Credit Shocks and Equilibrium Dynamics in Consumer Durable Goods Markets

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Abstract

This paper studies the equilibrium dynamics in consumer durable goods markets after aggregate credit shocks. We introduce two novel features into a general-equilibrium model of durable consumption with heterogeneous households facing idiosyncratic income risk and borrowing constraints: (1) different qualities of durable goods trade on secondary markets at market-clearing prices; and (2) households endogenously choose when to trade them or scrap them. The model successfully matches several empirical patterns that we document using data on U.S. car markets around the Great Recession. After a tightening of the borrowing limit, debt-constrained households postpone the decision to scrap and upgrade their low-quality cars, depressing mid-quality car prices. In turn, this effect reduces wealthy households’ incentives to replace their mid-quality cars with high-quality ones, thereby decreasing new-car sales. We further use our framework to study the effects of collateral constraints and aggregate income shocks, and to evaluate targeted fiscal stimulus policies such as the Car Allowance Rebate System in 2009 (“Cash for Clunkers”).

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1 Introduction

Expenditures on consumer durable goods are a large, highly volatile, and procyclical component of GDP. Car markets around the Great Recession represent a stark example of this volatility: Figure 1 shows that new-car registrations in the U.S. dropped from approximately 20 million in 2007 to 12 million in 2009, and recovered to 20 million by 2013.

The objective of this paper is to study the equilibrium dynamics of consumer durable goods markets in response to macroeconomic shocks, with a special focus on car markets. We set off our analysis by examining different data sources in order to gain a broad picture of household vehicle holdings and of aggregate dynamics in car markets. These data reveal several interesting patterns that complement the drop in new-car sales during the Great Recession displayed in Figure 1, most notably: (1) vehicle scrappage declined from 2007 to 2009; (2) the cost of replacing a used car with a new one increased from 2007 to 2009, as used-car prices dropped and new-car prices remained quite stable; and (3) the fraction of households who replaced a used vehicle with a new one declined during the Great Recession.

These facts motivate us to develop a macroeconomic model of durable consumption that features a notion of endogenous illiquidity, stemming from equilibrium dynamics in secondary markets. We use this novel framework to analyze the transmission mechanism of macroeconomic shocks to durable-goods purchases. The model includes the key elements to study this transmission: it allows for borrowing constraints that may affect households’ vehicle holdings; it incorporates transaction costs that may trigger household inertia; and it is set in general equilibrium, because households’ car purchases depend both on interest rates and on car prices, which displayed large fluctuations in the Great Recession.

Our model is an incomplete-markets framework with uninsurable income shocks and durable adjustment. Durable goods feature two additional characteristics that are important to account for their equilibrium dynamics. First, the model features a quality ladder for durable goods: cars differ in quality, because of depreciation, and thus are imperfect substitutes. Second, households choose when to replace their cars by trading them at market-clearing prices on secondary markets or by scrapping them.

We parametrize the model to match several aggregate statistics of car markets and household-level cross-sectional moments on car ownership in the U.S., as well as empirical targets about household income and wealth. Our economy features a negative correla-
tion between wealth and car quality, consistent with evidence that Gavazza, Lizzeri, and Roketskiy (2014) report. Wealthier households tend to own new, high-quality cars; when the quality of these cars depreciates, wealthy households sell them and replace them with newer, higher-quality ones. Low-wealth households tend to own old, low-quality cars, scrapping them when their quality deteriorates further and replacing them with mid-quality, used cars. Thus, secondary markets play the fundamental role of reallocating used cars from higher-income households to lower-income ones.

In this setting, we consider a shock that permanently tightens credit limits, as in other recent papers that propose this shock as a plausible exogenous source of macroeconomic dynamics consistent with the experience of the Great Recession (e.g., Guerrieri and Lorenzoni, 2017; Huo and Ríos-Rull, 2016). Also as in these papers, a tighter borrowing constraint advises all households to increase their savings, leading to a sharp decline in the equilibrium real interest rate. Importantly, in our model, low-wealth households, for whom the borrowing constraints becomes tighter, decide to postpone the scrappage of their old, low-quality vehicles. Because these low-wealth households are natural buyers of mid-quality, used cars, their decision to postpone scrappage leads to a decrease in the demand for these mid-quality cars, thereby lowering their price. Hence, high-wealth households, who normally trade in moderately used cars to replace them with new, high-quality ones, suffer an increase in the replacement cost of their vehicles and decide to delay their replacement. Thus, even though the change in the borrowing limits does not directly affect these wealthy
households, the equilibrium dynamics in secondary markets prompt them to postpone their new-car purchases. In our calibration, this negative feedback effect of secondary markets on the primary market quantitatively dominates the positive effect of low interest rates on purchases of new durable goods. Hence, the model predicts a large decrease in new-car sales, as well as in scrappage and used prices, consistent with the empirical evidence we document.

The distinctive feature of our model is the endogenous illiquidity of durable goods, arising from the equilibrium dynamics in secondary markets. In our framework, the price of used durables falls in response to a credit tightening, that is, when the marginal value of liquidity is highest. This equilibrium effect makes durable goods a poor store of value and amplifies their cyclical dynamics. Importantly, this mechanism is key to account for the positive comovement of scrappage and new sales observed in the data. We show that this implication of our model is in stark contrast to the implications of models that do not account for secondary-market equilibrium, in which unconstrained households respond to the credit shock and to the decline in interest rates by increasing their stock of durable assets, leading to a negative comovement between scrappage and new-durables purchases.

We further enrich our model to study several interactions between durable-goods markets and the macroeconomy, which allows us to quantify the importance of equilibrium dynamics in durable-goods markets during the Great Recession. Specifically, we consider borrowing constraints that depend on the value of households’ durable holdings—that is, collateral constraints—as well as aggregate income shocks. We show that these additional features increase the volatility of used-car prices and new-car sales, improving the quantitative performance of the calibrated model against the data. Most notably, the calibrated model with both a credit tightening and an aggregate income decline accounts for almost the entire drop in new-car sales and two-thirds of the drop in used-car prices.

Finally, we use our model to evaluate the effects of targeted fiscal stimulus, in the form of car-replacement subsidies, similar to the “Cash for Clunkers” program implemented in the U.S. in 2009. We show that secondary markets play an important role in the transmission of these policy interventions. Specifically, we find that general-equilibrium effects dampen the stimulus of these subsidies on the demand for new cars, by depressing the trading and prices of used-car markets. Hence, these subsidies are less effective than models that do not consider general-equilibrium effects would predict.
The rest of the paper is organized as follows. Section 2 highlights our contribution to the literature. Section 3 documents the key empirical patterns on vehicle prices and households' vehicle replacement during the Great Recession. Section 4 introduces our model, which we parametrize in Section 5. Section 6 considers the effects of macroeconomic shocks, such as an aggregate tightening of the borrowing limit as well as a negative aggregate income shock. Section 7 studies durable-replacement subsidies. Section 8 concludes.

2 Related Literature

This paper contributes to several strands of literature. First, since at least Mankiw (1982) and Bernanke (1985), understanding the dynamics of durable-goods expenditures has been a key question in macroeconomics. Starting from Caballero (1993) and Eberly (1994), the literature focused on models of durables adjustment in the presence of transaction costs, which lead to inaction and lumpy adjustment. Among these contributions, Leahy and Zeira (2005) is particularly related to our paper, as they study the cyclical effects of the timing of lumpy durable goods purchases in general equilibrium. Recently, Kaplan and Violante (2014), Berger and Vavra (2015) and Guerrieri and Lorenzoni (2017) embed households’ fixed adjustment costs into a Bewley (1986)-Huggett (1993)-Aiyagari (1994) general-equilibrium framework with uninsurable idiosyncratic risk.\(^1\) We enrich this framework with a quality ladder for durables, which households can trade at market-clearing prices on secondary markets (or scrap).\(^2\) We obtain that the illiquidity of durable goods is an equilibrium outcome that varies with the aggregate state of the economy, rather than a fixed parameter. Moreover, we show that this endogenous illiquidity is essential to account for the positive comovement of car scrappage and new-car sales observed during the Great Recession.\(^3\)

Second, our mechanism shares some features with those of other papers in which agents

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\(^1\)Relatedly, Huo and Ríos-Rull (2016) and Favilukis, Ludvigson, and Van Nieuwerburgh (2017) study the effects of financial shocks in models of housing with incomplete markets.

\(^2\)Caplin and Leahy (2006) develop a tractable equilibrium model of durable-goods markets with fixed adjustment costs by approximating the distribution of durable-goods holdings.

\(^3\)Our paper is also related to Adda and Cooper (2006), who empirically study the aggregate dynamics of car sales; Oh (2017), who studies durable replacement and second-hand markets in a representative-agent business-cycle model; Rampini (Forthcoming), who analyzes how durability affects durable-goods financing in a model with collateral constraints; Chafwehé (2017), who considers secondary markets for durables in a stationary partial-equilibrium model with incomplete markets.
are simultaneously buying and selling assets. Stein (1995) builds a housing-market model in which households’ down-payment constraints amplify shocks to house prices, thereby reducing housing demand. Anenberg and Bayer (2013) show that the cost of simultaneously holding two homes varies endogenously over the cycle, driving fluctuations in trade volume. Garriga and Hedlund (2017) study housing markets in the Great Recession, using an incomplete-markets model with search frictions that make housing illiquid. Our paper differs from these contributions in that our equilibrium notion of illiquidity stems from the imperfect substitutability across durables of different qualities that trade at market-clearing prices. Vehicles represent an ideal setting to measure relative price movements across goods of different qualities. However, the key insights from our analysis should apply to housing markets as well, as households move up and down a “property ladder.” Relatedly, Landvoigt, Piazzesi, and Schneider (2015) emphasize spillover effects across partially segmented housing markets during the 2000-2005 housing boom.

Third, the literature on consumer durable goods has investigated the role of secondary markets in allocating new and used goods (see, among others, Rust, 1985; Anderson and Ginsburgh, 1994). Most related are the empirical/quantitative papers of Adda and Cooper (2000), who study how government subsidies toward the replacement of old cars with new ones in France affect the time-series of new-vehicle sales; Stolyarov (2002), who investigates resale rates across different car vintages; and Gavazza, Lizzeri, and Roketskiy (2014), who provide a quantitative welfare analysis of secondary markets. We contribute to this literature by introducing (other) incomplete asset markets and macroeconomic shocks, thus studying the interactions between markets for durables and the rest of the economy.

The paper also contributes to the literature that studies capital replacement and, more generally, markets for capital goods. Among other papers, Cooper and Haltiwanger (1993) shows that the replacement of depreciated machines can create endogenous fluctuations in productivity and output of a single producer; Cooper, Haltiwanger, and Power (1999) explore aggregate investment fluctuations due to plants’ discrete replacement of their capital stock. However, none of these papers consider equilibrium in the market for used capital in presence of aggregate dynamics and thus fluctuations due to endogenous changes in the resale price of capital. Hence, our paper complements the recent work of Lanteri (2018),

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4Chen, Esteban, and Shum (2013) study the effects of the secondary market for automobiles on manufacturers’ incentives in the primary market.
who studies capital reallocation in an equilibrium model of partially irreversible investment with endogenous resale price of capital, whereas we focus on consumer durables.

Finally, our mechanism of delayed upgrading of durable goods during the Great Recession is consistent with the concurrent analysis of Dupor, Li, Mehkari, and Tsai (2018), who study the effect of households’ income expectations on their car purchases during the Great Recession, and with the evidence of Jaimovich, Rebelo, and Wong (2017), who show that households traded down in the quality of their non-durable consumption as well in that period.

3 Empirical Patterns

The goal of this section is to document key empirical facts on household adjustment of their vehicle stock and on the cost of replacement during the Great Recession. Appendix A describes in more detail the datasets we exploit and also provides some additional, complementary patterns to those that we report in this section.

(1) Vehicle scrappage decreased.

We obtain the yearly aggregate stock of registered vehicles in the U.S. from the Federal Highway Administration (FHWA), the yearly inflow of sales of new vehicles from the U.S. Bureau of Economic Analysis, and the yearly inflow of new-vehicle leases from the National Automobile Dealers Association (NADA). We combine these sources to construct the yearly outflow of vehicle scrappage using the accounting identity:

\[ \text{Scrappage}_t = \text{Stock}_t - \text{Stock}_{t-1} + \text{New Sales}_t + \text{New Leases}_t. \]

Figure 2 displays the resulting series of vehicles scrapped in the U.S. for the period 2005-2014, showing that vehicle scrappage bottomed out during the Great Recession, declining by approximately sixteen percent in 2009 relative to 2007. Together, Figures 1 and 2 imply that the total stock of vehicles remained approximately constant during the period, whereas the age of the stock increased during the recession, as we document in Appendix A.

(2) The cost of replacing a used vehicle with a new one increased.

We use NADA prices to compute the replacement cost of new vehicles, calculated as
the difference between the retail price of a new (i.e., age-0) vehicle model and the four-year-old trade-in price of the same vehicle model. The four panels of Figure 3 display these replacement costs. The top-left panel displays the average replacement cost for all vehicles in the NADA dataset; the other panels display it for three popular vehicles in the U.S.: the Honda Civic (top-right panel), the Toyota Camry (bottom-left panel), and the Honda Accord (bottom-right panel). All these panels show that the cost of replacing a used car with a new one spiked during the Great Recession, increasing in 2009 by approximately 20 percent relative to the pre-recession years.

Figure 4 highlights the key reason the cost of replacement increased during the Great Recession. Its top-left panel displays the average price of a new and of a four-year-old vehicle in the NADA dataset; the other three panels display the prices of a new and of a four-year-old vehicle for the same models for which Figure 3 displayed the costs of replacement. These panels show that, while new-car prices were quite stable throughout the 2003-2012 period, used-car prices were substantially more volatile, dropping by approximately 20 percent during the Great Recession.

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5We use prices in the Eastern region in the figures, but we obtained the same results when using prices from other regions, or we average prices across regions.

6As we explain in Appendix A, the NADA dataset reports prices recorded in the month of July of each year. Hence, 2008 prices do not incorporate the events of Fall 2008 following the bankruptcy of Lehman Brothers.

7We use manufacturers’ suggested retail prices (MSRPs) to approximate the transaction prices of new vehicles.
Figure 3: The figure displays the linearly detrended (=year 2003) cost of replacing a 4-year old vehicle with a new one of the same model during the period 2003-2012. The top-left panel displays the average cost computed using all vehicles in the NADA dataset. The other panels display the costs of replacing three popular models: the Honda Civic (top-right panel), the Toyota Camry (bottom-left panel), and the Honda Accord (bottom-right panel).

(3) Vehicle replacement decreased.

We use the Consumer Expenditure Survey (CEX) to measure households’ vehicle replacement. The CEX data are well-suited for this purpose, because they report information about households’ vehicles, including their acquisition date and whether they were acquired new or used.

We use these data for two main purposes. First, we seek to understand the quantitative importance of replacement for new-vehicle sales by calculating the share of households who replaced used vehicles with new ones, among all households who acquired new vehicles. This share equals approximately 50 percent (and in Appendix A we explain that this cars, following Table 1 of Hoekstra, Puller, and West (2017), which establishes that the average difference between MSRP and transaction prices is approximately $200 for new vehicle purchases during July 2009-April 2010. Moreover, in Appendix A we perform a robustness check that exploits customers’ incentives (cash rebates) published in the magazine Ward’s AutoWorld, buttressing our finding that new-car prices were significantly less responsive to the recession than used-car prices.

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Figure 4: The figure displays linearly detrended (=year 2003) prices of new and used (i.e., 4-year old) vehicles during the period 2003-2012. The top-left panel displays the average new price and the average used price computed using all vehicles in the NADA dataset. The other panels display new and used prices of three popular models: the Honda Civic (top-right panel), the Toyota Camry (bottom-left panel), and the Honda Accord (bottom-right panel).

Overall, these empirical patterns seem to suggest the following narrative for the decline in new-vehicle sales during the Great Recession: households delayed the scrappage of their (old) cars, thereby decreasing the demand for used cars and depressing their price; in turn, the decline in used-car prices increased the cost of replacing used cars with new ones, thereby reducing the demand for new cars. In the next section we formalize this idea in a Bewley (1986) economy in which households can acquire durable goods of different qualities, subject to a borrowing constraint. We show that a tightening of the borrowing limit induces
Figure 5: Share of households who replaced at least one used vehicle with a new one during 2003-2012.

...a larger fraction of constrained, lower-income households to decrease their demand for used cars, thereby triggering a decrease in secondary-market prices; this decrease leads to an increase in the cost for higher-income households to replace their used cars with new ones, and thus decreases new-car sales.

4 Model

We build a framework to study households’ durable adjustment when their durables depreciate over time and they face uninsurable, idiosyncratic income risk. Households derive utility from a non-durable consumption good and a durable good (i.e., a car). The key features of our framework are that depreciation of durables implies that different vintages are imperfect substitutes, with newer durable goods yielding higher utility; households can trade these vintages in secondary markets, or scrap them.

In this section, we describe the stationary equilibrium of the model, in which all aggregates and prices are constant over time; in Section 5 we calibrate the model and describe its key quantitative properties; in Section 6 we consider the effects of aggregate shocks.
4.1 Environment

Preferences. A continuum of unit-mass of infinitely lived households, indexed by \( i \), has preferences represented by a utility function defined over infinite sequences of non-durable consumption \( c_{it} \) and durable consumption (i.e., car services) \( d_{it} \):

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, d_{it}),
\]

where \( E_0 \) is the expectation operator, \( \beta \in (0, 1) \) is the discount factor, and \( u(c_{it}, d_{it}) \) is the per-period utility function.

Durable Goods. We consider a finite number \( N \) of different car qualities \( q_n \), with \( q_1 > q_2 > ... > q_N \). New cars are of quality \( q_1 \) and cars depreciate stochastically over time. Specifically, a car of quality \( q_n \), for \( n = 1, ..., N - 1 \), becomes a car of quality \( q_{n+1} \) in the following period with probability \( \pi_n \).

Each household owns at most one car.\(^8\) Hence, \( N + 1 \) possible car ownership statuses exist, the first \( N \) corresponding to the \( N \) car qualities, and we refer to the \( N + 1 \)th as the status of a household without a car. In period \( t \), household \( i \) enjoys utility from its durable \( d_{it} \) according to

\[
d_{it} = d(n, \theta_i) \equiv \begin{cases} 
q_n & \text{if household } i \text{ owns a car of quality } q_n \\
\theta_i & \text{if household } i \text{ does not own a car,}
\end{cases}
\]

where \( \theta_i \) is a household-specific type, constant over time, drawn from a distribution \( F_{\theta}(\theta) \), determining household \( i \)'s relative preference for living without a car. Hence, we allow for ex-ante heterogeneity in households' net utility enjoyed from a car, for example, because of the heterogeneous distance of households’ residence from their workplaces or heterogeneous quality of public transport in the cities where they live, which we take as exogenous.

Income. In every period \( t \), each household \( i \) receives idiosyncratic stochastic income \( w_{it} \)

\(^8\)Stochastic depreciation induces heterogeneity in quality, conditional on car age. However, we abstract from explicitly modelling horizontal differentiation of new-car qualities, because our main focus is the timing of replacement of used cars and its equilibrium response to aggregate shocks. Moreover, computational tractability prompts us to abstract from the possibility of owning multiple vehicles. Although several U.S. households own more than one vehicle, multiple vehicles do not seem to affect the main mechanism that we focus on, namely delayed replacement.
(denominated in units of the non-durable good), which evolves over time according to a Markov process with transition $F(w_t, w_{t+1})$.

**Technology.** New cars are produced by perfectly competitive firms using a linear technology with the non-durable good as the only input. Let $p_1$ be the constant marginal cost of new cars in terms of non-durables. Perfectly competitive firms operate a scrappage technology that gives $p_N$ units of the non-durable good for each scrapped car, regardless of its quality. A car of quality $q_N$ must be exogenously scrapped in the current period.

**Markets.** Households can trade cars at equilibrium prices $p_n$. Households who sell their cars of quality $q_n$ incur transaction costs $\lambda(p_n)$. Technology determines $p_1$ and $p_N$, whereas cars of quality $q_n$ trade at their market-clearing prices $p_n$, $n = 2, \ldots, N - 1$. For notational convenience, we let $p_{N+1} = 0$.

Households can borrow and save by trading one-period non-contingent bonds $b_{i,t+1}$ at their equilibrium price $p_b$, subject to a borrowing constraint

$$b_{i,t+1} \geq \phi,$$

where $\phi \leq 0$ is the debt limit.

**Government.** The government issues a constant level of non-contingent bonds $b_G$ and imposes lump-sum taxes $\tau$ on all households to finance interest payments on its debt. Hence, the budget constraint of the government is

$$b_G(1 - p_b) = \tau.$$  

In Section 7, we study a deficit-financed stimulus policy that subsidizes households’ car replacement.

**Timing.** At the beginning of each period, households receive their income and observe the depreciation shock to their durables. Next, they make trading, production, consumption, and saving decisions. The non-durable good is the numeraire of our economy.
4.2 Household Problem

We now describe households’ problem in recursive form. Let $V(b, w, n; \theta)$ be the value function of a household of type $\theta$ with bond holdings $b$, income $w$, and car quality $q_n$. This function satisfies the following Bellman equation:

$$
V(b, w, n; \theta) = \max_{c, b', \tilde{n}} u(c, d(\tilde{n}, \theta)) + \beta E[V(b', w', n'; \theta) | \tilde{n}, w],
$$

subject to stochastic transitions for income and car quality, the borrowing constraint (3), and the budget constraint:

$$
c + p_{\tilde{n}} + p_b b' + \tau = w + p_n - \lambda(p_n) I(\tilde{n} \neq n) + b,
$$

where the indicator function $I(\tilde{n} \neq n)$ equals one when households trade cars, and zero otherwise. The left-hand side of the budget equation (6) reports household expenditures: non-durable expenditures $c$, durable expenditures $p_{\tilde{n}}$ on car $\tilde{n}$, bond purchases $p_b b'$, and lump-sum taxes $\tau$. The right-hand side reports household resources: income $w$, the proceeds $p_n$ of the sale of car $n$, net of transaction costs $\lambda(p_n)$, and bond holdings $b$.

The Bellman equation (5) makes it explicit that household preferences for durables depend on their type $\theta$. Similarly, our notation highlights that the car $\tilde{n}$ that households choose could differ from the car $n'$ that they own at the beginning of the following period, because of depreciation.

The policy functions $b' = g_b(b, w, n; \theta)$ and $\tilde{n} = g_n(b, w, n; \theta)$ for future bond holdings and for car choice, respectively, solve the dynamic program (5).

4.3 Stationary Competitive Equilibrium

We now define the stationary competitive equilibrium of this economy. Clearing in the bond market requires

$$
\int g_b(b, w, n; \theta) dm(b, w, n; \theta) = b_G,
$$

where $m(b, w, n; \theta)$ is the beginning-of-period stationary cumulative distribution of households over individual states (i.e., bond holdings $b$, income $w$, and car quality $q_n$) and type $\theta$. The left hand-side is the aggregate net demand of assets from households, whereas the
right-hand side is the level of government debt.

Clearing in the market for cars of quality $q_1$ requires:

$$\int I(g_n(b, w, n; \theta) = 1)dm(b, w, n; \theta) = \int dm(b, w, n = 1; \theta) + x,$$

(8)

where $x$ is the endogenous aggregate production of new cars. The left-hand side is the aggregate demand for cars of quality $q_1$, which comes from all households whose policy function is to hold a car of quality equal to $q_1$ (thus, the indicator function $I$ for their choices). The right-hand side is the aggregate supply of cars of quality $q_1$, which is the sum of the equilibrium flow of new production $x$ and of the stock of existing cars of quality $q_1$ which did not depreciate from the previous period.

Clearing in the market for cars of a given quality $q_{\bar{n}}$, for $\bar{n} = 2, \ldots, N - 1$, requires

$$\int I(g_n(b, w, n; \theta) = \bar{n})dm(b, w, n; \theta) \leq \int dm(b, w, \bar{n}; \theta).$$

(9)

The left-hand side is the aggregate demand for cars of a given quality $q_{\bar{n}}$, which comes from all households whose policy function is to hold a car of quality $q_{\bar{n}}$ (thus, the indicator function $I$ for their choices). The right-hand side is the aggregate supply of cars of quality $q_{\bar{n}}$. If households do not scrap any car of quality $q_{\bar{n}}$ in equilibrium, equation (9) holds with equality; if households scrap some cars of quality $q_{\bar{n}}$ in equilibrium, equation (9) holds with strict inequality, and $p_{\bar{n}} = p_N$; that is, $q_{\bar{n}}$-cars trade at the scrappage value.

Definition 1 A **Recursive Stationary Competitive Equilibrium** is (i) a value function $V(b, w, n; \theta)$ and associated policy functions $g_b(b, w, n; \theta)$ and $g_n(b, w, n; \theta)$, (ii) a stationary distribution $m(b, w, n; \theta)$, and (iii) a vector of prices $(p_b, p_2, \ldots, p_{N-1})$, such that

1. $V(b, w, n; \theta)$ satisfies the Bellman equation (5);

2. The stationary distribution $m(b, w, n; \theta)$ is consistent with the type distribution $F_\theta(\theta)$, the exogenous income and car depreciation shocks, and with household policy functions $g_b(b, w, n; \theta)$ and $g_n(b, w, n; \theta)$;

3. The bond market clears—i.e., equation (7) holds;
Table 1: Parameter Values

<table>
<thead>
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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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4. Car markets clear—i.e., equation (8) determines the flow $x$ of production of new cars, and equation (9) holds.

5 Calibration

We now describe our choices of functional forms and parameter values for preferences, income process, credit market, car production, and trading. Table 1 reports the numerical values of the parameters.

Preferences. We follow Berger and Vavra (2015) and choose the following per-period utility function: $u(c_{it}, d_{it}) = \left(\frac{c_{it}^{\alpha}d_{it}^{1-\alpha}}{1-\gamma}\right)^{1-\gamma}$. We set $\alpha = 0.95$ to match the expenditure share on vehicles, which equals approximately five percent, according to Personal Consumption Expenditure data from the U.S. Bureau of Economic Analysis. We set the curvature of the per-period utility $\gamma = 2$, which is within the range Aiyagari (1994) considers.

A period in the model coincides with a year, consistent with the frequency of our data. Hence, we set $\beta = 0.945$, which, along with the calibrated degree of idiosyncratic risk discussed below, results in a real interest rate of approximately 2.5 percent, thereby matching its 2007 value.

Income. We assume income follows an AR(1) process in logs: $\log(w_{i,t+1}) = \rho \log(w_{i,t}) +$
\( \epsilon_{i,t+1} \). We set the persistence of the process to \( \rho = 0.9 \). The innovations \( \epsilon_{i,t+1} \) are i.i.d. across households and over time, normally distributed with mean \(-0.5\sigma^2_w\) and standard deviation \( \sigma_{\epsilon} = 0.2 \), following Flodén and Lindé (2001)’s estimates using PSID data. These parameters imply that mean income equals one—that is, a normalization—and the cross-sectional standard deviation \( \sigma_w \) of the log of income equals \( \frac{\sigma_{\epsilon}}{\sqrt{1-\rho^2}} = 0.63 \). We further discretize this process with a three-valued Markov chain, using the method in Rouwenhorst (1995).

**Bond Market.** We follow Guerrieri and Lorenzoni (2017) and set the level of government debt to match the ratio of liquid assets to GDP, which equals 1.78 in 2006. Moreover, we set the borrowing constraint \( \phi \) to target a fraction of constrained households approximately equal to 10 percent, consistent with the fraction that Kaplan and Violante (2014) report. The resulting value of \( \phi = -1 \) implies households can borrow up to the average annual income.

**Cars.** A large number of car qualities (and thus a large number of endogenous prices) makes computation cumbersome, because our model features rich household heterogeneity as well as aggregate dynamics. Hence, we choose a parsimonious structure for car qualities by setting \( N = 4 \).

We set the values of the depreciation probabilities and of the car qualities to match some statistics on the average lifetime of cars and their average price depreciation reported in Jacobsen and van Benthem (2015), which we briefly recall now. Specifically, we set the depreciation probabilities as follows: \( \pi_1 = 1/3 \), which implies that, on average, high-quality cars depreciate after three years; \( \pi_2 = 1/10 \), which implies that, on average, cars are of medium quality for 10 years; and \( \pi_3 = 1/2 \). These depreciation parameters allow our model to closely match two statistics: the average lifetime of cars, approximately equal to 15 years, and the average scrappage rate of 15-year-old cars, approximately equal to 10 percent.

We normalize \( q_1 = 1 \) and we set \( q_2 = 0.3 \) and \( q_3 = 0.1 \) (we do not need to specify a value for \( q_4 \), as households scrap these cars). These quality levels, along with the aforementioned depreciation probabilities, allow the model to match closely the price decline of a three-year-old car and, thus, are consistent with average replacement costs in our car-price dataset. We set the marginal cost \( p_1 \) of producing new cars equal to 0.45 in order to match the ratio
of average new car prices to household income. We set the scrappage value \( p_N \) to 0.036 to match the average residual value of cars older than 15 years. Given these values for \( q_3 \) and \( p_N \), some households scrap cars of quality \( q_3 \) in the stationary equilibrium of our model, consistent with the evidence on scrappage rates for old cars. This scrappage of quality-\( q_3 \) cars implies their price equals \( p_N \).

Moreover, we parametrize the type distribution determining the utility of not owning any car to a two-type distribution, with values \( \theta_i \in \{0, 1\} \), with probabilities \( \rho = 0.9 \) and \( 1 - \rho = 0.1 \), respectively. Thus, households with \( \theta_i = 1 \) choose not to own a car, whereas households with \( \theta_i = 0 \) choose to own one; their probabilities allow the model to match the empirical share of households with no car, which equals approximately 10 percent in the 2000-2010 American Community Survey.

**Transaction Costs.** We specify the transaction-cost function to include a fixed cost and a cost proportional to the car value—that is, \( \lambda_0 + \lambda_1 p_n \). We use NADA prices to calibrate these parameters to match the difference between retail and trade-in prices across cars of different vintages, implying that empirical retail prices map into the prices paid by buyers in the model and trade-in values map into the prices obtained by sellers. We obtain \( \lambda_0 = 0.03 \) and \( \lambda_1 = 0.15 \).

5.1 Properties of the Stationary Equilibrium

We now describe the main features of the stationary equilibrium of the calibrated model, with a greater focus on durable goods.

Because the utility function displays complementarity between non-durable consumption and durable consumption, households enjoy both a higher level of non-durable consumption and higher-quality cars as their liquid wealth (i.e., bonds \( b_d \)) and income increase. Thus, the correlation between non-durable consumption and car values is positive: it equals 0.51. Similarly, the correlation between liquid wealth and car values equals 0.49. The presence of transaction costs and the discreteness of the set of car qualities induce inaction in durables adjustment, thereby reducing these correlations relative to their values in models in which households can freely adjust their durable stock.\(^9\) These correlations are also

\(^9\)This procedure abstracts from other potential sources of trading costs, such as time and search costs, which unfortunately we cannot measure in our data.

\(^{10}\)In Section 6.1.1, we compute the stationary equilibrium of the economy without transaction costs.
Figure 6: Thresholds for car replacement. The solid line represents the threshold for upgrading from quality $q_3$ to quality $q_2$ (or $q_1$). The dashed line represents the threshold for upgrading from quality $q_2$ to $q_1$. The dotted line corresponds to the borrowing limit $\phi$.

lower than the correlation between bond holdings $b_{it}$ and non-durable consumption $c_{it}$, both of which households can adjust at no cost, and which equals 0.83. Moreover, the correlation between income $w_{it}$ and beginning-of-period car values is 0.38, whereas the correlation between income and the value of the car chosen for the same period ($p_{it}$) is higher and equals 0.56. This difference reflects the fact that income shocks are persistent and, thus, provide information about future wealth and consumption, inducing durable adjustment. The correlation between income and non-durable consumption, which is freely adjustable, is 0.88.

Figure 6 displays households’ main replacement policies as a function of their bond holdings (on the vertical axis) and income (on the horizontal axis): the dashed line is the threshold for replacing cars of quality $q_2$, and the solid line is the threshold for replacing cars of quality $q_3$. These thresholds represent the minimum level of bond holdings that triggers households with a given level of income to upgrade their car. For instance, households with a $q_2$-car ($q_3$-car) upgrade to a higher-quality $q_1$ car ($q_2$ or $q_1$ car) if their bond holdings and their income lie above the dashed (solid) line, whereas they keep their current car if their bond holdings and their income lies below the dashed line.

The figure identifies three sets of households. The first one, above the dashed line,
comprises the richest households, who replace their cars as soon as they depreciate from \( q_1 \) to \( q_2 \). In the stationary equilibrium of our economy, approximately four percent of households upgrade from \( q_2 \) to \( q_1 \) in each period. The second set, the one between the solid line and the dashed line, comprises mid-wealth households. The majority of these mid-wealth households replace their cars of quality \( q_3 \) (or \( q_4 \)), buying cars of quality \( q_2 \) from the richest households. The minority of mid-wealth households—that is, those with low liquid wealth and high income—replace their cars of quality \( q_3 \) with cars of quality \( q_1 \): because the persistence of income shocks is high, these households expect their wealth and non-durable consumption to increase in the near future; thus, they choose to avoid paying the transaction costs multiple times and upgrade directly to cars of the highest quality.

In the stationary equilibrium, approximately two percent of households upgrade directly from a car of quality \( q_3 \) to a car of quality \( q_1 \). The figure shows that the solid threshold for upgrading from \( q_3 \) to a higher-quality car (either \( q_2 \) or \( q_1 \)) coincides with the borrowing limit (i.e., the horizontal dotted line at \( \phi = -1 \)) for households with sufficiently high income.

The third set, the one above the dotted line of the borrowing limit \( \phi = -1 \) and below the solid line, comprises households with low income and high debt. These households keep their low-quality cars and will upgrade them only after they deleverage and move away from the borrowing constraint. We recall that, in our stationary equilibrium, ten percent of households are borrowing-constrained (this fraction is a calibration target). The vast majority of these constrained households, approximately 85%, have a low income realization and thus are in this third region.

Finally, the stationary equilibrium of the economy features no household who downgrades to lower-quality durables: all households, including those who own cars of quality \( q_1 \), either hold on to their cars or upgrade to higher-quality cars.

### 6 Macroeconomic Shocks

In this section, we study the effects of macroeconomic shocks, such as an aggregate tightening of the borrowing limit as well as a negative aggregate income shock. Specifically, we compute the transitional dynamics of our model economy that starts from the steady state characterized in Section 5.1, receives unexpected aggregate shocks (described in more details in the following sections), and reaches a new steady state over time, thereby following
several recent papers that assume households did not foresee the aggregate shocks of the Great Recession (e.g., Guerrieri and Lorenzoni, 2017; Huo and Ríos-Rull, 2016). Along the transition path, we assume households have perfect foresight about aggregate variables.

When the economy is out of steady state, the value function, the distribution of households over individual states, and the equilibrium prices for bonds and cars change over time. Hence, we solve for the sequences $\{V_t\}_{t=0}^T$, $\{m_t\}_{t=0}^T$, $\{p_{b,t}, p_{2,t}, \ldots, p_{N-1,t}\}_{t=0}^T$, consistent with household optimization and market clearing, where $t = 0$ is the period in which the shock hits and households learn about it, and $T$ is the period in which the economy reaches its new steady state. Appendix B describes the numerical algorithm, explaining a widely applicable novel method we develop to overcome the challenge of clearing multiple markets when heterogeneous agents make discrete choices.

We start by considering the effects of a credit shock that tightens households’ borrowing limit. In our view, this case allows us to illustrate in the cleanest way the key economic mechanisms that lead to declines in new-car sales, scrappage, and used-car prices. Moreover, we show that it can quantitatively account for a sizable fraction of the declines observed in the data. We will further enrich the model to include additional realistic features of the Great Recession, such as aggregate income shocks and a borrowing constraint that depends on the value of households’ durable holdings—that is, a collateral constraint. We show how these richer versions of the model improve the quantitative performance of the calibrated model.

6.1 Credit Shock

We now analyze the aggregate dynamics of our economy following a tightening of the borrowing limit for all households.\footnote{Benmelech, Meisenzahl, and Ramcharan (2017) and Guerrieri and Iacoviello (2017) emphasize the empirical relevance of aggregate credit-market conditions for vehicle sales.} We model this credit tightening as an unexpected shock that hits the economy in its stationary equilibrium; when the shock hits, households learn about current and future credit limits.

We parametrize the path of the borrowing limit to match the sharp decline of the real interest rate during the Great Recession, which dropped from 2.5 percent in 2007 to −2 percent in 2010 (we measure the real interest rate as the difference between the annualized return on three-month Treasury bills and the growth of the GDP-deflator). In practice, we
Figure 7: The left panel displays the dynamics of the borrowing constraint and the right panel displays the dynamics of the interest rate. The economy is in the stationary equilibrium at $t = -1$, and households learn about the new path of the borrowing limit at $t = 0$. The horizontal axis displays time $t$.

match this decline by gradually decreasing the credit limit from $\phi = -1$ in the pre-shock stationary equilibrium at $t = -1$ to $\phi_t = -0.4$ at $t = 2$, thus changing by 0.2 in each period $t = 0, 1, 2$. After the shock, the credit limit stays permanently at its new, tighter level, as in Guerrieri and Lorenzoni (2017).\footnote{We have also studied the effects of a temporary credit shock that reduces the credit limit for a limited number of periods and then gradually reverts back to the initial level. However, the behavior of the interest rate is at odds with that observed in the 2008-2010 data, as it jumps above its steady state value when the credit limit gradually moves back to its steady-state value.}

Figure 7 displays the sharp decline in the interest rate (right panel) as the tighter borrowing limit (left panel) changes all households’ consumption-saving trade-off. More specifically, the shock forces low-wealth households, whose debt is close to the old borrowing constraint, to reduce their debt to satisfy the new, tighter borrowing limits. Simultaneously, wealthier households want to increase their precautionary savings, foreseeing that they will face tighter credit conditions in the future, should their income decrease. Because the aggregate demand for savings increases, the top-right panel shows that the real interest rate $r_t \equiv 1/p_{b,t} - 1$ has to fall to clear the bond market. The drop in the interest rate is particularly swift when the borrowing limit changes in periods $t = 0, 1, 2$, because households need to satisfy the increasingly tighter borrowing constraints; the interest rate then stabilizes around its new steady state level of 1.9 percent—that is, 600 basis point lower
than its value of 2.5 percent in the old steady state—when the borrowing limit stays at its new long-run level.

Figure 8 displays striking patterns on car markets, most notably while the borrowing limit becomes increasingly tighter in $t = 0, 1, 2$. Specifically, the credit tightening advises all households to postpone expenditures on durable goods, thereby holding on to their current cars and delaying their replacement. These incentives are stronger for low-wealth households, as their initial debt was close to the old borrowing limit. Because these households usually own cars of quality $q_3$, and they postpone their replacement, scrappage falls (top-left panel) and the demand for used cars of quality $q_2$ falls as well.\(^{13}\) The lower demand for

\(^{13}\)The credit shock also leads to a fall in demand for cars of quality $q_1$ from households with low-wealth
cars of quality $q_2$ induces a decrease in their trading volume (top-right panel) and in their equilibrium price (bottom-left panel).

The softening of used-car markets spurs wealthy households to postpone the replacement of their cars, as well. These households usually own cars of quality $q_2$ and would upgrade them with new ones in normal times—that is, in the pre-shock stationary equilibrium. However, these wealthy households now face a high replacement cost, because they can trade in their $q_2$-cars at fire-sale prices $p_2$ only; moreover, they anticipate that use-car values will recover after the economy adjusts to the new credit conditions. Hence, new-car sales decline on impact at $t = 0$ (bottom-right panel). This finding is striking because (i) the tighter credit limits are not binding for new-car buyers, and (ii) the real interest rate falls sharply, making durable-goods purchases more attractive for unconstrained households.

Quantitatively, the credit shock accounts for declines in new-car sales and in scrappage of approximately seven percent, and in used prices of approximately ten percent relative to their respective values in the stationary equilibrium at $t = -1$. Hence, our quantitative analysis suggests that the credit shock alone, disciplined to match the dynamics of the interest rate observed in the data, can account for sizable fractions of the decline in new car sales, of the increase in scrappage, and of the increase in the replacement cost of used cars that we documented in Sections 1 and 3.

When the borrowing limit stays at its new long-run level, the protracted delays in car replacement during the previous periods prompts a spike in scrappage, used trade, and new sales at $t = 3$. In Section 6.2, we will show that these spikes become more muted once we introduce additional features of credit markets, such as collateral constraints. From $t = 4$, car markets adjust close to their new, long-run equilibria, which features lower new-car sales, lower scrappage, lower used-car prices, but a higher volume of trade on the secondary market for quality-$q_2$ cars compared to the old stationary equilibrium.\footnote{An implication of our assumption of stochastic quality depreciation is that the echo effects of aggregate shocks are quantitatively less important, relative to models of durables replacement with deterministic quality (or age) transitions.}

6.1.1 Inspecting the Mechanism: Endogenous Illiquidity of Durable Goods

One distinctive feature of our model is the endogenous illiquidity of durable goods, which implies the volume of trade in secondary markets and used-car prices drop as credit con-
The Role of Equilibrium in Secondary Markets. To understand the role of secondary markets, we compare the response to the credit shock in general equilibrium with the outcome that would arise in absence of market clearing in the used-car market. Specifically, we assume households can trade cars with the rest of the world at the prices prevailing in the initial stationary equilibrium. Hence, while the bond price adjusts after the shock to clear the bond market, the price of used cars $p_2$ does not.

The left panel of Figure 9 displays the path of car scrappage, whereas the right panel displays the path of new-car sales in this partial-equilibrium case. In the absence of the endogenous response of used-car prices, the economy features a strong negative comovement between scrappage and new-car purchases. This comovement arises because borrowing-
constrained households still decide to postpone scrappage of their low-quality cars, as the left panel shows. Their demand for used cars falls, but, in the absence of secondary-market clearing, this shift in demand does not translate into a lower price of used cars; hence, wealthy households experience no change in the cost of replacing their used $q_2$ cars with higher-quality ones. In addition, the decrease in the equilibrium real interest rate stimulates their car-replacement activity, inducing a large increase in new-car sales, approximately equal to 60 percent, relative to their value in the initial stationary equilibrium, as the right panel shows.\footnote{The increase in purchases of durable goods in response to a credit tightening is consistent with some findings of Guerrieri and Lorenzoni (2017), most notably their Figure 15.}

This counterfactual case highlights that equilibrium in secondary markets plays a key role in generating a decline in purchases of new durables and a positive comovement of scrappage and new sales, consistent with the U.S. data during the Great Recession that we reported in Sections 1 and 3.

**The Role of Transaction Costs.** A large literature emphasizes the role of transaction costs in explaining consumer inertia in durable-goods markets (e.g., Caballero, 1993; Atanasio, 2000; Berger and Vavra, 2015). We now study how transaction costs affect our economy and its response to the credit shock. To this goal, we remove transaction costs by setting $\lambda_0 = \lambda_1 = 0$. We first describe the key patterns of car replacement in the stationary equilibrium to facilitate the comparison with the stationary equilibrium of the economy with transaction costs of Section 5.1; we then discuss the response of this economy to the credit shock.

As we recount in Section 5.1, the stationary equilibrium of the economy with transaction costs features no households downgrading the quality of their cars, and approximately four percent of households replacing cars of quality $q_2$ with cars of quality $q_1$. These patterns change significantly in an economy with no transaction costs. First, one percent of households downgrade from $q_1$ to $q_2$. Second, 0.3 percent of households downgrade from $q_2$ to $q_3$. Third, the volume of trade of used cars is higher: more than seven percent of households replace a cars of quality $q_2$ with a car of quality $q_1$. Overall, these patterns of trading allow households to achieve a higher correlation between non-durable consumption and car values: this correlation coefficient equals 0.64, versus 0.49 in the economy with...
Figure 10: Credit shock in the absence of transaction costs. The economy is in stationary equilibrium at $t = -1$. Households learn about the new path of the borrowing limit $t = 0$. The horizontal axis displays time $t$. The top-left panel displays scrappage; the top-right panel displays the volume of trade of used cars; the bottom-left panel displays the price $p_2$ of quality-$q_2$ cars; and the bottom-right panel displays sales of new cars. The solid line displays the baseline case and the dashed line displays the case without transaction costs ($\lambda_0 = \lambda_1 = 0$).

The absence of transaction costs implies that quality downgrading plays an important role in the response of the economy to the credit shock. Figure 10 illustrates how the economy without transaction costs (dashed lines) behaves in response to this aggregate shock, and compares it with that of our baseline calibration with transaction costs (solid lines). Without transaction costs, the shock leads to a substantially larger decline in scrappage, in used prices, and in new-car sales relative to those of the baseline case; however, the volume of trade of used cars increases above its initial level, whereas it drops in the economy with transaction costs (as we observe in the data). The key reason for these
equilibrium dynamics is that, in order to increase their liquid assets, many middle- and low-wealth households who own cars of quality $q_2$ respond to the credit shock by selling them, temporarily downgrading to cars of quality $q_3$. This force leads to an increase in supply (and thus in trading volume) of used cars of quality $q_2$, driving their price down further, as well as the incentives of wealthy households to replace their $q_2$-cars with new ones. This downgrading effect is so strong that cars of quality $q_3$ are in excess demand at the scrappage value $p_N$; thus, they temporarily trade at a higher price than $p_N$ while the economy adjusts to the shock.

By contrast, the volume of downgrading that the aggregate shock induces is quantitatively small in our baseline calibration with transaction costs, because households anticipate that downgrading implies they incur transaction costs twice: first, when they downgrade; and second, when they will re-upgrade their cars in the near future, once the economy stabilizes towards its long-run stationary equilibrium.

Overall, this counterfactual case suggests that by preventing larger downgrading of durable goods than that observed in the data, transaction costs play an important role in dampening the effect of the shocks on secondary-market prices and, thus, on new-car sales.

### 6.2 Collateral Constraint

Our baseline case considers a constant credit limit $\phi$ that applies to all households, independent of their durable holdings. We now study a specification of the model in which household borrowing limits depend on the expected resale value of their durables—that is, a collateral constraint. This analysis encompasses the case of car loans, although it applies more generally. We show that this modification reinforces the main mechanism of our model, which links a credit tightening to a drop in new durable purchase through a drop in resale prices; moreover, this modification smooths the recovery of durable-goods markets once the borrowing limit stays at its new long-run value.

To introduce a role for durables as collateral, we replace equation (3) with the following constraint:

$$b_{i,t+1} \geq \phi_t \left( \chi_0 + \chi_1 E_t \left[ p_{q_i,t+1} | q_{it} \right] \right),$$

where the term $\phi_t$ denotes the exogenously time-varying level of the credit limit. Notice that this collateral constraint allows for both uncollateralized debt, though the term $\chi_0$, 

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Figure 11: Shock to the collateral constraint. The economy is in stationary equilibrium at $t = -1$. Households learn about the new path of the borrowing limit $t = 0$. The horizontal axis displays time $t$. The top-left panel displays scrappage; the top-right panel displays the volume of trade of used car; the bottom-left panel displays the price $p_2$ of quality-$q_2$ cars; and the bottom-right panel displays sales of new cars. The blue line displays the baseline case and the red line displays the case with collateral constraint.

and collateralized car loans, through the term $\chi_1 E_t [p_{q_{t+1}} | \tilde{q}_d]$. Both these components of the credit limit are affected by the aggregate shock $\phi_t$.

Equation (10) highlights that the expected collateral value depends on the chosen car quality. Higher quality implies a higher expected equilibrium resale value and, thus, a larger borrowing capacity. To match our empirical targets with this alternative specification, we set $\chi_0 = .8$, $\chi_1 = .85$ and consider a shock that, as in our baseline formulation, changes $\phi_t$ from a steady-state value of $-1$ to a new steady-state value of $-0.4$.

Figure 11 illustrates the transitional dynamics of our variables of interest. Qualitatively, the collateral constraint does not change the outcomes relative to the baseline shock. Quan-
titatively, we observe a substantially larger decline in scrappage and new-car sales than in the baseline case, whereas the dynamics of used prices are broadly similar. Most notably, the drop in new-car sales (bottom-right panel) is almost twice as large as that obtained in the baseline case, thereby accounting for approximately half of the decline in new-car sales displayed in Figure 1. The reason for this larger drop is that the presence of resale values in the credit constraints induces an amplification effect: expected low resale prices for used cars further tighten credit limits for new-car buyers after the aggregate shock hits.

This amplification effect is closely related to a standard financial accelerator effect à la Kiyotaki and Moore (1997), with one important difference. In our model, in contrast to most of the existing literature, there is a clear distinction between new-investment prices and resale values. While resale values drop in bad times, new prices do not. This difference makes new purchases particularly unattractive during downturns, inducing a stronger amplification mechanism than in models with a single price of durable (or capital) goods.

6.3 Aggregate Income Shock

We now further enrich our model by considering the joint effect of a tightening of the credit limit and of a negative aggregate income shock. Thus, this case includes additional realistic features of the Great Recession, during which both credit conditions and household incomes deteriorated. We parametrize the credit shock in the same way as in our baseline case of Section 6.1. Moreover, we approximate the output decline induced by the Great Recession by assuming that all households receive an exogenous negative shock equal to two percent of their income for two years, which in our calibration coincide with 2008 and 2009.\textsuperscript{16}

Figure 12 displays the transitional dynamics of our key variables of interest. The aggregate income shock amplifies the effects of the credit shock on car markets. In particular, relative to the baseline credit shock, it leads to larger declines in scrappage and in new-car purchases, which drop by approximately 35 percent relative to their initial values in the stationary equilibrium. Hence, through a combination of aggregate credit and income shocks, our model successfully accounts for a large fraction of the empirical decrease in car sales during the Great Recession, displayed in Figure 1.

\textsuperscript{16}For models that endogenously generate output declines following similar financial shocks, see for instance Guerrieri and Lorenzoni (2017) and Huo and Ríos-Rull (2016).
Figure 12: Credit and income shock. The economy is in the stationary equilibrium at $t = -1$, and households learn about the new path of the borrowing limit and income at $t = 0$. The horizontal axis displays time $t$. The top-left panel displays scrappage; the top-right panel displays the volume of trade of used cars; the bottom-left panel displays the price $p_2$ of quality-$q_2$ cars; and the bottom-right panel displays sales of new cars. The solid line displays the baseline case with a credit shock only; the dashed line displays the case with credit and income shocks.

The income decline affects the patterns of car replacement through two different channels. First, it directly amplifies our mechanism of delayed quality upgrade by making financial conditions even tighter for poor households close to the scrappage threshold. Second, it induces wealthier households to postpone their car replacement, regardless of the feedback from the secondary market: they hold on to their intermediate-quality cars until after the recession. Hence, both demand and supply in the used-car market cars drop on impact, inducing a sizable decline in the volume of cars traded.

In Appendix C.1, we perform an analysis of the separate roles of secondary-market prices and transaction costs, similar to that of Section 6.1.1, in this case with both credit
and aggregate income shock hitting the economy. The results are consistent with those of the baseline case with only a credit shock. Notably, this decomposition confirms that accounting for equilibrium in secondary markets is key to explain the decline in new-car sales observed during the Great Recession.

We also study an aggregate income shock in the absence of credit tightening, under two alternative specifications. First, we consider a shock that hits all households symmetrically (as in the case analyzed above). Whereas the income shock acts as a powerful amplifier for the credit tightening, we find that an aggregate income decline alone cannot account for the empirical patterns described in Section 3. When the borrowing limit does not change, the income shock induces price effects that do not match those observed in the data: the combination of low current income and expectations of higher future income lead to an increase in the real interest rate; moreover, the decrease in both demand and supply of used cars imply the income shock alone cannot generate a sizable drop in used-car prices. For these reasons, our model suggests that a quantitatively successful explanation of the dynamics described in Section 3 involves a combination of tighter credit conditions and an income decline.

Second, in Appendix C.2, we explore the effects of an income shock that hits low-income households only, inspired by the empirical literature on the skewed effects of recessions (e.g., Guvenen, Ozkan, and Song, 2014). We find that a temporary income loss for low-income households leads to similar qualitative effects as the credit tightening, suggesting our main mechanism of delayed scrappage and replacement is general and may apply to several empirically relevant cases in which aggregate shocks affect the income-wealth distribution asymmetrically. Quantitatively, however, also this version of the income shock, in isolation, seems less powerful than the combination of credit shock and aggregate income shock illustrated in Figure 12.

7 Policy Evaluation: Durable-Replacement Subsidies

We now use our framework to study the effects of a fiscal intervention aimed at stimulating durable replacement during a credit crunch, similar to the car-replacement stimulus implemented in the U.S. in 2009 (the Cars Allowance Rebate System, commonly referred to as
“Cash for Clunkers”). Similar subsidies are common across different countries and across several recession episodes (e.g., Adda and Cooper, 2000, provide a structural evaluation of two interventions in France in the 1990s).

In our framework, secondary markets play an important role in the transmission of these policy interventions. Thus, we introduce a durable-replacement subsidy immediately after the credit-supply shock discussed in Section 6.1 hits the economy. Specifically, in the first year in which the credit shock hits, the government offers a subsidy equal to 10 percent of the price of a new car to owners of cars of quality $q_3$ who choose to scrap their cars and replace them with a new car (i.e., of quality $q_1$) in that year.\(^{17}\) We assume the government initially finances this policy by running a deficit; after ten years, the government raises lump-sum taxes in order to gradually reduce the debt to its initial steady-state value. Formally, taxes follow the following rule:

$$
\tau_t = \begin{cases} 
\tau^*_t & \text{if } t < 10, \\
\tau^*_t + \psi(b_{Gt} - b^*_G) & \text{if } t \geq 10,
\end{cases}
$$

where $\tau^*_t$ and $b^*_G$ are taxes and government debt in the baseline case analyzed in Section 6.1; we set $\psi = 0.06$ to achieve convergence of government debt to its steady-state value within thirty years from the policy implementation.

In Figure 13, we compare the dynamics of the key variables of interest under the policy (dashed line) with those obtained in the baseline case with no subsidies (solid line). The direct effect of the policy is to attenuate the fall in scrappage and in new-car sales while the subsidies are available. However, general-equilibrium effects dampen the stimulus of these subsidies. Most notably, the policy induces a further decline in the price of used cars (quality $q_2$) and a larger fall in the volume of trade, relative to the baseline case, because, in

\(^{17}\)The Car Allowance Rebate System offered subsidies between $3,500 and $4,500, depending on car models, that is, approximately 10 percent of the average new car price. However, these subsidies were only available during the months of July and August 2009 and, thus, our yearly calibration does not allow us to exactly match the timing aspect of the policy. Moreover, the Car Allowance Rebate System did not involve a minimum age requirement in order for scrapped vehicles to qualify for the subsidy (this aspect differs from the related French policies studied by Adda and Cooper, 2000). Eligibility depended largely on fuel efficiency, as well as on other attributes that our model abstracts from. In practice, however, most scrapped cars were relatively old. For simplicity, we focus on an eligibility criterion based on our notion of car quality, but given the size of the subsidy, extending eligibility to higher-quality cars would not affect the results.
the baseline case, most households who scrap their $q_3$-cars replace them with $q_2$-cars rather than new $q_1$-ones. However, the stimulus leads households to substitute away from cars of quality $q_2$ and towards cars of quality $q_1$. As a result, demand for cars of quality $q_2$ falls, triggering a drop in their price and their volume of trade. In turn, the fire-sales $p_2$ prices urge wealthy households—who, in the absence of the policy, would trade in their $q_2$ cars for $q_1$ cars—to delay these replacement purchases. Hence, the subsidies are less effective than models that do not include general-equilibrium effects would predict.

Overall, this analysis highlights that these subsidies generate two types of substitution: (1) substitution from $q_2$-cars to $q_1$-cars, which seems broadly consistent with the results

Figure 13: Credit shock and durable-replacement subsidy. The economy is in the stationary equilibrium at $t = -1$, and households learn about the new path of the borrowing limit and policy at $t = 0$. The horizontal axis displays time $t$. The top-left panel displays scrappage; the top-right panel displays the volume of trade of used cars; the bottom-left panel displays the price $p_2$ of quality-$q_2$ cars; and the bottom-right panel displays sales of new cars. The solid line displays the baseline case, the dashed line displays the case with replacement subsidies.
of Hoekstra, Puller, and West (2017), who find that households tended to purchase less expensive and smaller new vehicles during the period of the Car Allowance Rebate System, and (2) intertemporal substitution in scrappage and demand for new cars only from the near future, which is consistent with the empirical evidence of Mian and Sufi (2012) and Hoekstra, Puller, and West (2017).

8 Conclusions

In this paper, we propose a novel general-equilibrium model of endogenous illiquidity of consumer durable goods to account for the aggregate dynamics of durable expenditures. Our equilibrium notion of illiquidity stems from the imperfect substitutability across durables of different qualities, which trade at market-clearing prices. Aggregate shocks lead to changes in the relative prices of durables of different qualities, affecting the replacement cost of higher-quality goods. We show that our model matches several striking patterns of U.S. car markets during the Great Recession.

We use our framework for policy analysis, evaluating the efficacy of durable stimulus. We show that this targeted fiscal stimulus induces substitution in demand away from used durables and towards new ones, thereby depressing secondary-market prices. These low used-car prices dampen the overall effectiveness of the stimulus, because they reduce the replacement of intermediate-age cars that do not qualify for the subsidy.

We believe that car markets represent an ideal setting in which to study our mechanism, since we can measure relative price movements across goods of different qualities quite accurately. Nevertheless, in future research, we hope to apply the key insights of our mechanism to housing markets as well, in which households climb a “property ladder” as their income increases.
References


APPENDICES

A Data and Additional Empirical Patterns

In this appendix, we describe in more detail the datasets we used in Section 3 and we provide some additional empirical patterns that complement those we reported in Section 3.

A.1 Data Sources

In addition to the aggregate data that we used to construct the annual number of scrapped cars displayed in Figure 2, we use two data sources in Section 3. The first one is a rich dataset on new- and used-car prices obtained from NADA. The second one is the Consumer Expenditure Survey. We now describe these datasets in more detail.

NADA Prices. This dataset is an unbalanced panel, reporting historic values of different vintages of vehicle models. It includes two price series—retail and trade-in—for 10 U.S. geographic regions—California, Central, Desert, Eastern, Midwest, Mountain, New England, Northwest, Southeast, and Southwest. \(^{18}\) Retail prices represent “the typical selling price” of a transaction between a dealer (as a seller) and a user (as a buyer) for a used vehicle, based on clean conditions; trade-in prices represent “the typical price for a vehicle at trade-in”—that is, a transaction in which a buyer sells an older model to a dealer, using the proceeds as partial payment on a new purchase. NADA prices are updated monthly, based on transaction records at dealerships. We obtained these price data for the month of July for every year from 2003 to 2012.

CEX. The CEX is a quarterly survey of U.S. households that, among other things, reports information about households’ vehicles at the time of the interview, such as the model, the age, whether it is owned or leased, the acquisition date (although it is often missing), and whether it was acquired new or used.

We use these data from 2003 to 2012 (for comparability with our NADA prices mentioned above) to compute some aggregate statistics on households’ vehicle holdings and

\(^{18}\)The States included in each region are available at the following link: http://www.nada.com/b2b/Portals/0/assets/pdf/NADA_Regions%20Datasheet_2013.pdf.
transactions. More specifically, the CEX surveys are quarterly, with most households interviewed for four quarters. We define a vehicle replacement when we observe that a household disposes of a vehicle it previously possessed (either owned or leased) and acquires another vehicle, even if these two events happen in different quarters. This definition mechanically implies households surveyed for fewer quarters are less likely to replace a vehicle than households surveyed for all four quarters. Hence, we restrict our analysis to households surveyed for at least three quarters, computing our statistics at the annual level.

Although the CEX data are useful to understand households’ decisions regarding their vehicles, we should point out that their use poses some challenges. Most importantly, the sample size of each CEX survey is not large—on average, approximately 7,000 households per quarterly survey; because we further restrict our analysis to households surveyed for at least three quarters, we have approximately 5,600 households per year. Moreover, households trade their vehicles infrequently, which implies that the aggregate statistics we construct based on these CEX data are noisy.\(^\text{19}\)

### A.2 Additional Empirical Patterns

1. **The decline in used-car prices was due to a decline in their demand.**

   We use the CEX data to investigate the behavior of households in secondary car markets, which can shed some light on the decline in used-car prices documented in Figure 4. To this goal, we calculate the fraction of households who replaced a used, old car with another used, but younger, car. The left panel of Figure A1 shows this fraction declined during the Great Recession, thereby suggesting a decline in the demand for used cars was the main reason for the decline in used-car prices, rather than an increase in their supply.\(^\text{20}\) The right panel of Figure A1 further reinforces the idea that the increase in the supply of used cars during the Great Recession was likely modest, by displaying the fraction of households who sold cars but did not simultaneously purchased another one. Although this fraction increased during the Great Recession, the magnitude of the overall increase from 2007 to 2010 was modest (approximately .009 percent) and thus it is smaller than the decline

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\(^\text{19}\)Households’ vehicle sales most likely follow vehicle purchases, rather than vice versa. Hence, our procedure could miss households’ replacement when households purchase a vehicle in the last quarter in which they are surveyed (because the subsequent sales are not recorded).

\(^\text{20}\)Figure 5 and the left panel of Figure A1 together suggest the decline in used-car prices did not trigger a substitution from new cars to lightly used (i.e., pre-owned) vehicles.
Figure A1: The left panel displays the fraction of households with cars who replaced at least one used vehicle with another used one during 2003-2012. The right panel displays the fraction of households with cars who disposed of a used vehicle without acquiring another one during 2003-2012.

in replacement purchases documented in the left panel of Figure A1; more generally, the level it reached during those years was lower than the level it reached pre-2008, whereas replacement purchases clearly bottomed during the Great Recession.

(5) The average age of registered vehicles increased during the Great Recession.

Consistent with the decline in scrappage and in new-vehicle registrations we document, we also observe an increase in the average age of the stock of registered vehicles. Figure A2 shows the time series of the average age of all light vehicles in operation. The source of these data is R.L. Polk Co.

Before the Great Recession, this average age was approximately ten years, and it was increasing by approximately 0.1 year per year. Starting in 2008, the average age of the vehicle stock increases more rapidly, by approximately 0.3 year per year, reaching 11.4 years by 2013.

(6) Manufacturers’ cash rebates were limited during the Great Recession.

We complement the analysis of NADA prices with a dataset on cash rebates offered by car manufacturers on purchases of new vehicles. These rebates were advertised on the specialized magazine Ward’s AutoWorld. We find that, despite some fluctuations over

\footnote{We are grateful to Charles Murry for graciously sharing these data with us.}
Figure A2: The figure displays the average age of all light vehicles in operation in the U.S., between 2005 and 2014.

time, these rebates did not increase substantially during the Great Recession.

Specifically, Figure A3 displays the average manufacturer rebate on a new Toyota Camry, one of the popular car models that we analyzed in Section 3, during the period 2006-2011. We focus on the Toyota Camry because we have consistent and large availability of data on rebates over time.\(^{22}\) The figure shows that the rebate exhibits limited variation over time. While the rebate is larger in 2009 than in 2007, rebates were even larger in both 2006 and 2011—i.e., two years of economic expansion. Moreover, the overall variation in the rebates shown in the figure is small relatively to the price of a new Toyota Camry—i.e., between one and three percentage points of the overall price—and substantially smaller than the volatility of used transaction prices discussed in Section 3.

While these data on rebates seem to confirm the robustness of our finding that new-car prices did not change as much as used-car prices during the Great Recession, we should acknowledge one limitation of these rebate data. They report manufacturers’ rebates only, and not those offered by car dealers. Thus, it is possible that dealers sold at a discount their new-car inventories when the financial crisis hit. Nevertheless, monthly data on Industrial Production on Motor Vehicles (available from the Federal Reserve Bank of St. Louis) show a very sharp contraction in new-car production in Fall of 2008, consistent with the fact

\(^{22}\)We construct the series in the figure by averaging over geographic locations and model trims. We find a similar pattern if we focus on single trims of this car.
Figure A3: The figure displays the average cash rebate (in dollars) offered by Toyota on the purchase of a new Toyota Camry between 2006 and 2011.

that, on new-car markets, most of the adjustment came from quantities produced, and not from prices.

B Solution Algorithm

In this appendix, we describe our algorithm to solve for the stationary equilibrium and for the transitional dynamics following unexpected aggregate shocks. We emphasize our novel method to find market-clearing prices in the presence of heterogeneous agents making discrete choices, which seems applicable to a large class of models. We use this method to solve for the stationary equilibrium and for the transitional dynamics of our model.

B.1 Stationary Equilibrium

We now provide the key steps to solve for the stationary equilibrium of the model (see Definition 1 in Section 4.3).

1. We discretize the set of possible states for bonds $b$ (using a fine grid with $N_b = 600$ nodes) and income $w$, using the method in Rouwenhorst (1995) with three nodes.

2. We guess a bond price $p_b$ and car prices $p_n$ for $n = 2, ..., N - 1$. 

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3. We solve for the value function $V$ on the discretized state-space by iterating on the Bellman equation \((5)\). We let households choose bonds on a continuum by interpolating the continuation value. We obtain the policy functions $g_b$ and $g_n$.

4. We compute the stationary distribution $m$ on the discretized state-space using a non-stochastic simulation: we start from an initial distribution and then iterate on the law of motion of the distribution implied by the policy functions and by the transition probabilities of the income and depreciation shocks. We allocate households to grid points for bonds according to the distance between their desired level of bonds and the two closest grid points, following the method in Young (2010).

5. We compute excess demand for bonds and for cars, and update prices using a Quasi-Newton method until all markets clear. We describe the details of the market-clearing procedure in subsection B.3.

**B.2 Transitional Dynamics**

We now provide the key steps to solve for the transitional dynamics, assuming the economy is initially in the stationary equilibrium and households learn about the new aggregate conditions at $t = 0$. To compute the equilibrium dynamics, we need to solve for sequences of prices $\{p_{b,t}, p_{n,t}\}_{t=0}^{T-1}$, policy functions $\{g_{b,t}, g_{n,t}\}_{t=0}^{T-1}$, and household distributions $\{m_t\}_{t=1}^{T-1}$ such that households maximize utility, all markets clear in each period, and the distribution evolves according to households’ policy functions, to the transition probabilities of the idiosyncratic income and to the depreciation shocks. We apply a sequential solution algorithm as that described by Ríos-Rull (1998).

1. We compute both the initial and the final stationary equilibrium using the algorithm described above.

2. We set the number of periods (years), $T = 30$, by which we assume the economy converges to the final stationary equilibrium.

3. We guess a sequence of bond prices and car prices $\{p_{b,t}, p_{n,t}\}_{t=0}^{T-1}$ for $n = 2, \ldots, N - 1$.

4. We initialize the algorithm parameter $S = 0$ that we use in the following steps.
5. We obtain a sequence of policy functions \( \{g_{b,t}, g_{n,t}\}_{t=S+1}^{T-1} \), by iterating backward in time from \( t = T - 1 \) to \( t = S + 1 \) and solving the household maximization problem in each period, using interpolation of the continuation value. Notice that at \( t = T - 1 \), the continuation value is simply given by the value function \( V \) associated with the final stationary equilibrium.

6. Taking as given all prices, decision rules, and value functions from \( t = S + 1 \) onwards, we look for the prices \( p_{b,S} \) and \( p_{n,S} \) and associated decision rules \( g_{b,S} \) and \( g_{n,S} \) such that all markets clear at \( t = S \), given the distribution of households \( m_S \). We look for market-clearing prices using a Quasi-Newton method, described in more detail in subsection B.3.

7. We update the distribution of households using the obtained policy functions and compute \( m_{S+1} \), using a non-stochastic simulation. We allocate households to grid points for bonds according to the distance between their desired level of bonds and the two closest grid points, following Young (2010).

8. We iterate on steps 4-7 by sequentially setting \( S = 1, \ldots, T - 1 \), hence clearing markets one period at a time and obtaining a new sequence of prices.

9. We compute a convex combination of the guessed price sequence and the newly obtained price sequence and restart from step 4. We continue this procedure until convergence of the price sequence.

**B.3 Market-Clearing Method**

Our model features heterogeneous agents making a discrete choice over car quality. The discreteness of the policy functions generates a challenge in clearing markets: the excess demand function for a given car quality is a step function, leading to either inaccuracy or failure of standard root-finding methods.

To explain this problem and our proposed solution, we now use a simplified version of our model in stationary equilibrium, in which only two car qualities exist, \( n = 1, 2 \). Thus, we only need to solve for the relative price of cars of quality \( q_2 \), \( p \equiv p_2/p_1 \). Car scrappage is exogenous and so is the bond price. Moreover, let us restrict attention to heterogeneity
in income $w$ and wealth $b$, by assuming all households have the same no-car utility type $\theta$. Thus, we consider the discretized space for the state $(b, w, n)$.

First, we introduce some convenient notation. Let us consider all households with a given income realization $\bar{w}$ who own cars of a given quality $\bar{n}$. These households differ in their wealth $b$, which we discretized on a grid $\{b_j\}$ for $j = 1, ..., N_b$, where $j$ denotes a grid point.

Let $m_j(\bar{w}, \bar{n})$ be the fraction of households at grid point $j$ at the beginning of the period. Let $b^*(\bar{w}, \bar{n}; p) \in [\phi, b_{N_b}]$ be the threshold for wealth such that only households with wealth above $b^*(\bar{w}, \bar{n}; p)$ choose a car of quality $q_1$, given a relative price $p$; that is

$$g_n(b, \bar{w}, \bar{n}) = \begin{cases} 
2 & \text{if } b \leq b^*(\bar{w}, \bar{n}; p) \\
1 & \text{if } b > b^*(\bar{w}, \bar{n}; p).
\end{cases} \tag{B1}$$

Notice that, in general, $b^*(\bar{w}, \bar{n}; p)$ does not coincide with any grid point for $b$. Let $b_J$ and $b_{J+1}$ be the two neighboring grid points, such that $b_J < b^*(\bar{w}, \bar{n}; p) < b_{J+1}$.\footnote{In the interest of simplifying notation, we avoid explicitly expressing $J$ as a function of $(\bar{w}, \bar{n})$, but it is understood that each income and car-quality state has associated thresholds and neighboring grid points.}

Total demand for cars of quality 2 coming from households with income $\bar{w}$ and car $\bar{n}$ equals $\sum_{j=1}^J m_j(\bar{w}, \bar{n})$, that is, the mass of households whose wealth is below the threshold. Under standard continuity properties of the value function $V$, the threshold is a continuous function of the price $p$. Hence, for small changes in $p$, the threshold $b^*(\bar{w}, \bar{n}; p)$ is still between the same grid points. Accordingly, no change occurs in the total quantity demanded by households with income $\bar{w}$ and car $\bar{n}$. A sufficiently large price change, instead, implies the threshold crosses one of the closest grid points, either $b_J$ or $b_{J+1}$, leading to a discrete change in the quantity demanded. This point shows that total demand conditional on a given realization of income and car quality is a step function.

Aggregate demand for cars of quality $q_2$ is the sum of demands coming from all discrete income and car-quality values. Because the sum of multiple step functions is also a step function, aggregate demand is a step function. Moreover, the total amount of cars of quality $q_2$ is fixed at the beginning of the period. Hence, total excess demand (demand minus supply) is also a step function with respect to the price.

Finding a zero of a step function is problematic for numerical non-linear equation solvers.
Moreover, the simple approach to stop at a price that gives the minimum absolute excess demand can be quite inaccurate even with a large number of grid points.\footnote{In our model, this approach does not achieve a market-clearing error below $10^{-3}$, even with 1,000 grid points for bonds. Furthermore, this issue cannot be easily solved by using Monte-Carlo simulation instead of a non-stochastic simulation. One can use similar arguments to show that a Monte-Carlo simulation also leads to an excess demand that takes the shape of a step function. Moreover, the size of the market-clearing error guaranteed by this approach equals the inverse of the number of agents used in the simulation. This relation leads to a substantially higher computational cost than our proposed method, for a given desired level of accuracy.}

We propose an intuitive, efficient, and easily applicable solution to obtain a continuous excess demand function, achieving accuracy in market clearing. The key idea is that continuity can be achieved by making the distribution of households close to the threshold depend on the distance between the threshold and the neighboring grid points.

Specifically, we compute the threshold associated with a given guessed price. Next, we take the beginning-of-period distribution $m$ and we move a fraction of agents from grid point $J$ to $J+1$, proportionally to the the distance between the threshold and grid point $b_{J+1}$:

$$m_{J\rightarrow J+1} = \frac{b_{J+1} - b^*(\bar{w}, \bar{n}; p)}{b_{J+1} - b_{J-1}} m_J. \quad \text{(B2)}$$

We rationalize this choice as follows. We interpret each mass point $m_J$ as a discrete approximation of the true distribution of households with a level of wealth in a neighborhood of grid point $b_J$. We propose an alternative, continuous approximation of this distribution, which we construct by distributing households at grid point $b_J$ over the interval $[b_{J-1}, b_{J+1}]$. If we distribute these households using a uniform distribution, a fraction $\frac{b_{J+1} - b^*(\bar{w}, \bar{n}; p)}{b_{J+1} - b_{J-1}}$ of households are at grid point $b_J$ according to the discrete approximation of the distribution, but are instead to the right of the threshold $b^*(\bar{w}, \bar{n}; p)$ under the uniform approximating distribution.\footnote{Alternative closed-form expressions for the mass of agents who move between grid points can be found assuming other approximating distributions, for instance a truncated normal. This alternative assumption leads to quantitatively negligible differences in the solution.} Hence, they should make the same car-quality choice as households at grid point $b_{J+1}$.

Symmetrically, we move a fraction of agents from grid point $J+1$ to $J$ as follows:

$$m_{J+1\rightarrow J} = \frac{b^*(\bar{w}, \bar{n}; p) - b_J}{b_{J+2} - b_J} m_{J+1}. \quad \text{(B3)}$$

We get a new distribution $\tilde{m}$, which coincides with $m$, except at the grid points that are
closest to the thresholds, in particular $\tilde{m}_j = m_j + m_{j+1\rightarrow j}$ and $\tilde{m}_{j+1} = m_{j+1} + m_{j\rightarrow j+1}$.  

Next, we use the modified distribution to evaluate aggregate demand for car quality $q_2$. Thanks to the continuity of $b^*$ with respect to the price, it is easy to prove that the expression $\sum_{j=1}^{J} \tilde{m}_j(\bar{w}, \bar{n})$ is a continuous function of $p$. Hence, total excess demand is a continuous function of the price, allowing us to find a zero with arbitrary accuracy with standard non-linear solvers.

In the interest of consistency in the treatment of all the markets, we also use $\tilde{m}$ to clear the bond market. Moving agents to close grid points for bonds is similar to the way we deal with the discreteness of the grid and with the continuity of the bond policy function $g_b$, following the simulation method Young (2010) proposed.

While we referred to a simplified model, the method generalizes to the richer model of Section 4. In practice, our algorithm to clear markets both for the stationary equilibrium and the transitional dynamics works as follows:

1. We guess prices for bonds and car qualities.

2. We solve the household problem and compute all the thresholds for wealth such that households are indifferent between any two car qualities chosen in equilibrium, for each levels of income, car quality, and type.

3. We transform the distribution by shifting households close to the thresholds proportionally to the distance between the thresholds and the neighboring grid points as in equations (B2) and (B3).

4. We use the transformed distribution to evaluate excess demand for bonds and car qualities.

5. We update prices using a Quasi-Newton method until markets clear.

6. We use the obtained policy functions and the transition probabilities of the idiosyncratic shocks to update the transformed distribution associated with equilibrium prices and get the next beginning-of-period distribution.
C Additional Numerical Results

In this appendix, we provide two sets of additional results. First, we perform the decomposition of the role of equilibrium in secondary markets and the role of transaction costs in the model with both credit and aggregate income shocks of Section 6.3. Second, we show that the key mechanism highlighted in the paper does not arise only in presence of credit-supply shocks, but more generally, in the presence of other shocks that affect the wealth-income distribution asymmetrically.

C.1 Credit and Income Shock: Inspecting the Mechanism

We study the separate roles of used durable prices and transaction costs in the economy hit by a credit tightening and an aggregate income shock, as in Section 6.3. The results of this decomposition are very similar to our findings in the presence of a credit shock only (Section 6.1.1), thereby emphasizing that accounting for equilibrium in secondary markets is crucial even in the presence of aggregate income changes.

First, we re-compute the transitional dynamics assuming that the secondary market does not clear, and cars can be traded at their initial prices. Figure C1 displays the resulting equilibrium dynamics. As we found in the case of a credit shock only, scrappage declines substantially and new-car production increases in response to the shocks. Hence, equilibrium in the secondary market is key to induce a fall in new car sales, consistent with the evidence during the Great Recession. Relative to Figure 9, the aggregate income shock further decreases scrappage and dampens the initial increase in car sales, which peak three years after the initial shocks at over fifty percent above the steady-state value.

Second, we re-compute the transitional dynamics with credit and aggregate income shock, clearing both credit and car markets, but setting the transaction costs equal to zero, as we did in Figure 10 for the baseline case without aggregate income shocks. Figure C2 displays the results. The dashed line represents the dynamics in the absence of transaction costs, whereas the solid line reproduces the dynamics obtained in Figure 12 in the presence of transaction costs. Similar to our findings of Section 6.1.1, the absence of transaction costs induces a spike in downgrading activity in the recession, leading to a temporary increase in the trading volume of used cars, a more sizable decline in used prices, and a larger fall in scrappage and new production, compared to the economy with calibrated transaction
Figure C1: Credit and income shock in the absence of secondary-market clearing. The economy is hit by the same credit and income shock as in Figure 12. The bond market clears. However, the market for used cars does not clear—that is, cars can be traded with the rest of the world at the prices prevailing in the initial stationary equilibrium. The left panel displays scrappage and the right panel displays sales of new cars.

costs.

C.2 Skewed Income Shock

We now show that the key mechanism highlighted in the paper arises also in presence of skewed income shocks, even without shocks to credit supply. The empirical literature on the skewed effects of business cycles (e.g., Guvenen, Ozkan, and Song, 2014) motivates us to study the effects of a shock that decreases the income of low-income households only over a period of two years. We assume the income realization of low-income households (i.e., households whose income is the lowest point in our grid) decreases by ten percent for two years. The persistence of the shock over two years implies that this shock affects the income process of all households, either directly because of its current realization, or indirectly because of the possibility of a transition to the low-income shock in the second period. For simplicity, we focus on the equilibrium in the car market and abstract from bonds-market clearing, but the results are robust to general-equilibrium effects from the interest rate.

Figure C3 illustrates the effects of this shock to low-income households on the key variables of interest. The qualitative effects are similar to those arising after the credit
tightening analyzed in Section 6: low-income households, who are temporarily hit by the income shock, choose to postpone the scrappage of their low-quality cars, inducing a decline in used-car prices and in the volume of used-car trade; in turn, this equilibrium effect induces higher-income households to postpone the replacement of their intermediate-quality cars, leading to a decrease in new-car sales. However, the equilibrium dynamics are quantitatively smaller than those reported in Section 6: the drop in new-car sales is less than two percent. Hence, this analysis suggests that skewed income shocks may have contributed to the empirical patterns described in Section 3, but they are unlikely to be their main driver. Nevertheless, they could be potentially relevant to account for the dynamics of
Figure C3: Income shock to low-income households. The economy is in the stationary equilibrium at \( t = -1 \), and households learn about the new income path at \( t = 0 \). The horizontal axis displays time \( t \). The top-left panel displays scrappage; the top-right panel displays the volume of trade of used cars; the bottom-left panel displays the price \( p_2 \) of quality-\( q_2 \) cars; and the bottom-right panel displays sales of new cars. The solid line displays the baseline case (credit shock), and the dashed line displays the case of an income shock to low-income households.

durable-goods purchases during other business-cycle episodes in which credit markets were not as affected as during the Great Recession.