is \( 1/2 \) and we also have that the unskilled wage should satisfy Assumption 1, \( W_N < a - cD \), otherwise there would be no interaction between high-tech firms in their demand for skilled labor.

**Assumption 2** (skill distance) The skill distance is such that \( D \in \left[ \hat{D}^a, \sqrt{2} \hat{D}^a \right] \).

We then have:

**Lemma 1** Under Assumption 2, the zero profit condition satisfies \( a - 2cD \leq W_N^{ZP_a} (D) \leq a - cD \).

**Proof.** From (25) \( a - 2cD \leq W_N^{ZP_a} (D) \leq a - cD \) if and only if \( \sqrt{\frac{2c(cD^2M - K)}{M}} \in [0, cD] \), which is true under Assumption 2.  

Now consider the market-clearing condition in the high-tech market. Aggregate demand for \( z \) by skilled workers in North is now \( M \left[ \int_0^{D_c} z_M (\delta) \, d\delta + \int_{D_c}^D z_M (\delta) \, d\delta \right] /D \) where the skilled wage is given by (14). Evaluating the integrals gives aggregate demand for \( z \) by skilled workers, unskilled workers and to cover the fixed costs of the high-tech firms as:

\[
Z_N^{da} (W_N, D) = \frac{M (a - W_N)^2 + 4c (D (M + L_N) (W_N - 1) + K)}{8cD}
\]  
(27)
It is easy to confirm that, when Assumption 1 and Condition W hold, \( \partial Z_N^{da}(W_N, D) / \partial W_N > 0 \) and \( \partial^2 Z_N^{da}(W_N, D) / \partial W_N^2 < 0 \) so that \( Z_N^{da}(W_N, D) \) is an increasing, concave function in \([Z, W_N]\) space for any given skill distance \( D \). Aggregate supply of \( z \) is \( Z(J) / 2D \), which from (10) is

\[
Z_N^{sa}(W_N, D) = M \left( a - \frac{cD}{2} \right) \tag{28}
\]

Equations (27) and (28) define a material balance condition in the high-tech market \( MB^a \),

\[
Z_N^{da}(W_N, D) - Z_N^{sa}(W_N, D) = 0 \tag{29}
\]

and it is easy to confirm that, when Condition W holds \( \frac{dW_N}{dD}|_{MB^a} > 0 \). The \( MB^a \) condition describes an upward-sloping locus in \([D, W_N]\) space. Aggregate demand in North for the high-tech good is increasing in the unskilled wage and decreasing in the skill distance while aggregate supply is independent of the unskilled wage and decreasing in the skill distance. If the unskilled wage increases there will be an increase in the demand for \( z \). Reducing the skill distance increases supply but, since it is easy to show that \( \partial Z_N^{da}(W_N, D) / \partial D < \partial Z_N^{sa}(W_N, D) / \partial D < 0 \), would increase demand even more. The only way to restore material balance, therefore, is to increase the skill distance, which is equivalent to the exit of high-tech firms.

Since \( ZP^a \) and \( MB^a \) are both upward-sloping in \([D, W_N]\) space it would appear that confirming the existence of an autarchy equilibrium might pose some problems. However, rather than working with the \( ZP \) and \( MB \) conditions we can create an alternative form of the material balance condition

\[
\frac{2}{M} \Pi (J) + 8cD \left( D_N^{N^a}(W_N, D) - Z_N^{sa}(W_N, D) \right) = 0
\]

which, when solved for \( W_N \) gives:

\[
W_N^{MB^a}(D) = 1 + \frac{DM (2 + 2a - cD) - K}{2DL_N} \tag{30}
\]

It is easy to verify that

\[
\frac{dW_N^{MB^a}(D)}{dD} < 0 \tag{31}
\]

so that this alternative material balance condition defines a downward-sloping locus in \([D, W_N]\) space. Moreover, it is trivial that if \( ZP^a \) holds then this alternative material balance condition implies \( MB^a \). From (26) and (31) it follows that in principle, under Assumptions 1 and 2,
an autarchy equilibrium satisfying Condition W exists. Although equations (25) and (30) are too complex to admit of an explicit solution, this autarchy equilibrium without trade is actually similar to the core-periphery equilibrium established in Abdel-Rahman and Wang (1997). From that analysis we know that an autarky equilibrium satisfying Condition W exists but, as in the previous section, there are parameter restrictions that are necessary to guarantee such existence.

For the zero-profit condition to be consistent with Assumption 1 it is necessary that $W_{N}^{ZPa} (D) \leq a - cD$, i.e., Assumption 2 holds. For there to be an autarchy equilibrium satisfying Assumptions 1 and 2 as well as Condition W, it is therefore necessary that $W_{N}^{MBa} \left(\sqrt{2}\bar{D}^{a}\right) < W_{N}^{ZPa} \left(\sqrt{2}\bar{D}^{a}\right)$ and that $W_{N}^{MBa} \left(\bar{D}^{a}\right) > W_{N}^{ZPa} \left(\bar{D}^{a}\right)$. Define
\[
\alpha = \frac{L_{N} + M}{L_{N} - M} + \frac{(4L_{N} - 3M)}{2\sqrt{2}(L_{N} - M)} \tag{32}
\]
We then have:

**Lemma 2** Suppose that Assumptions 1 and 2 and Condition W hold. Then $W_{N}^{MBa} \left(\sqrt{2}\bar{D}^{a}\right) < W_{N}^{ZPa} \left(\sqrt{2}\bar{D}^{a}\right)$ if $a > \alpha$ and $L_{N} > M$.

**Proof.** Define $\Delta W_{N} (a, D) = W_{N}^{ZPa} (D) - W_{N}^{MBa} (D)$. Then substituting from (17) gives $\Delta W_{N} (a, \sqrt{2}\bar{D}^{a}) = \left(4\left(L_{N} (a - 1) - M(a + 1)\right) - 4\sqrt{2}\bar{D}^{a}(4L_{N} - 3M)\right)/4L_{N}$. It follows that $\partial\Delta W_{N} (a, D)/\partial a = 1 - M/L_{N} > 0$ iff $L_{N} > M$. This implies that there is a lower limit on $a$ above which $\Delta W_{N} (a, \sqrt{2}\bar{D}^{a}) > 0$. Solving $\Delta W_{N} (a, \sqrt{2}\bar{D}^{a}) = 0$ for $a$ and simplifying gives (32). ■

**Lemma 3** $W_{N}^{MBa} (\bar{D}^{a}) - W_{N}^{ZPa} (\bar{D}^{a}) > 0$ if $a \in (\alpha, \bar{a})$.

**Proof.** Simple substitution gives $\Delta W_{N} (\bar{a}, \bar{D}^{a}) = 0$. Also, $\partial\Delta W_{N} (a, D)/\partial a > 0$. ■

Lemmas 1-3 then imply:

**Theorem 2** Under Assumptions 1 and 2, there exists a unique autarchy equilibrium $EA = \{D^{a}, W_{N}^{a}, P_{N}^{a}\}$ in North satisfying (25),(30) and (9) with the unskilled wage binding the skilled wage in North, provided that $a \in (\alpha, \bar{a})$ and $L_{N} > M$.

Whether or not the unskilled wage imposes a binding constraint on the skilled wage, it follows from (20) and (30) that in autarchy the unskilled wage is higher in North than in South.
3.2 Equilibrium with Free Trade and Immobile Labor

Consider now an integrated North-South world economy in which there is free trade but no labor mobility and the equilibrium (ET) that such an economy might sustain. In the absence of frictions or transport costs, trade results in price parity for the agricultural good $x$, that is, $p_N = p_S = p$. However, in contrast to a Samuelsonian perfectly competitive world of homogeneous labor where factor price equalization would hold, our skill-matching framework with heterogenous workers will result in unskilled wage disparity since the unskilled wages depend on the unskilled workers’ productivity as well as the equilibrium relative price of the food product (in units of the high-tech good).

The impact of free trade on South is easily described. South exports the agricultural good in return for the high-tech good, so that trade increases aggregate demand for $x$ in South. Since the aggregate supply of $x$ is perfectly inelastic, this leads to an increase in the $x$ price in South and, from (9), an increase in the unskilled wage.

Now consider North. Once again, in equilibrium we must have that the $x$ market clears, the $z$ market clears and that high-tech firms break even. Further, we need to distinguish between cases where the unskilled wage imposes a binding constraint on high-tech wage bargaining and cases where it does not.

3.2.1 Unskilled Wage Non-Binding

When the unskilled wage in North is non-binding (i.e., Condition W does not hold), the zero-profit condition for high-tech firms in North is unaffected by trade, so that equations (16) and (17) continue to characterize the equilibrium. By contrast, aggregate demand for $z$ is affected since $z$ is now exported to South. Aggregate demand for $z$ in North is, from (7), (9) and (18)

$$Z_N^d = ((a - 1) M + (p \cdot \gamma_N - 1) L_N + (p \cdot \gamma_S - 1) L_S) / 2$$

(33)
Aggregate supply of \( z \) is given by (19). Equating demand and supply gives the equilibrium post-trade price of \( x \) as:

\[
\bar{p}_N^t = \frac{L_N + L_S}{L_N \gamma_N + L_S \gamma_S} + \frac{(a + 1) M - \sqrt{cKM}}{L_N \gamma_N + L_S \gamma_S}
\]  

(34)

We then have:

**Theorem 3** Under Assumptions 1 and 2, if Condition W does not hold before and after integration of the world economy then free trade with no labor mobility reduces the unskilled wage and the food price but raises the welfare of the unskilled in North.

**Proof.** Denote the autarchy and post-trade food prices by \( p^a \) and \( p^t \) respectively. The autarchy equilibrium in the \( z \) market requires, from (18) and (19)

\[
((a - 1) M + (p^a \gamma_N - 1) L_N) / 2 = aM - \sqrt{cKM} / 2
\]

while the post-trade equilibrium requires

\[
((a - 1) M + (p^t \gamma_N - 1) L_N + (p^t \gamma_S - 1) L_S) / 2 = aM - \sqrt{cKM} / 2
\]

(36)

It follows that

\[
p^a \gamma_N L_N = p^t \gamma_N L_N + (p^t \gamma_S - 1) L_S
\]

(37)

so that

\[
p^a - p^t = \frac{(p^t \gamma_S - 1) L_S}{\gamma_N L_N} > 0
\]

(38)

Thus, the unskilled wage, \( \gamma_N p_N \), in North must also decrease as a result of free trade. Moreover, from the utility function and the demand functions, indirect utility for an unskilled worker is

\[
v(p_N) = 2 \ln \left( \frac{\gamma_N p_N + 1}{4p_N} \right)
\]

It is obvious that \( v \) is decreasing in \( p_N \) so that the utility of unskilled workers in North increases with free trade. ■

The intuition behind this result is simply explained. Opening North and South to trade increases aggregate demand for the high-tech good in North to meet import demand in South. However, given that Condition W does not hold the aggregate supply of the high-tech good
is perfectly inelastic as a result of which the increased demand places an upward pressure on the high-tech price. Since this price is the numeraire, this means that the relative price of food must fall, diverting demand by unskilled workers in North from the high-tech good to food consumption. The fall in the relative price of food leads directly to a reduction in the unskilled wage rate. Under the log-linear utility where the income and substitution effects exactly offset each other, the resulting increase in consumption implies that the utility of unskilled workers in North increases despite the narrowing in the North-South wage differential. Although this result concerning the welfare of the unskilled in North depends crucially on the utility functional form, the findings regarding the responses of prices, wages and inequalities to trade are all qualitatively unchanged under a general constant-elasticity-of-substitution preference specification.

Since trade reduces the unskilled wage rate in North we can utilize Theorem 1 to conclude:

**Theorem 4** Under Assumptions 1 and 2, a free trade equilibrium without labor mobility exists in which the unskilled wage does not bind the skilled wage in North.

Straightforward differentiation of equations (9) and (34) yields:

**Theorem 5** Under Assumptions 1 and 2, if Condition W does not hold before and after integration of the world economy then the unskilled wage in North under free trade with no labor mobility increases with the skilled productivity in North and the population of the unskilled in South. If it is further assumed that the North-South unskilled productivity differential is sufficiently small, then the unskilled wage in North under free trade also increases with its own population.

**Proof.** The result concerning the skilled productivity measured by $a$ is trivial, as a higher value of $a$ increases the relative price of $x$ under free trade and thus the unskilled wage in North. Let $\psi_i$ ($i = N, S$) be an indicator function with $\psi_N = 1$ and $\psi_S = 0$. Differentiating $\tilde{p}_N^i$ with respect to $L_i$ gives $\frac{\partial \tilde{p}_N^i}{\partial L_i} = \frac{(\gamma_N - \gamma_S)[(1-\psi_i)L_N-\psi_iL_S]+[(a+1)M-\gamma_N\lambda]}{(L_N\gamma_N+L_S\gamma_S)^2}$, which is always positive for $i = S$ and is positive for $i = S$ when $\gamma_N - \gamma_S \to 0$. □

The positive effect of the skilled productivity on the unskilled wage in North is as expected, consistent with findings in Grossman and Helpman (1992) and Stokey (1991). The results concerning the influences of population size deserve further comments. As in Stokey, an increase in the population in South raises the wage in North, as a result of a higher food price under free
trade caused by a greater demand. However, this in turn lowers the welfare of the unskilled in North. When the population of the unskilled in North increases, there is a peer-group induced North-South productivity differential effect through which the free trade food price as well as the unskilled wage in North tend to reduce, thus countering the positive demand effect on the food price and generating an ambiguous effect on the unskilled wage in North.

3.2.2 Unskilled Wage Binding

Now suppose that the unskilled wage imposes a binding constraint (i.e. Condition W holds). As in the previous section, the zero-profit condition for the high-tech firms in North is unaffected\(^{11}\) being characterized in this case by (24) so that, from (9) we have the zero-profit condition

\[
p_{N}^{2dP} (D) = \frac{a - 2c D}{\gamma_N} + \frac{1}{\gamma_N} \sqrt{\frac{2c (cD^2 M - K)}{M}} \tag{39}
\]

Aggregate demand for \(z\) is, from (7), (9) and (27) given by

\[
Z^{d^t} (p, D) = \frac{M (a - p \gamma_N)^2 + 4c (D ((M + L_N) (p \gamma_N - 1) + (p \gamma_S - 1) L_S) + K)}{8c D} \tag{40}
\]

while aggregate supply of \(z\) is, as before, given from (28) as \(Z^{s^t} (p, D) = M (a - \frac{4c D}{2})\). Equations (40) and (28) define a material balance condition in the high-tech market (\(MB^t\)):

\[
Z^{d^t} (p, D) - Z^{s^t} (p, D) = 0 \tag{41}
\]

As in the autarky case, we can define an alternative material balance equation for the \(z\) market given by \(\frac{2c}{3} \Pi^t (J) + 8c D \left( Z^d (p, D) - Z^s (p, D) \right) = 0\). Substituting, simplifying and solving for the relative price of food gives

\[
p_{N}^{MB^t} (D) = \frac{L_N + L_S}{L_N \gamma_N + L_S \gamma_S} + \frac{DM (2 + 2a - cD) - K}{2D (L_N \gamma_N + L_S \gamma_S)} \tag{42}
\]

While comparison of (42) and (30) would appear to be ambiguous, we can, in fact, state the following:

\(^{11}\) This assumes, of course, that the unskilled wage in North is not driven so low that Condition W does not hold, an assumption that leaves our qualitative conclusions unaffected.
Theorem 6 If there exists an autarchy equilibrium $EA = \{D^a, W_N^a, p_N^a\}$ in North with the unskilled wage binding the skilled wage, then a free trade equilibrium without labor mobility $ET = \{D^f, W_N^f, p_N^f\}$ also exists satisfying: $D^f < D^a, W_N^f < W_N^a$ and $p_N^f < p_N^a$.

**Proof.** Define $Z^a(p^a, D) = \frac{M(a - p^a\gamma_N)^2 + 4\epsilon(D(M + LN)(p^a\gamma_N - 1) + K)}{8\epsilon D}$. Then we can rewrite the material balance equation in autarchy as:

$$MB^a : Z^a(p^a, D) = M \left( a - \frac{cD}{2} \right) \tag{43}$$

Define $Z^t(p^t, D) = \frac{M(a - p^t\gamma_N)^2 + 4\epsilon(D((M + LN)(p^t\gamma_N - 1) + (p^t\gamma_N - 1)L_N) + K)}{8\epsilon D}$. Then the material balance equation with trade becomes:

$$MB^t : Z^t(p^t, D) = M \left( a - \frac{cD}{2} \right) \tag{44}$$

Suppose that $p^a = p^t$ for a given $D$. Then it is clear that $Z^t(p^t, D) > Z^a(p^a, D)$ so that equations (43) and (44) cannot both hold. Now suppose that $p^a \neq p^t$. We know that $\partial Z^t(p^t, D) / \partial p^t > 0$ so for (43) and (44) both to hold for a given $D$ it must be the case that $p^t < p^a$. It follows that $MB^t$ lies below $MB^a$ in $[D, p]$ space so that if $MB^a$ intersects the ZP equation in autarchy then $MB^t$ will intersect the ZP equation with free trade. Since $p^t < p^a$ it follows from (9) that $W_N^f < W_N^a$. Finally, since the ZP equation is upward-sloping in $[D, p]$ space, a reduction in $p$ must also lead to a reduction in $D$. \[ \square \]

Figure 3 illustrates Theorem 6. Opening North and South to trade increases aggregate demand for $z$ in North. To meet this additional demand requires an increase in supply, which requires a decrease in the skill distance for the high-tech firms and reconfirms that trade leads to international production specialization. For this to be consistent with the free-entry, zero-profit condition requires that the total wage costs of each high-tech firm falls. It is clear from Figure 2 that a reduction in $D$ increases the wage paid to high-tech workers whose best outside opportunity is the second-nearest high-tech firm - workers more than skill distance $D_c$ from their current employer. It is necessary, therefore, that the food price $p$ and the agricultural wage rate $W_N$ fall since this reduces the best outside opportunity for high-tech workers within skill distance $D_c$ of
their current employer. Provided that $W_N$ falls sufficiently, the latter effect more than offsets the former, with the result that trade leads to the entry of additional high-tech firms but a reduction in the agricultural wage rate in North, consistent with findings under free trade with the unskilled wage not binding the skilled wage in North (see Theorem 3). Moreover, by examining equation (42), it is straightforward that the results in Theorem 5 remain qualitatively unchanged.

(Figure 3 near here)

Theorem 6 also allows us to comment on the impact of trade on wage inequalities. Trade reduces the maximum skill distance $D$ and lowers the unskilled wage rate, as a result of which trade reduces the within-the-skilled-group wage inequality as illustrated in Figure 4. This finding is entirely new to the literature on North-South trade and integration. It is not the case, however, that all skilled workers benefit from trade. As Figure 4 illustrates, workers with pre-trade skill distances greater than $D_o$ certainly benefit from trade. By contrast, those with skill distances near zero find that their wage rates fall. The reasoning is simple to see. For the former group trade makes the second-best alternative - employment in a type-$2D$ firm - more competitive and so improves their bargaining position, while for the latter, trade makes the second-best alternative - the agricultural wage - less competitive and so worsens their bargaining position. Importantly, since all skilled workers in our economy are on equal ground in the quality sense, those who are better matched are not more skilled - they are worse-off because their gains from luck (in matching) are lower as free trade creates greater parity in outside alternatives among skilled workers.

(Figure 4 near here)

In summary, whether Condition W holds or not, free trade always narrows the inequality in unskilled wages between North and South. If Condition W does not hold then trade unambiguously widens unskilled/skilled wage disparities in North, an outcome that is consistent with much of the available evidence. By contrast, if Condition W holds, wage inequalities between skilled and unskilled workers in North generally increase only for skilled workers being employed
at sufficiently large skill distances. Since the unskilled wage is more likely to bind in middle-income countries, the result obtained by Fischer and Serra (1996), that developed countries are more likely to vote for protection against middle- than low-income countries, may be modified if between-the-skilled-and-unskilled equity is a concern in North.

We are now ready to conclude:

**Theorem 7** Under Assumptions 1 and 2, free trade without labor mobility reduces between-North-and-South inequality of the unskilled. If Condition W does not hold before and after integration of the world economy, then free trade without labor mobility widens the between-skilled-and-unskilled inequality in North without altering the within-the-skilled-group wage inequality. If Condition W holds before and after integration of the world economy, then free trade without labor mobility narrows the within-the-skilled-group wage inequality while leading to an ambiguous change in the between-skilled-and-unskilled inequality in North.

Finally, we would like to discuss briefly the case where skilled workers obtain a larger share of the total potential gains from matching than high-tech firms. This case may be regarded as one with a stronger (economy-wide) labor union, as compared to the benchmark case above. A direct consequence of this is the presence of the within-the-skilled group wage inequality even when the unskilled wage in North does not bind. Yet, such an inequality would not be affected by free trade. When the unskilled wage in North binds, free trade would still narrow the within-the-skilled group wage inequality as in the case with a symmetric sharing rule.

4 Extensions

Our analysis thus far has assumed, in particular, that there are no trade frictions between North and South. The impact of introducing such frictions in the form, for example, of positive transport costs on the exports of North, South or both is easily identified.

Suppose that transport costs are of Samuelson’s “iceberg” type and apply solely to the high-tech good: so that if a quantity Z is exported an amount $\tau Z$ is received in South, with $0 < \tau < 1$. Then they will adversely affect the profitability of the high-tech firms in North since some of the output of each firm is effectively lost. This moves the ZP locus in Figure 3 to the right, as a result of which such trade frictions lead to an increase in the skill distance $D^t$, a decrease in the
unskilled wage rate $W^R_N$ in North and the agricultural price $p^t$, as a result of which they lead to also lead to a decrease in the unskilled wage rate in South.

If, by contrast, the transport costs apply to the agricultural good they serve to drive a wedge between the agricultural price in North and South. In these circumstances, if the transport costs are high enough they might actually serve to cut off trade altogether. More generally, such trade frictions mean that the agricultural price in North will be higher and in South will be lower than the agricultural price in the frictionless ET equilibrium. As a result, there is an upward pressure on wage bargaining in the high-tech firms with the result that opening to trade will lead to less entry of new high-tech firms as compared with the ET equilibrium.

5 Conclusions

We have developed a two-sector general-equilibrium model of North-South trade featuring a high-tech sector in North that depends crucially on horizontal skill matching, in addition to a food-processing sector that can be operative in both countries. Our analysis establishes conditions under which an autarchy equilibrium and a free trade equilibrium without labor mobility exist. We have also shown that the nature of this equilibrium and the impact of trade are affected by whether or not the unskilled wage offers an effective outside alternative in the matching framework, influencing the outcome of the skilled wage bargaining process in North. We have analyzed the effect of bilateral free trade agreement on wages, income inequalities and social welfare in each country. In particular, we have shown that whether the unskilled wage is binding or not, free trade with no labor mobility always reduces the unskilled wage and the food price but raises the welfare of the unskilled in North. In addition, the unskilled wage in North under free trade increases with the skilled productivity in North and the population of the unskilled in South.

With respect to equity issues, free trade tends to narrow between-North-and-South inequality of the unskilled. When the unskilled wage does not bind the skilled wage in North, free trade widens the between-skilled-and-unskilled inequality in North while leaving the within-the-skilled-
group wage inequality unaffected. By contrast, when the unskilled wage does bind the skilled wage in North, free trade narrows the within-the-skilled-group wage inequality while resulting an ambiguous effect on the between-skilled-and-unskilled inequality in North. The skilled workers who are most likely to lose from trade are those whose best alternative to employment with their current high-tech firm is employment in the unskilled sector, i.e. relatively “captive” skilled workers. Those most likely to gain are the skilled workers whose best alternative is employment in another high-tech firm.

Given these results, it is natural to inquire from a normative point of view whether it is beneficial to adopt a redistributive policy reallocating from the skilled to the unskilled in North under the free trade regime. For example, this may be done by a progressive income tax and a subsidy to the unskilled. Our analysis makes clear that the effectiveness of this redistributive policy will depend crucially on skill matching and the bargaining threat points facing the skilled workers.

Looking forward to future research, it would be interesting to incorporate horizontal skill matching in the labor market into the conventional vertical product differentiation model of North-South trade in a two stage game. This would enable us to study the interactions between skill matching selection by firms and quality improvements in production over time, as well as their implications for the patterns of North-South trade in the short- and long-run.
References


Figure 1: Skill Matching and Productivity in the High-Tech Industry

Figure 2: Skilled Wage Determination
Figure 3: Impact of Trade on Equilibrium

Figure 4: Impact of Trade on Skilled Wage