Bargaining with Private Information and the Option of a Compulsory License∗

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Abstract. We study the bargaining problem between a country and a multinational firm over the firm’s entry to sell a patented product in the country’s market, when the country has the threat of issuing a compulsory license. We examine how private information about the firm’s payoffs affects the timing and nature of entry in the case where the compulsory license expands the surplus for some firm types. Efficiency calls for all firms with sufficiently high valuations to enter immediately, and those with lower valuations to wait for a compulsory license. Equilibrium play begins with a sequence of offers by which the country successively screens the firms, inducing a flow of entry from firms with relatively high valuations. We show that if the maximum initial valuation is below a critical level, there will be a bargaining pause equilibrium in which negotiations lie dormant for a finite period, with all remaining firms being licensed at the earliest possible opportunity for the country to issue a compulsory license. If the maximum initial valuation exceeds the critical value, there will be a licensing delay equilibrium in which the country will continue to bargain beyond the earliest possible date for a compulsory license to be issued, with bargaining concluding in finite time with the licensing of all remaining firms. Two types of inefficiency arise—too many firms are forced to wait for the license, and some firms who do not wait for the license are forced to delay agreements.

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1 Introduction

1.1 Motivation

We consider the bargaining problem between a country and a multinational firm over the terms on which the firm will offer a newly patented product for sale in the country’s domestic market. We assume that the firm has private information about the surplus that will be created by its entry into the market, as well as about the magnitude of the spillover effects that entry will have on the firm’s other markets. The country makes offers to the firm over the terms on which the firm will enter. In contrast to the standard dynamic bargaining problem associated with the Coase Conjecture, we assume that after an exogenously given period of time, the country has the option of issuing a compulsory license, allowing another firm to sell the product in the country’s domestic market in return for exogenously-set royalty payments to the patent holder.

Our problem is motivated by the Trade Related Intellectual Property (TRIPS) Agreement of the World Trade Organization (WTO). Prior to the TRIPS agreement, patent protections were ineffective in lower-income countries. Firms were accordingly reluctant to enter such markets, while the countries often obtained access to patented products by purchasing from imitators producing copies of the patented products. The TRIPS agreement requires all WTO members to provide a minimum level of patent protection, precluding access to patented products via imitators. In response, and at the insistence of developing countries, the TRIPS agreement includes provisions for compulsory licensing: If a firm does not introduce a newly-patented product into a low-income WTO member within a reasonable period of time, the country can force the firm to issue a license allowing the sale of the product in the country.

The TRIPS agreement strengthened patent protection among WTO members, in principle making entry more attractive and ensuring access to patented products without resorting to compulsory licensing. However, it is well documented that newly patented drugs become available in middle and low income countries only after significant delays, and in many cases are not available at all. For example, Danzon, Wang and Wang [8] studied the pattern of market entries for 85 new chemical entities in 25 major markets from 1994-1998. They found that only about half of the potential entries actually occurred. Similar evidence of launch delays is found in a more extensive study by Cockburn, Lanjouw, and Schankerman [7], who examine 642 drugs in 76 countries over the period 1983-2002. There is also ample evidence that compulsory licenses are important in ensuring access. Beall and Kuhn [3] identify 24 cases between 1995 to 2011 in which compulsory licensing was used as a threat in bargaining between governments and patent holders of pharmaceuticals. Compulsory licenses were issued in half of the cases, and in the remaining cases the threat of a compulsory license
typically prompted either price reductions by the producer or the issuance of a voluntary license.

Entry into a new market can have spillover effects on a firm’s existing markets, and the evidence suggests that access to newly patented products is especially problematic when these spillover effects are large. Countries with lower prices or smaller markets experience fewer new-product launches and longer launch delays. As Goldberg [11] argues, this reluctance to enter lower income markets may reflect “global reference pricing,” whereby high income countries use the prices at which pharmaceuticals are sold in other markets in setting the maximum price allowed in their own market. Similarly, arbitrage between markets may cause entry into a lower price market to have a detrimental effect on profits in high income markets.¹ Cockburn, Lanjouw, and Schankerman [7] find that entry of newly patented drugs occurs later in markets where per capita income is lower, the products are subject to price controls, and patent protection is weaker. We expect prices to be lower in low income markets, and weaker patent protection may lead to greater arbitrage opportunities, both leading to larger spillovers to high income markets.²

Compulsory licences play an especially important role when spillovers are important. Beall and Kuhn [3] find that the threat of a compulsory license was used in negotiations over products where the demand was high in both the middle/low income country and in the high income countries, so that pricing spillovers to high income markets are likely to have been important. For example, 16 of the 24 cases studied by Beall and Kuhn involved drugs for HIV/AIDS. More recent cases have tended to involve drugs for non-communicable diseases such as cancer and heart disease, as discussed by Cherian [5].³

The seller in our bargaining problem (in this case a country selling market access) makes a sequence of increasingly attractive offers to a privately informed buyer (a firm contemplating entry), with the objective of extracting surplus from the highest valuation buyers. In addition to the induced deadline for the bargaining to end, bargaining under the shadow of a compulsory license differs from a standard bargaining problem in two important ways. First, because of the spillovers to the firm’s existing markets, the total surplus arising out of an agreement may be negative. Second, we expect spillovers not to arise under compulsory licensing, and hence the total surplus may expand when the bargaining deadline is reached, perhaps even moving from negative to positive.

¹For example, arbitrage within the EU is legal due to the adoption of regional exhaustion of property rights. Gansland and Maskus [10] estimate that pharmaceutical prices in Sweden declined by 12% following its entry into the EU, which allowed imports of patented pharmaceuticals from lower price markets. Illegal arbitrage also exists, as in the case of imports of patented pharmaceuticals into the US from Canada in violation of US patent law.

²We view price controls as an indication of the type of bargaining over the terms of entry that we model, though we simplify our model by assuming that the terms of bargaining include only the price of entry.

³The TRIPs agreement contains separate provisions for entry in the case of a national emergency, such as an epidemic, that does not require delay. The requirement of a reasonable period of time before a compulsory license can be issued, and hence our analysis, applies to non-emergency cases of entry, as would arise with drugs for cancer, heart conditions, and so on.
1.2 Related Literature

The literature on bargaining under incomplete information, with the Coase Conjecture (Coase [6], Gul, Sonnenschein and Wilson [12]) as its centerpiece, is extensive. We refer to Ausubel, Cramton and Deneckere [2] for an introduction to the literature, and focus here on the two most closely related papers.

Fuchs and Skrzypacz [9] consider a bargaining problem with an exogenously fixed deadline at time $T$. In each period, the seller makes an offer, and the buyer accepts or rejects. If no acceptance occurs before the terminal time $T$, the game ends with the buyer and seller receiving shares $\alpha$ and $\beta$ of the discounted surplus, where $\alpha + \beta \leq 1$, with both positive. In the limit as the length of a time period approaches 0, (focusing on the case $\alpha + \beta < 1$, so that some surplus is dissipated upon hitting the deadline), there exists a positive density on $[0, T)$ describing the probability of reaching an agreement before the deadline, and there is also an atom of trade at time $T$. Fuchs and Skrzypacz motivate their analysis as explaining the high frequency of agreements that occur at the deadline in labor negotiations. As $T \to \infty$, the atom at time $T$ disappears and the expected time to trade and the expected price converge to 0, giving a version of the Coase conjecture.

The problem of bargaining with the threat of a compulsory license that we consider here differs in several ways. First, the uninformed party may issue the compulsory license at the deadline, but may also continue to negotiate, with the option of issuing the license at a later date. In particular, we treat the WTO requirement of a “reasonable amount of time” for negotiation as an exogenously imposed limit representing the earliest time at which the country can issue a compulsory license. Second, as noted above, the negative spillover from entry on profits in other markets may be sufficiently large that the joint surplus from entry is negative for some firms, precluding entry in the absence of a compulsory license. Third, the spillover to high income markets is mitigated when a compulsory license is issued, so that the joint surplus from a compulsory license may exceed that from entry. Finally, we relax the assumption that the ratio of the deadline payoff to the entry payoff is common across all types.

Bond and Saggi [4] study the effect of compulsory licensing when the firm and country engage in an alternating offers bargaining game where the parties have complete information about payoffs. They show that the equilibrium of

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\(^4\)Spier [16] considers a case of pre-trial negotiation in which the deadline is similarly endogenously imposed, in the form of a decision by the plaintiff to go to trial. Spier’s model differs in that the payoff to be divided does not shrink over time, so that discounting is not sufficient to induce immediate settlement in the complete information case.

\(^5\)This is the counterpart of setting $\alpha + \beta > 1$ in Fuchs and Skrzypacz. If the spillover is due to reference pricing by high income countries, the surplus may expand under a license because the price charged by a compulsory licensee is likely to have less of an impact on the reference price than a price set by the patent holder. In the case of arbitrage, the surplus can expand because the compulsory license requires that the licensee sell only in the domestic market. The country has an incentive to monitor this, since export sales by the licensee could lead to a complaint to the WTO by the parent country of the patent holder. The anticipation of a larger surplus in the future can also induce delay in Acharya and Ortner [1] and Merlo and Wilson [13].
the complete information bargaining game results in the constrained (that the country receive at least as high a payoff as provided by a compulsory license) efficient outcome. In the case where the compulsory license increases joint surplus, the constrained efficient outcome can result in either immediate entry or a compulsory license, with the latter occurring if and only if the discounted joint surplus from issuing a compulsory license at the first opportunity exceeds the joint surplus from immediate entry.\textsuperscript{6} The emphasis of their analysis is on how the option of a compulsory license affects the payoffs of the respective parties. The focus in the present paper is instead on how the existence of private information affects the efficiency of the entry decision.

\subsection{Preview}

Bargaining begins with a succession of offers from the country, designed to induce entry by firms who generate a relatively high social surplus. The unique equilibrium takes one of two forms, depending on the parameters specifying the upper bound on the value of entry and the delay required until a compulsory license can be issued. If the upper bound on valuations is relatively low, the successive skimming of high valuation firms continues to a point at which entry ceases and both parties wait until enough time has passed that a compulsory license can be issued, at which point the remaining firms are subjected to a license. We refer to this as a bargaining pause equilibrium. Alternatively, if the upper bound on the value of entry is sufficiently high, then the country finds it optimal to continue bargaining for a finite period of time after the deadline, and then issuing a license. We refer to this as a licensing delay equilibrium.

The bargaining pause equilibrium leads to two sources of inefficiency. Some firms enter only after a delay, whereas the efficient outcome calls for immediate entry, and some firms are subject to licensing that would enter voluntarily and immediately in the efficient solution. The licensing delay equilibrium has an additional inefficiency due to the delay in issuing a license.

An obvious comparison for our model is to one with an exogenously fixed bargaining deadline and no compulsory licensing, as examined by Fuchs and Skrzypacz \cite{9}. With a deadline but no compulsory license, the equilibrium outcome leads to a payoff for the country identical to that it would receive if it simply waited until the deadline and then issued a single take-it-or-leave-it offer. Our country fares better in the bargaining pause equilibrium than if it waited until the licensing opportunity and issued a single take-it-or-leave-it offer, coupled with licensing for those firms that rejected the offer. The ability to issue a license thus endows the country with some bargaining power beyond that created by the induced deadline.

\textsuperscript{6}Bond and Saggi \cite{4} also consider the case where spillovers arise under a compulsory license. Spillovers may render the total surplus from a compulsory license negative, so that the efficient outcome prescribes no entry, while the country’s payoff from a license may be positive, so that constrained efficiency is inconsistent with no entry. In this case, the bargaining outcome will call for preemptive entry, in which the firm enters just before the compulsory licensing deadline, delaying the negative payoff from entry as long as possible and then entering to avoid the even lower payoff from licensing.
On the other hand, the payoff to the country in the licensing delay equilibrium is lower than would be obtained by waiting until the first licensing opportunity, and then making a take-it-or-be-licensed offer. The fact that the country can delay issuing the license weakens its bargaining power relative to the case where it is committed to issuing the license at a fixed deadline. Nonetheless, because the total surplus under the compulsory license is larger than under entry when the firm’s valuation is sufficiently low, bargaining will terminate in finite time with a license, and with a positive payoff to the country.

2 The Model

2.1 The Bargaining Problem

We consider a firm facing the option of entering a country. The firm is characterized by a (fixed) parameter $q$ describing the quality of its product. If the firm does not enter, the country earns a payoff $0$ and the firm earns profit $\hat{\pi}^N$ in its existing markets.\(^7\) If the firm enters, it pays an entry price $p$ to the country, produces a product of quality $q$, and earns a total (discounted present value) profit of

$$\pi^N(q) + \pi^S(q) - p. \quad (1)$$

The function $\pi^S(q)$ gives the profit earned in the country of entry, as a function of the quality of the firm’s good. To keep things simple, we assume that $\pi^S(q)$ is the total surplus created in the country, and that the firm extracts all of this surplus. The function $\pi^N(q)$ gives the profit earned in the firm’s existing markets, as a function of the quality of the firm’s product $q$. Obviously, the firm’s profits in its existing markets will also depend on other variables, such as the price set in those markets, but we suppress the notation for these other variables, just as we did in defining $\hat{\pi}^N$ above. The country’s payoff from entry is the entry price $p$ paid by the firm.

The difference $\hat{\pi}^N - \pi^N(q)$ reflects the spillover effects of the firm’s entry. For example, entering the new market may trigger reference pricing provisions in existing markets. Alternatively, the prospect of customers arbitraging across markets may constrain the prices the firm can charge in its other markets. The expansion in output required to enter the country may increase the firm’s costs of serving its other markets. An important feature of the compulsory licensing problem we consider is the possibility that these spillovers may be sufficiently large that they swamp the gain from entry, so that the firm would never voluntarily enter the market. Indeed, a significant fraction of patented pharmaceutical products are never introduced into low and middle income country markets, prompting developing countries to insist on the ability to issue compulsory licenses in return for their agreement to provide patent protection.

\(^7\)Discussions of firms entering countries often refer to the firm’s existing market as the “North” and the market to be entered as the “South,” motivating our superscripts.
To simplify the notation, let
\[
\begin{align*}
v &= \pi^S(q) \\
s &= \hat{\pi}^N - \pi^N(q) \\
x &= v - s = \pi^S(q) + \pi^N(q) - \hat{\pi}^N.
\end{align*}
\]

Then the value \(v\) is the surplus generated by entry in the country of entry. The spillover \(s\) is the loss in the firm’s existing markets caused by the firm’s entry into the country in question. The total surplus created by entry is \(x\).

The firm is privately informed of the value of \(q\). The values of the entry surplus \(v\) and the spillover \(s\) (and hence the total surplus \(x\)) depend on the firm’s quality \(q\), and hence are similarly known to the firm but unknown to the country. We assume that \(v\) and \(s\) are increasing, linear functions of \(q\)—higher quality products generate higher surplus in the country of entry, but also greater spillover effects—and that \(q\) is drawn from a uniform distribution.\(^8\) Moreover, we assume that the surplus \(v\) is more responsive to quality than is the spillover \(s\), so that the total surplus \(x\) is also increasing in \(q\). Finally, we assume that the country’s surplus from entry \(v\) is positive throughout, but we keep the problem interesting by allowing the total surplus \(x\) to be negative for small values of \(q\) and positive for large values of \(q\).

The value of the total surplus, \(x\), will be a key variable, and it is convenient to reformulate the uncertainty in terms of \(x\) and then summarize these collected assumptions:

**Assumption 1**

1.1. \(x\) is uniformly distributed on \([-a, b]\), where \(a, b > 0\).

1.2. \(v(x) = v_0 + \xi x\), where \(\xi \geq 1\) and \(v(-a) \geq 0\) (i.e., \(v_0 > a\xi\)).

The assumption \(a > 0\) ensures that, in the absence of a compulsory license, the efficient outcome will call for some types of firm (for which \(x < 0\)) to not enter the country, with entry on the part of all firms for which \(x > 0\).

We divide time into periods of length \(\Delta\), representing the length of time for which the country is committed to its current offer. In each period, the country has the option to make an offer to the firm, which the firm may either accept or reject. If the country’s offer of entry at price \(p\) in period \(n\) is accepted by the firm, the country obtains a payoff \(e^{-r\Delta_n}p = \beta^n p\) and the firm receives \(e^{-r\Delta_n}(x - p) = \beta^n (x - p)\), where \(\beta = e^{-r\Delta}\) is the discount placed on a period’s delay.\(^9\)

In each period, starting at an exogenously given time \(C\) and hence period \(C/\Delta\), the country can either issue a compulsory license or offer a price and

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\(^8\)The convenient feature of the uniform distribution is that truncations of the distribution have the same shape as the original distribution. Fuchs and Skrzypacz [9] exploit the same property while working with the more general distribution function \(F(x) = x^\alpha\), with the uniform as a special case.

\(^9\)Though we do not impose such a restriction, equilibrium prices will be positive. Hence, the possibility that a firm for whom \(x < 0\) would collect an entry subsidy and then renege on entry does not arise.
wait until the next period (in the event of a rejection) to face the same choice again. Negotiations can potentially continue indefinitely, with the acceptance of an offer or the issuance of a compulsory license ending the bargaining game.

Our modeling of the compulsory license reflects the provisions of the Trade Related Intellectual Property (TRIPS) Agreement of the World Trade Organization (WTO). Article 31 of the TRIPS Agreement allows countries to force patent holders to enter into licensing agreements, provided that there is a reasonable amount of time for an agreement to be reached between the patent holder and a licensee before a compulsory license can be issued, and provided that adequate remuneration be provided to the patent holder. We thus assume that the country can force the issuance of a license at any time on or after some fixed time $C$, as long as the patent holder receives a royalty payment of $R$. The issuance of a compulsory license can be the subject to dispute resolution under the WTO, and so it is reasonable to interpret the TRIPS guidelines as constraints on $C$ and $R$ that must be met by the country if it grants a compulsory license, and we accordingly treat $C$ and $R$ as exogenously given.\footnote{Standards for the earliest license time $C$ and the royalty rate $R$ have emerged out of interpretations of the TRIPS agreement. The WTO itself does not identify a minimum time required before a compulsory license can be issued. However, it does require adherence to the standards of the Paris Convention on Industrial Property. Article 5 of the Paris convention specifies a minimum time period of four years from the application for a patent or three years from the granting of a patent, whichever is greater. With regard to compensation, the TRIPS agreement contains strict rules concerning royalties, and the WHO [14] provides specific guidelines on what represents adequate remuneration in the case of compulsory licenses. Patent holders have received royalties ranging from .5% of the generic price to 2% of revenues from the sale of the product in cases where compulsory licenses were issued. These rates are substantially below market rates, which are closer to 5% of revenues. These constraints and the existence of the WTO obligation leave little room for additional bargaining if a compulsory license is issued, so we view it as quite reasonable to interpret $R$ as being fixed and not too large (cf. Assumption 2).}

If a compulsory license is issued, the payoff to the country is $\alpha v - R$, where $\alpha > 0$ and $R$ is the royalty payment. We interpret $\alpha < 1$ as reflecting the case in which the quality of the licensee’s product is below that of the patent holder. The possibility of $\alpha > 1$ could arise if the licensee is able to avoid regulatory or marketing costs that the patent holder would face in entering the market. We assume that a compulsory license gives rise to no spillovers. For example, a licensed counterpart of a firm’s product is often deemed irrelevant for reference pricing purposes. Similarly, the licensed product may not be a viable competitor in the world market, and indeed the licensee is prohibited from exporting the product under WTO rules. Production by a licensee typically has no impact on the firm’s costs. As a result, compulsory licensing may generate a larger surplus than does unlicensed entry.

It is clear that a license will be of no use to the country if either $\alpha$ is too small or $R$ too large. To focus on cases where compulsory licenses will be a credible threat for the country, we assume:

**Assumption 2**

\[
R \leq \alpha \frac{1}{2} [v(-a) + v(0)]
\]
\[2.2\] \( \alpha \xi \in (\frac{1}{2}, 1) \).

Assumption 2.1 puts an upper bound on the ratio \( R/\alpha \) which ensures that, if told nothing about the firm’s type other than that the total surplus generated by unlicensed entry is negative, the country would prefer to license rather than forego the product.

Turning to Assumption 2.2, if \( \alpha \xi \geq 1 \), then for every value of the surplus \( x \), the surplus from issuing a license is larger than the surplus from unlicensed entry.\(^{11}\) We are interested in cases in which the compulsory license does not always expand the surplus, and hence assume \( \alpha \xi < 1 \), ensuring that entry without a license yields a larger surplus than does the license for large values of \( x \).

The requirement in Assumption 2.2 that \( 1/2 < \alpha \xi \) ensures that the license does not dissipate too much of the country’s value. The ability of a licensee to produce a high quality version of the product is a significant determinant of the value of \( \alpha \). Initially, TRIPs required that licensees had to be located in the country issuing the license, which made the compulsory license an ineffective threat for low income countries without a domestic pharmaceutical industry. A subsequent WTO declaration in 2003 allowed the issuance of a compulsory license to import the product from a foreign firm that was capable of producing it. This change resulted in a substantial increase in the use of compulsory license as a threat, and motivates our assumption that \( \alpha \) is not too small.\(^{12}\)

### 2.2 Benchmark Payoffs

The country always has the option of eschewing licensing and continuing to bargain with the firm, and the resulting payoff will be a useful benchmark. To calculate this payoff, let there be a finite collection of periods \( \{0, 1, \ldots, N\} \) and let \( V^N_n(x_n) \) be the expected payoff to the country from the continuation of the bargaining game without a compulsory license, given that period \( n \) has been reached without an agreement and given that the set of remaining types is \([-a, x_n]\).\(^{13}\) Then this function must satisfy

\[
V^N_n(x_n) = \max_{x_{n+1}} \left( \frac{x_n - x_{n+1}}{x_n + a} \right) P_{n+1}(x_{n+1}) + \beta \left( \frac{x_{n+1} + a}{x_n + a} \right) V^N_{n+1}(x_{n+1}),
\]

\[2\]

\(^{11}\)This is obvious for \( x < 0 \), and for \( x \geq 0 \) follows from \( \alpha v(x) = \alpha [v_0 + \xi x] > \alpha \xi x > x \).

\(^{12}\)In practice, royalty payments may vary with the revenues from the product. We can allow for this possibility by expressing the royalty payment as \( R(x) = R_0 + \mu x \), where \( \mu > 0 \). With this specification, the skimming property in Lemma 2 will continue to hold as long as \( \mu < 1 \). Our results for the case of a constant royalty payment will then be unaffected if Assumption 2 is modified to maintain the bounds on the profitability of a compulsory license. In particular, the fact that the royalty is simply a transfer between country and firm means that the threshold values obtained with a constant royalty payment are unaffected. Therefore, we have opted for the simpler specification of the royalty payment.

\(^{13}\)A straightforward successive skimming result, established in Lemma 2 for the case of a compulsory license, ensures that in each period \( n \) the set of set of types who have not accepted an offer will be an interval \([-a, x_n]\).
where \( p_{n+1}(x_{n+1}) \) is a function specifying the price \( p_{n+1} \) the country must set in order to induce firms (and only such firms) to enter if their valuation exceeds \( x_{n+1} \). Hence, the country’s task in period \( n \), given \( x_n \), is to choose \( x_{n+1} \) and \( p_{n+1} \) to maximize this expression, recognizing that \( x_{n+1} \) and \( p_{n+1} \) are linked by the firms’ optimization. The following Proposition combines Sobel and Takahashi’s [15] Theorem 6 with a limiting (in the length of the horizon) argument. (The online appendix presents a proof, which is a straightforward backward induction argument.) Throughout, “unique” equilibria are unique up to some freedom in how a measure-zero set of firms breaks indifferences.

**Proposition 1** Let Assumption 1.1 hold, let there be no compulsory licensing, and let the firm and country bargain for periods \( n \in \{0, 1, \ldots, N\} \). Then there exists a unique perfect Bayesian equilibrium, with a value function \( V^N_n \) satisfying (2). Taking the limit as \( N \to \infty \) yields the expected payoff to the country from the infinite-horizon bargaining problem with no compulsory licensing,

\[
V^\infty_0(x) = \left( \frac{\sqrt{1 - \beta} + \beta - 1}{2\beta} \right) \frac{x^2}{x + a}.
\]

In comparison, a country able to make a monopoly offer, given that types on the interval \([-a, x]\) have not entered, would receive payoff

\[
V^m(x) = \frac{1}{4} \frac{x^2}{x + a}.
\]

Finally, we calculate the expected payoff to the country from a compulsory license, given that types on the interval \([-a, x]\) have not entered. This payoff is (using the convention that values in the bargaining game with a compulsory license are denoted by \( W \), and benchmark values by \( V \)):

\[
W_{CL}(x) = \alpha \frac{v(-a) + v(x)}{2} - R = v_0 + \frac{\alpha \xi (x - a)}{2} - R.
\]

The expected surplus will expand on the issuing of a license if \( W_{CL}(x) > (-a + x)/2 \). Indeed, when \( x < 0 \) a compulsory license is the only hope for a positive surplus. If \( R \) is sufficiently small, then the country has an incentive to license the product, even though entry could not be mutually profitable in the absence of a license.

Comparing these payoffs gives the following result, proven in Section 5.1:

**Lemma 1** Let Assumptions 1–2 hold. Then for all \( x > 0 \),

\[
V^\infty_0(x) < V^m(x) < W_{CL}(x).
\]

Hence, the country fares better under a use-it-or-lose-it choice of imposing a compulsory license immediately than under a similar opportunity to set a monopoly price (it is here that we use Assumption 2.2’s implication that \( \alpha \) is
not too small), and fares better under both than from forever surrendering the licensing or monopoly pricing option.

Since $\beta = e^{-r\Delta}$, $V_0(\alpha) = \frac{2}{3}e^{x-\Delta}$, $V_{\infty}(x)$ falls as the period length $\Delta$ is reduced. The country has an incentive to gradually reduce the price over time in order to extract surplus from the highest valuation entrants. However, this ability to price discriminate declines as $\Delta$ is reduced, since $\Delta$ represents a bargaining friction that defines the period of time over which the country is committed to its current offer. The payoff from continuing to bargain approaches the monopoly payoff of $V_m(x) = \frac{1}{2}x^2 + a$ as the bargaining friction becomes arbitrarily large, and approaches 0 as the bargaining friction becomes arbitrarily small. In particular, the entry outcome approaches the efficient one in which all firms with $x > 0$ will enter at time 0.14

2.3 Efficient Outcome

If a compulsory license is issued, the total payoff is the sum of the payoff to the country, $\alpha v - R$, and the payoff $R$ to the firm, or

$$\alpha v.$$ 

Since the compulsory license may expand the surplus, we cannot simply apply the characterization of efficiency (i.e., immediate entry if $x > 0$ and no entry if $x < 0$) that applies when the compulsory license is not an option. The discounted value of the payoff from a compulsory license offered at the earliest opportunity, period $N = C/\Delta$, is $\alpha v \beta^N$, so the efficient outcome is now entry at time 0 if

$$x > \alpha v \beta^N,$$

and the issuing of a compulsory license if

$$x < \alpha v \beta^N.$$ 

Assumptions 1.2 and 2.2 then ensure that there is a unique value $x^*$ solving $x^* = v(x^*) = \alpha(\xi v_0 + \xi x^*) \beta^N$ such that the efficient outcome calls for entry at time 0 if $x > x^*$ and compulsory licensing if $x < x^*$.

Figure 1 illustrates. Unlike the case with no licensing, the country gains access to the product even if $x < 0$. Moreover, the form of access changes if $0 < x < x^*$, switching from entry to a compulsory license.15

14 This is the Coase conjecture (cf. Gul, Sonnenschein and Wilson [12]): in the absence of a deadline, the ability of a country facing to price discriminate vanishes as period length shrinks to zero.

15 Since the compulsory license is a WTO obligation, the firm and country can rely on the WTO dispute settlement process to enforce an agreement. Agreements reached outside the compulsory license rules would lack this enforcement process. We accordingly assume that it is not possible to negotiate the issuance of a compulsory license before the deadline, even if (because $\alpha v > x$) it would be efficient to do so. In a similar vein, we do not allow the firm to buy out the country’s option of a compulsory license when $x < 0$. Since such agreements would not be a WTO obligation, they would have to be enforced in the domestic court if the country reneged, which would be problematic. In addition, an agreement by a developing country to forego access to patented medicines in return for a cash payment would likely be politically unacceptable.
Figure 1: Illustration of the efficient outcome under compulsory licensing. A compulsory license is issued when \( x < x^* = \alpha v_0 \beta^n / (1 - \alpha \xi \beta^N) \), and entry occurs at time 0 if \( x > x^* \).

3 Equilibrium

We now consider the equilibrium of the bargaining game between the firm and country. The country faces a basic tension between the desire to extract surplus from firms that should (in the efficient solution) enter at time 0 and the desire to postpone entry for cases where the surplus is largest with a compulsory license. We will show that this tension leads to one of two types of equilibria. One type of equilibrium involves a bargaining pause: the country makes offers to the firm until some period \( Z < N = C / \Delta \), and then makes no further offers until it issues a compulsory license in period \( N \). The second type of equilibrium involves a licensing delay: bargaining extends up to or beyond period \( N \), with a finite period \( Z \geq N \), at which time a compulsory license is issued.

3.1 A Preliminary Result

We first establish a successive skimming property that commonly arises in bargaining under incomplete information, implying that the entrants in each period are an interval of the highest valuation firms remaining in the market. Suppose that the country offers a sequence of prices \( p_n \) for periods \( n \in \{0, 1, .., Z - 1\} \) and issues a compulsory license if the firm has not entered by period \( Z \geq C / \Delta \).

The firm chooses either to enter at the period \( n \) that maximizes the value of entry, or to wait for the compulsory license and receive the royalty payment, \( R \). The firm’s optimal payoff will be

\[
\max \left( \max_{n < Z} (x - p_n) \beta^n, R \beta^Z \right).
\]

The following Lemma follows immediately from the observation that the payoff gain to the firm of delaying the decision to enter at \( n < Z \) to instead either enter at \( n' > n \) (given by \( (x - p_n) \beta^n - (x - p_{n'}) \beta^{n'} \)) or incur a compulsory license at \( Z \) (given by \( (x - p_n) \beta^n - R \beta^Z \)) is decreasing in \( x \).

**Lemma 2** If a firm of type \( x \) generates a higher surplus from entry at period \( n \) than from entering at any later period, then so does any firm of type \( \hat{x} > x \). If a firm of type \( x \) generates a higher surplus from entry at period \( n \) than waiting to be licensed, then so does any firm with surplus \( \hat{x} > x \).
The key implication is that in equilibrium, the types of firms remaining in period $n$ will be an interval of the form $[-a, x_n]$.

### 3.2 A Three Period Example

This section presents a three-period version of the problem, with the option of a compulsory license appearing (only) in the final period. This provides the basic tools for examining the general problem, and allows a simple illustration of a bargaining pause equilibrium and the inefficiencies that arise in such an equilibrium. It also provides a characterization of the benefits of continuing to bargain in the penultimate period that will be useful in analyzing the licensing delay equilibrium.

Looking forward to the general analysis of a bargaining pause equilibrium, in which a compulsory license is issued in period $N$, we refer to the periods as $N-2$, $N-1$ and $N$.

#### 3.2.1 Period $N$

The country enters period $N$ with the remaining interval of possible firm types given by $[-a, x_N]$. In period $N$, the country has the option of issuing a compulsory license or of making a final offer of $p_N$ to the firm. Lemma 1 ensures that country’s optimal action is to issue the compulsory license, which yields the country a payoff of $W_{CL}(x_N)$.

#### 3.2.2 Period $N-1$

The interval of possible firm types in period $N-1$ is given by $[-a, x_{N-1}]$. A firm of type $x$ will enter in period $N-1$ if $x - p_{N-1} \geq \beta R$. The entrant who is indifferent between entering at $N-1$ and waiting for a compulsory license will be the highest valuation firm remaining at $N$, giving $x_{N-1}$, so $p_{N-1} = x_{N-1} - \beta R$.

The payoff to the country from $N-1$ onward will be

$$W_{N-1,CL}(x_{N-1}) = \max_{x_N} \left[ \left( \frac{x_{N-1} - x_N}{x_{N-1} + a} \right) (x_N - \beta R) + \beta \left( \frac{x_N + a}{x_{N-1} + a} \right) W_{CL}(x_N) \right].$$

(5)

The optimal choice of $x_N$ will be given by

$$x^*_N(x_{N-1}) = \begin{cases} \frac{x_{N-1} + \alpha v_0 \beta}{2 - \alpha \beta \xi} & \text{for } x_{N-1} \geq \tilde{x}_{N-1} := \frac{\alpha v_0 \beta}{1 - \alpha \beta \xi} \\ x_{N-1} & \text{for } x_{N-1} < \tilde{x}_{N-1}. \end{cases}$$

If $x_{N-1} > \tilde{x}_{N-1}$, and hence some entry occurs in period $N-1$, then the country chooses a price

$$p^*_{N-1}(x_{N-1}) = \frac{x_{N-1} + \alpha v_0 \beta}{2 - \alpha \beta \xi} - R \beta.$$

(6)

---

16We use here the inequality $1 - \alpha \beta \xi > 0$, implied by Assumption 2.2. If this inequality fails, then there are no circumstances under which entry occurs in period $N-1$, because the country finds it more valuable to wait and license all firms.

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Firms with \( x \in [\hat{x}_{N-1}, x_{N-1}] \) will enter. If \( x_{N-1} < \hat{x}_{N-1} \), there will be no entry in period \( N-1 \) and all firms currently in the market will be issued a compulsory license next period, giving us our first glimpse of a bargaining pause equilibrium. Observe that the critical value \( \hat{x}_{N-1} \) is the value of \( x \) at which the total surplus from entry equals the discounted value of the payoff from waiting for a compulsory license.\(^\text{17}\)

### 3.2.3 Period \( N-2 \)

In period \( N-2 \), a firm of type \( x \) will enter if \( x - p_{N-2} \geq \max[\beta(x - p_{N-1}), \beta R] \). Letting \( \Gamma = \frac{1}{2-\alpha \beta ^2 \xi} \), the price can be expressed as a function of the marginal entrant in \( N-2 \), given by \( x_{N-1} \), as follows:

\[
p_{N-2}(x_{N-1}) = \begin{cases} 
(1 - \beta(1 - \Gamma))x_{N-1} + \beta^2(\alpha v_0 \Gamma - R) & \text{for } x_{N-1} \geq \hat{x}_{N-1} \\
\hat{x}_{N-1} - \beta^2 R & \text{for } x_{N-1} < \hat{x}_{N-1}.
\end{cases}
\]

The demand curve is piecewise linear, with demand becoming less elastic as next period’s threshold entrant falls below the value at which entry ceases to occur at \( N-1 \).

The value function for the country in period \( N-2 \) is defined analogously to (2), which can be solved to yield the policy function\(^\text{18}\)

\[
x_{N-1}^*(x_{N-2}) = \begin{cases} 
\max \left[ \frac{x_{N-2}(1 - \beta(1 - \Gamma))}{2 - \beta^2(1 - \Gamma)^2}, \hat{x}_{N-1} \right] & \text{for } x_{N-2} \geq \hat{x}_{N-2} := (2 - \beta)\hat{x}_{N-1} \\
\frac{x_{N-2} + \alpha \beta^2 v_0}{2 - \alpha \beta^2 \xi} & \text{for } \frac{\alpha \beta^2 v_0}{1 - \alpha \beta^2 \xi} =: \hat{x}_{N-2} < x_{N-2} < \hat{x}_{N-2} \\
x_{N-2} & \text{for } x_{N-2} < \hat{x}_{N-2}.
\end{cases}
\]

We can now use the solutions for \( x_{N-1}^*(x_{N-2}) \) and \( x_N^*(x_{N-1}) \), describing the marginal entrant (if any) in periods \( N-1 \) and \( N-2 \), to describe the equilibrium entry pattern as a function of the initial upper bound \( x_{N-2} \) on the distribution of types. Figure 2 illustrates.

First, if the initial upper bound on the firm type distribution \( x_{N-2} \) falls short of \( \hat{x}_{N-2} \), then there is no entry at \( N-2 \) (from (7)). Furthermore, there is no entry at \( N-1 \) because the threshold for entry at \( N-1, \hat{x}_{N-1} \), exceeds that at \( N-2 \). Thus, in this case the firm waits until period \( N \), at which point a compulsory license is issued. This is a bargaining pause equilibrium.

Next, suppose \( x_{N-2} \) falls in the interval \([\hat{x}_{N-2}, \hat{x}_{N-2}^*]\). Then entry will occur at \( N-2 \) if the firm’s realized type satisfies \( x \geq \frac{x_{N-2} + \alpha \beta^2 v_0}{2 - \alpha \beta^2 \xi} \), which is shown by the area \( A \) above the solid line in the interval \([\hat{x}_{N-2}, \hat{x}_{N-2}^*]\) in Figure 2. For

\(^{17}\)It is straightforward to show that \( W_{N-1, CL}(x_{N-1}) \) is continuous, convex, and differentiable in \( x_{N-1} \) on \([-a,b]\).

\(^{18}\)Due to the kink in the demand function, the objective function is continuous but not differentiable at \( x_{N-1} = \hat{x}_{N-1} \). The fact that \( (2 - \beta) > (2 - \beta(2 - \Gamma)) \) ensures that the objective function is strictly concave in \( x_{N-1} \) and there will be a unique maximum.
Figure 2: Equilibrium entry. The horizontal axis measures $x_{N-2}$, the upper bound on the prior distribution over firms’ types, while the vertical axis measures $x$, the firm’s realized type. We are thus interested in values below the diagonal (noting the smaller scale on the vertical axis). The equilibrium outcome entails entry in period $N-2$ for realizations in region A, entry in period $N-1$ in region B, and a compulsory license issued in period $N$ in regions C and D. In contrast, efficiency calls for immediate (period $N-2$) entry in regions B and C.

values of $x$ below the solid line, there is delay until a compulsory license is issued at $N$. This is again a bargaining pause equilibria.

Finally, for $x_{N-2} > \hat{x}_{N-2}$, there will be entry in period $N-2$ for high-valuation firms (values of $x$ above the solid line), entry in period $N-1$ for intermediate valuations (values of $x$ in the area B between the solid line and the dashed line), and otherwise a compulsory license. Here, the bargaining continues up until the date licensing becomes an option, at which point the country issues a license. Had we allowed subsequent periods, we would find cases of a licensing delay equilibrium.

3.2.4 Implications

The efficient outcome is for all firms with $x < \hat{x}_{N-2}$ (corresponding to the area D in Figure 2) to wait for the license and for the remaining firms (areas A, B, and C) to enter immediately (cf. Figure 1). In contrast, we have the following
bargaining outcomes:

- \( b = x_{N-2} < \hat{x}_{N-2} \). In this case, all firms are licensed, which is the efficient outcome.

- \( b = x_{N-2} \in (\hat{x}_{N-2}, \hat{x}_{N-2}) \). In this case, the entry decision is efficient for firms with values of \( x \) in the portions of regions A and D in this interval. For parameter values in the portion of region C associated with this interval, bargaining results in inefficient delay until a compulsory license is issued.

- \( b = x_{N-2} > \hat{x}_{N-2} \). For values of \( b \) in this range, there is an inefficient delay from entry at \( N-2 \) to entry at \( N-1 \) for firms in region B, and inefficient delay from entry at \( N-2 \) to a compulsory license at \( N \) for firms in region C.

Similarly, with an arbitrary number of periods, the efficient outcome will be that high valuation firms enter immediately and low valuation firms wait for a license, while bargaining will lead to (i) too many firms being forced to wait for the license, and (ii) some firms who do not wait for the license being forced to delay agreements.

### 3.3 Bargaining with an Infinite Horizon

We now examine the game in which there is an infinite horizon, with the country having the ability to issue a license at any point after some time \( C \). As in the case of the three period model, we expect two possible configurations for an equilibrium, depending on the parameter values. In each case, there will be an initial interval of periods during which the country makes offers that are accepted by some types of firms. The country moves through a declining sequence of such offers, successively skimming through the firms with relatively high valuations. One possibility is that the expected return from skimming is sufficiently low that the country stops making offers at some time \( K < C \). There will then be a bargaining pause interval over which no agreements are reached, until time \( C \) arrives and the compulsory license is issued. The other possibility is that the expected profits from continuing skimming are sufficiently large that the country continues to bargain until time \( K \geq C \).

#### 3.3.1 Bargaining Pause: Terminal Conditions

We begin with a bargaining pause equilibrium, in which the country makes an initial sequence of offers that are potentially accepted (if the firm has a sufficiently high valuation) up to some time \( K < C \). The offer at time \( K \) is the last offer that is possibly accepted, after which no further offers are accepted and no further entry occurs, until time \( C \), when a compulsory license is issued. We say in this case that bargaining ceases at time \( K \). To the extent possible, we index the variables in this discussion by time rather than period (in contrast
to the three-period example of Section 3.2.2), reflecting the fact that the period length $\Delta$ plays a only a nuisance role in this portion of the argument.

If the equilibrium is to take this form, the marginal firm that accepts the country’s offer at time $K$, denoted by $x_{K+\Delta}$, must be such that the country prefers to keep bargaining until time $K$ rather than ceasing earlier, but then prefers to wait for the license at time $C$ rather than to keep bargaining beyond $K$. This will be the case if the total surplus created by entry at time $K$ on the part of a firm of type $x_{K+\Delta}$ equals the total surplus from waiting for a compulsory license.

In the three period example of Section 3.2.2, bargaining occurs (i.e., the country makes an offer that is potentially accepted) in period $N-2$ if and only if the value of the highest remaining firm exceeds $\tilde{x}_{N-2}$, and bargaining occurs in period $N-1$ if and only if the value of the highest remaining firm exceeds $\tilde{x}_{N-1}$. The critical values $\tilde{x}_{N-2}$ and $\tilde{x}_{N-1}$ are those at which the total surplus from the highest remaining firm equals the total surplus earned by waiting for a compulsory license in period $N$. Applying a similar argument to the general case, we find that the country will be indifferent between either making an offer at time $K$ that makes at least some remaining firm willing to accept or waiting until time $C$ to license the remaining firms if $x_K$, the highest remaining valuation at time $K$, satisfies

$$x_K = \alpha v(x_K)e^{-r(C-K)},$$

(8)

where the left side is by definition the surplus available at time $K$ from the marginal firm and the right side is the surplus available from waiting for a license. If instead $x_k$ is the larger of these two surpluses, then the country will prefer to induce at least some possibility of agreement at time $K$, making an offer that will secure an agreement from the next highest slice of firms, and perhaps also at subsequent times. Alternatively, if $\alpha v(x_K)e^{-r(C-K)}$ is the higher of these two surpluses, then the country strictly prefers to license all firms remaining at time $K$, and bargaining will cease earlier than time $K$.

We define $\tilde{x}(D)$ to be the value of $x$ at which the surplus from entry equals the surplus from a issuing a compulsory license after a delay of time $D$, or

$$\tilde{x}(D) = \frac{\alpha v_0 e^{-r(D)}}{1-\alpha \xi e^{-r(D)}}.$$

(9)

This is a generalization of the expressions for $\tilde{x}_{N-1}$ and $\tilde{x}_{N-2}$ from the three-period example. Notice that this condition is a function of the length of the delay, but does not depend on period length. In order for some firms to accept offers at time $K$, with no subsequent acceptances and a compulsory license issued at time $C$, we must have $x_K > \tilde{x}(C-K)$, so that the country prefers to induce entry on the part of some firms, and the country must find it optimal to set a price so that $x_{K+\Delta} = \tilde{x}(C-K)$ (and hence $x_{K+\Delta} < \tilde{x}(C-(K+\Delta))$, ensuring that there is no further entry between time $K$ and time $C$.

In the three-period example, (6) gives the price that makes firm $\tilde{x}_{N-1}$ just indifferent between entering in period $N-1$ and waiting for a compulsory license.
in period N. An analogous argument indicates that in a bargaining pause equilibrium, a firm of type \(x_K = \tilde{x}(C - K)\) will be indifferent between entering at time \(K\) and waiting for a license at time \(C\) if the price \(P_K\) is given by

\[
p_K = g_K x_K + f_K,
\]

where

\[
g_K = \frac{1}{2 - \alpha \xi e^{-r(C-K)}} \quad f_K = \left(\frac{\alpha v_0}{2 - \alpha \xi e^{-r(C-K)}} - R\right) e^{-r(C-K)}.
\]

(10)

3.3.2 Licensing Delay: Terminal Conditions

We now turn to the case in which bargaining has reached a time \(K \geq C\) without an agreement or a license. Lemma 1 establishes that the country prefers issuing the compulsory license at first opportunity, namely time \(C\), rather than either bargaining forever or setting a monopoly price at time \(C\). However, this still leaves open the possibility that the country might choose to bargain for some time past \(C\) and then issue the compulsory license—a licensing delay equilibrium. Under what conditions would the country prefer imposing a compulsory license at time \(K \geq C\) than waiting until time \(K + \Delta\) (or later) to issue the compulsory license?

Our analysis of the three-period example in Section 3.2.2 has provided all of the tools to answer this question. Let \(W_{K,CL}(x_K, \Delta)\) be the solution to the value function for the optimization in which the country makes an offer at time \(K\), and then (by assumption) issues a compulsory license at time \(K + \Delta\). The corresponding maximization problem is presented in (5). It is straightforward that if \(W_{CL}(x_K) < W_{K,CL}(x_K, \Delta)\), then the country will not want to issue a compulsory license at time \(K\). It is almost as immediate that if \(W_{CL}(x_K) > W_{K,CL}(x_K, \Delta)\), the country will issue a compulsory license at time \(K\). In particular, the proof of the following lemma shows that \(W_{CL}(x_K) - W_{K,CL}(x_K, \Delta)\) is declining in \(x_K\). Hence, the relative advantage of offering a compulsory license sooner rather than later declines as the upper bound on the firms’ types grows. But then, if the country prefers to issue a compulsory license now rather than wait a period, it will surely prefer to issue the license now rather than wait a period, and then wait yet another period under less favorable terms. The function \(\tilde{x}\) in the following statement is taken from (9). Section 5.2 proves the following:

Lemma 3 Let Assumptions 1–2 hold and suppose time \(C\) has been reached without an agreement.

[3.1] For \(\Delta\) sufficiently large that \(e^{-r\Delta} < \frac{2\alpha \xi - 1}{(\alpha \xi)^2}\), the country will issue a license at time \(C\).

[3.2] For \(\Delta\) sufficiently small that \(e^{-r\Delta} > \frac{2\alpha \xi - 1}{(\alpha \xi)^2}\), there will exist a critical value \(x_{CL} \geq \tilde{x}(\Delta)\) such that the country will continue to bargain at time \(K \geq C\) if \(x_K > x_{CL}\) and will issue a compulsory license otherwise.

[3.3] \(\lim_{\Delta \to 0} x_{CL} = \tilde{x}(0)\).

Lemma 3 thus shows that the country will continue to bargain past time \(K\) if \(\Delta\) is sufficiently small and \(x_K\) is sufficiently large. Assumption 2.2 guarantees
that $\frac{2\alpha\xi - 1}{(\alpha\xi)^2} \in (0, 1)$, so there will exist values of $\Delta$ that satisfy each of these cases. When $\Delta$ is large, continuing to bargain is relatively unattractive because of the delay imposed before the country can issue the compulsory license. Thus, a sufficiently large value of $\Delta$ allows the firm to commit not to continue to bargain past its first opportunity to issue a compulsory license at time $C$.

Observe that while the threshold for terminating bargaining at some time $K < C$ depends on the delay in time $C - K$ until a compulsory license can be issued and is independent of the period length $\Delta$, the threshold for terminating bargaining at some time $K > C$ depends on the period length $\Delta$ and is independent of the time $K$. Lemma 3 thus allows a comparison between the value of the threshold for terminating bargaining before and after time $C$. For $K < C$, the threshold $\tilde{x}(C - K)$ for terminating bargaining satisfies $\tilde{x}(C - K) \leq \tilde{x}(\Delta)$, with equality holding in the last period before a compulsory license can be issued. The threshold is increasing in the length $C - K$ of the delay, because a greater delay until a compulsory license can be issued lowers the return to waiting for a compulsory license. For $K > C$, the threshold $x_{CL}$ satisfies $x_{CL} > \tilde{x}(\Delta)$ for $\Delta > 0$ because $W_{CL}(\tilde{x}(\Delta)) - W_{K, CL}(\tilde{x}(\Delta), \Delta) = (1 - e^{-r\Delta})W_{CL}(\tilde{x}(\Delta)) > 0$. When $\Delta$ is small, the country will find it profitable to continue to bargain if $x_K$ is sufficiently large because the cost of delaying the compulsory license is small. Interestingly, as $\Delta \to 0$, $x_{CL}$ approaches $\tilde{x}(0)$ because the cost of delaying the compulsory license goes to 0. This illustrates how the shrinking of period length erodes the ability of the country to commit to not continue bargaining.

The terminal condition for the price schedule that is consistent with issuing a compulsory license at time $K + \Delta$ is also obtained from (6) in the three period example, $p_K = g_K x_K + f_K$, where

$$g_K = \frac{1}{2 - \alpha \xi e^{-r\Delta}} \quad f_K = \left( \frac{\alpha \xi}{2 - \alpha \xi e^{-r\Delta}} - R \right) e^{-r\Delta}.$$  \hspace{1cm} (11)

### 3.4 The Equilibrium Path

The previous sections have established the terminal conditions for a bargaining pause at time $K < C$ with the highest remaining type of firm given by $\tilde{x}(K - C)$ and for a licensing delay equilibrium that terminates at time $K > C$ with highest remaining type $x_{CL}$. The final step in characterizing either type of equilibrium is to conjecture a price schedule for time $t \leq K$ of the form $p_t = g_t x_t + f_t$ and solve for the time path of prices and the initial upper bound of firm payoffs, $b$, that is consistent with reaching the appropriate terminal stock at time $K$.

Section 5.3 makes this argument precise, proving the following proposition,

**Proposition 2** Let assumptions 1–2 hold.

[2.1] Suppose the highest valuation buyer at time 0 solves

$$b = x_K \Pi_{n=1}^{K/\Delta} \frac{1}{(1 - e^{-r\Delta}(1 - g_{t+\Delta})) g_{K-n\Delta}} \quad \text{for some } K < C,$$

where

$$g_t = \frac{(1 - e^{-r\Delta}(1 - g_{t+\Delta}))^2}{2(1 - e^{-r\Delta}) e^{-r\Delta} g_{t+\Delta}} \quad f_t = e^{-r\Delta} f_{t+\Delta}.$$
for $t < K$ with $x_K = \hat{x}(C - K)$, and where $g_K$ and $f_K$ are given by (10). Then the discrete-time model with incomplete information and compulsory licensing has a unique equilibrium in which bargaining ceases at time $K$ with highest valuation buyer $x_K = \hat{x}(C - K)$, and with a compulsory license issued at time $C$. The equilibrium prices and cutoff valuations satisfy

$$x_{t+\Delta} = \frac{1-e^{-r\Delta}(1-g_t+\Delta)}{2(1-e^{-r\Delta})+e^{-r\Delta}g_t+\Delta} x_t \quad p_t = \frac{(1-e^{-r\Delta}(1-g_t+\Delta))^2}{2(1-e^{-r\Delta})+e^{-r\Delta}g_t+\Delta} x_t + e^{-r\Delta}f_{t+\Delta}. \quad (13)$$

[2.2] Suppose the highest valuation buyer at time $0$ solves

$$b = x_K \Pi_{n=1}^{K/\Delta} \frac{1}{(1-e^{-r\Delta}(1-g_t+\Delta)) g_{K-n\Delta}} \quad (14)$$

for some $K > C$ with $x_K = \hat{x}(\Delta)$, where $g_K$ and $f_K$ are given by (11). Then the discrete-time model with incomplete information and compulsory licensing has a unique equilibrium in which bargaining continues until time $K$, at which point a compulsory license is issued. The evolution of prices and threshold values is given by (13).

The discreteness of the time periods introduces a straightforward but notationally troubling integer problem into the specification of the equilibrium strategies. We have avoided this difficulty in the statement of Proposition 2 by assuming that the upper bound $b$ on the distribution of firm types fortuitously satisfies either (12) or (14). This difficulty disappears in the limit as $\Delta \to 0$, giving a limiting characterization for all values of $b$:

**Lemma 4** Let Assumptions 1–2 hold.

[4.1] If

$$C > \hat{C}(b) \equiv \log \left(1 - \frac{1}{2 - \alpha \xi} \log \left(\frac{\alpha v_0}{b(1 - \alpha \xi)}\right) \right)/r, \quad (15)$$

then in the limit as $\Delta \to 0$, the equilibrium cutoff values on the equilibrium path will be given by

$$x_t = b \exp \left[\frac{e^{rt}}{g_K}(e^{-rt} - 1)\right], \quad (16)$$

with $g_K$ determined from (10). Bargaining terminates at time $K < C$ with the highest remaining valuation firm $x_K = \hat{x}(C - K)$, and any remaining firms being licensed at time $C$.

[4.2] If $C < \hat{C}$, then in the limit as $\Delta \to 0$, the equilibrium cutoff values will be given by (16) with $g_K$ determined from (11). Bargaining continues until a compulsory license is issued at some time $K > C$ at which the highest remaining valuation firm is $x_K = \hat{x}(0)$.

[4.3] The limiting (as $\Delta \to 0$) payoff to the country in the bargaining pause equilibrium, when bargaining terminates at time $K$ with a delay of $D$ until a compulsory is issued, is $e^{-rK}W_{K,CL}(b, D)$. The limiting payoff to the country in a licensing delay equilibrium when bargaining terminates at time $K > C$ is $e^{-rK}W_{K,CL}(b, 0)$.
The payoff $W_{K,CL}(b, D)$ is the payoff to a take-it-or-leave-it offer, given that there is a delay of length $D$ until the compulsory license is issued. Lemma 4.3 shows that the present value payoff to the country in either a bargaining pause or licensing delay equilibrium is equal to that of waiting an making a take-it-or-leave-it offer at the time at which the bargaining would cease, with any remaining firms licensed at first opportunity.

## 3.5 Interpretation

We can collect and interpret these results, organizing the discussion around three points. First, what sort of bargaining outcomes should we expect to see in the presence of compulsory licensing? In the bargaining pause case, we can expect to see successive price offers from the country, designed to skim high-surplus firms into agreements, until some (endogenously determined, as a function of the parameters) time $K < C$. At this point, negotiations cease, and we will observe no further agreements until the opportunity to issue a compulsory license appears, at which point the remaining firms will be licensed. This bargaining hiatus reflects the fact that the relative value of a compulsory license is especially high for low-surplus firms, to the extent that once a sufficient interval of high-surplus firms have been skimmed into agreements, the equilibrium course of action is then for all parties to wait for a license. In the licensing delay case, the upper bound of buyer valuations is sufficiently high relative to the minimum delay to issue a compulsory license that the country finds it profitable to continue to make offers to firms for a finite period after the deadline.

Second, what are the efficiency implications of this bargaining process? In the bargaining pause equilibrium, the boundary between firms that enter and firms that are licensed is given by (from (9))

$$\frac{\alpha v_0 e^{-rD}}{1 - \alpha \xi e^{-rD}},$$

for some $D < C$. In contrast, the efficient boundary between entry and compulsory licensing in the continuous-time limit is given by the solution to $(\alpha v_0 + \xi x)e^{-rC} = x$, which we can solve for a boundary value of

$$\frac{\alpha v_0 e^{-rC}}{1 - \alpha \xi e^{-rC}}.$$

We then have two sources of inefficiency in the bargaining pause case. Because $D < C$, too many firms are subjected to compulsory licenses. In addition, the efficient solution for those firms that do enter is that they should enter immediately, while in equilibrium the country’s attempts to screen the firms gives delayed entry.

In the licensing delay equilibrium, the boundary between firms that enter and firms that are licensed is given by $\tilde{x}(0)$. Too many firms will receive a compulsory license in this case as well, and an additional inefficiency is introduced because the compulsory license is delayed beyond $C$ for those that do receive it.
In the absence of compulsory licensing, the presence of firms generating a negative surplus plays no role. Such firms never enter, and could just as well be deleted from the model. With compulsory licensing, such firms give rise to the possibility that licensing may yield a positive surplus even if entry does not. As we have seen the bargaining outcome allows some of these gains to be captured, but at the expense of subjecting too many firms to licensing.

Third, we can compare the payoff to the country in the bargaining outcome with what it would obtain by making a take-it-or-leave-it offer at time $C$, with rejection followed by compulsory licensing. In a bargaining pause equilibrium, the country’s equilibrium payoff will be equivalent to the payoff of a take-it-or-leave-it offer at time $K < C$ followed by a compulsory license at time $C$ for firms that do not accept the offer (Lemma 4.3). It is immediate that the payoff from the strategy of waiting until time $C$ to make the take-it-or-leave-it offer must be less than that from a take-it-or-leave-it offer at $K < C$ if there is entry in the former, because the country loses from having to delay entry. Thus we have the result that the payoff from the equilibrium we have calculated, under which the country bargains with the firm until time $K$ and then waits until time $C$ to issue a compulsory license, dominates the payoff from the delayed-until-$C$ monopoly strategy of waiting until time $C$ to make a take-it-or-leave-it offer followed by a license.

For the licensing delay equilibrium, on the other hand, the payoff to the country is less than that obtained from the “delayed-until-$C$ monopoly” strategy (a take-it-or-leave-it offer at time $C$, with rejection followed by compulsory licensing). The payoff is lower because the equilibrium payoff in the licensing delay equilibrium will be equivalent to that obtained by the delayed-until-$K$ monopoly strategy (of bargaining until $K$ and then issuing a license). The payoff from the delayed-until-$K$ monopoly strategy must be less than that obtained from the delayed-until-$C$ monopoly strategy. The lower payoff in the licensing delay equilibrium reflects the inability of the country to commit to issuing the license at the earliest opportunity.

4 Discussion

4.1 Two-dimensional Uncertainty

In this section, we relax our assumption that the surplus $x$ created by the firm upon entry into the country and the spillover $s$ are perfectly correlated. We identify conditions under which our equilibrium characterization from Section 3.3 continues to hold.

Since the skimming property in Lemma 2 depends only on $x$, the optimal entry decision is a function of $x$ alone. Letting $F(x)$ denote the marginal distribution of $x$, the payoff to the country under a compulsory license will be

$$W_{CL} = \int_{-a}^{x_N} \alpha(x + E(s|x))dF(x)/F(x) - R.$$
The solution to this problem would be identical to that obtained in Section 3.3 if we maintain Assumption 1.1 and replace Assumption 1.2 with $E(s|x) = v_0 + (\xi - 1)x$. Hence, the relationship between the spillover $s$ and the surplus $x$ from Assumption 1.2 is replaced by an analogous relationship between the expected spillover and the surplus. This assumption would be satisfied, for example, by the bivariate uniform density:

$$f(x, s) = \begin{cases} \frac{1}{(b-a)d} & x \in [-a, b], s \in [v_0 + (\xi - 1)x - \frac{d}{2}, v_0 + (\xi - 1)x + \frac{d}{2}] \\ 0 & \text{otherwise} \end{cases}$$

for $\xi \geq 0$.

### 4.2 Conclusion

We have examined bargaining between a firm and a country over the terms on which the firm enters the country’s market, when the country has the ability to issue a compulsory license (after some delay), and in the presence of private information on the part of the firm about both the payoff from entry and the payoff from a compulsory license. We have focused on the case in which entry has negative spillovers for the firm’s other markets, and where the issuance of a compulsory license may increase the surplus for some firm types. As a result, the efficient outcome is for higher valuation firms to enter immediately, while lower valuation firms should wait for a compulsory license to be issued.

We have identified two types of equilibria to the bargaining problem, a bargaining pause and a licensing delay equilibrium. For each type of equilibrium, the country begins by making successive price offers designed to skim high-surplus firms into agreements. If the maximum initial valuation is sufficiently low, the bargaining will reach a point prior to the delay required for a license to be issued at which the negotiations effectively cease and no more entry occurs. Instead, both parties wait until the opportunity to issue a license arises, at which point the remaining firms are licensed. This bargaining pause equilibrium gives rise to two sources of inefficiency. First, too many firms are subjected to compulsory licenses. Second, entry on the part of those firms that are not licensed is inefficiently delayed by the country’s attempts to screen the firms. The ability to issue the license endows the country with some bargaining power, even as time periods become arbitrarily short. In particular, the country’s payoff is higher than it would be if it simply waited until the first licensing opportunity, made a take-it-or-leave-it offer to the firm, and then licensed any rejecting firms.

We also show that if the maximum initial valuation is above a threshold value, the country will find it optimal to continue to bargain even after the time at which the option of issuing a compulsory license is available. In this licensing delay equilibrium, the firm will continue to bargain beyond the deadline until the maximum remaining valuation is sufficiently low, and then will issue the compulsory license. In addition to the two types of inefficiency in the bargaining pause equilibrium, there is an additional inefficiency due to the delay in issuing
the compulsory license to firms. In this case, the country’s payoff is lower than it would be if it could commit to making a take-it-or-leave-it offer to the firm at the earliest allowable point for the issuance of a license. The ability to delay the license weakens the country’s bargaining power, although it will still issue a compulsory license in finite time.

5 Appendix: Proofs

5.1 Proof of Lemma 1

To show that \( V^m(x) < W_{CL}(x) \), we need

\[
\frac{1}{4} x^2 < \alpha \frac{v(-a) + v(x)}{2} - R.
\]

This inequality is most difficult to satisfy if \( R \) attains its maximum of \( \alpha (v(-a) + v_0)/2 \) (cf. Assumption 2.1), in which case it can be rearranged as (using \( v(x) = v_0 + \xi x \))

\[
\frac{1}{2} x^2 < \alpha \xi x.
\]

Assumption 2.2, that \( \alpha \xi > 1/2 \) ensures that this inequality is satisfied.

To show that \( V_0^\infty < V^m(x) \), we need

\[
\sqrt{1 - \beta + \beta - 1} < 1/4,
\]

which, given \( \beta \in (0, 1) \), is a straightforward calculation.

5.2 Proof of Lemma 3

We examine the function \( W_{K,CL}(x_K, \Delta) \), which is the payoff to the country of making an offer to the firm at time \( K \) and then issuing a compulsory license at time \( K + \Delta \). This function is given by (5) with \( \beta = e^{-r \Delta} \).

Letting \( \tilde{x}(\Delta) \) be defined as in (9), Section 3.2.2 established:

- For \( x_K \leq \tilde{x}_\Delta \), the country’s optimal offer at time \( K \) induces no entry. Since a compulsory license would be issued at time \( K + \Delta \), \( W_{K,CL}(x_K, \Delta) = W_{CL}(x_K) e^{-r \Delta} \) for \( x_K \leq \tilde{x}_\Delta \).
- \( W_{K,CL}(x_K, \Delta) \) is increasing and strictly convex in \( x_K \) for \( x_K > \tilde{x}(\Delta) \)

In particular, we can write the country’s payoff function as

\[
W_{K,CL}(x_K, \Delta) = \begin{cases} \frac{(A_2 x_K + A_3)^2 + A_4 x_K + a(A_3 + a(A_1 - A_2) + A_4)}{A_4} \quad & \text{for } x_K \geq \tilde{x}_\Delta \\ W_{CL}(x_K) e^{-r \Delta} \quad & \text{for } x_K < \tilde{x}_\Delta \end{cases}
\]

for \( x_K \geq \tilde{x}_\Delta \),

(17)
where
\[ A_1 = 1 - \frac{\alpha \xi e^{-r\Delta}}{2} > 0, \quad A_2 = 1, \quad A_3 = \alpha v_0 e^{-r\Delta}, \quad A_4 = -e^{-r\Delta} R. \]

This function is convex in \( x_K \) since \( W_{CL}(x) \) is linear in \( x \) and
\[
\frac{\partial^2 W_{K,CL}(x_K, \Delta)}{\partial x_k^2} = \frac{(a(2A_1 - A_2) + A_3)^2}{(x_K + a)^3 2A_1} > 0. \tag{18}
\]
The sign comes from the fact that \( A_1 > 0 \) and \( a(2A_1 - A_2) + A_3 = (2A_1 - A_2)(a + \tilde{x}) > 0. \)

The country will issue a compulsory license at time \( K \) if \( \Omega(x_K) \equiv W_{CL}(x_K) - W_{K,CL}(x_K, \Delta) \geq 0 \). This condition will be satisfied for \( x \leq \tilde{x}(\Delta) \), since \( \Omega(x) = (1 - e^{-r\Delta}) W_{CL}(x) \) in that case. We also have that \( \lim_{x \to \infty} \Omega'(x) = -\frac{A_2^2}{2A_1} \), so \( \Omega(x) \) is decreasing in \( x \) if \( A_2^2 < 2A_1 \alpha \xi \). Evaluating yields the requirement that
\[ 1 - 2\alpha \xi + (\alpha \xi)^2 e^{-r\Delta} > 0. \]

We then have two cases. If \( 1 - 2\alpha \xi + (\alpha \xi)^2 e^{-r\Delta} < 0 \), which must hold for \( \Delta \) sufficiently large by Assumption 2, then \( \Omega(x) \) is increasing in \( x \) and \( \Omega(x) \geq 0 \) for all \( x \). In this case the country will issue a compulsory license at the earliest opportunity.

If \( 1 - 2\alpha \xi + (\alpha \xi)^2 e^{-r\Delta} > 0 \), which must hold for \( \Delta \) sufficiently close to 0, then \( \lim_{x \to \infty} \Omega(x) < 0 \), and there will exist an \( x_{CL} \geq \tilde{x}(\Delta) \) such that \( \Omega(x) < 0 \) for \( x > x_{CL} \).

### 5.3 Proof of Proposition 2

Equation (9) identifies a type, as a function of the delay \( D \) until a compulsory license can be issued, with the property that the country will be just indifferent between inducing entry by such a firm and waiting to license all remaining firms at time \( C. \) Now suppose that the bargaining process reaches time \( K \) with stock \( x_K \), that some firms accept at time \( K \), and that no firm accepts thereafter, with the remaining firms waiting to be licensed at time \( C. \) Let \( D = C - K \). Then the marginal firm, \( x_{K+\Delta} \), will be just indifferent between entering at time \( K \) and waiting to be licensed, and hence the price \( p_K \) must solve
\[
p_K = x_K + \Delta - e^{-rD} R. \tag{19}
\]
Using this expression for price, the country’s value function at time \( K \) is given by
\[
W_{K,CL}(x_K) = \max_{x_K+\Delta} \left( x_K - x_K + \Delta \right) \left( x_K + \Delta - \alpha v_0 e^{-rD} R + e^{-rD} \int_{x_K+\Delta}^{x_K+\Delta} (\alpha v(x) - R) dx \right). \]
Taking a first order condition, using \( v(x) = v_0 + \xi x \), and rearranging gives the unique solution
\[
x_{K+\Delta} = \frac{x_K + \alpha v_0 e^{-rD}}{2 - \alpha \xi e^{-rD}}. \tag{20}
\]

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This gives \( x_{K+\Delta} \) as an increasing function of \( x_K \). Moreover, we have
\[
x_{K+\Delta} \leftrightarrow \tilde{x}(D) \iff x_K \leftrightarrow \tilde{x}(D).
\]
This tells us that the solution for \( x_{K+\Delta} \) will be interior. As long as \( x_K \) has not already fallen below the threshold \( \tilde{x}(D) \) for waiting to be licensed, and yet some firms are to enter in period \( K \), then neither will \( x_{K+\Delta} \) fall strictly below the threshold \( \tilde{x}(D) \). This in turn ensures that the solutions we calculate below are appropriate, in this and previous periods.

If bargaining ceases at time \( K \), then we must have fallen below the threshold at time \( K + \Delta \), and hence
\[
\frac{\alpha v_0 e^{-r(C-K-\Delta)}}{1 - \alpha \xi e^{-r(D-\Delta)}} \geq x_{K+\Delta} = \frac{\alpha v_0 e^{-rD}}{1 - \alpha \xi e^{-rD}}.
\]
(21)

Inserting the expression (20) for \( x_{K+\Delta} \) into the indifference condition (19) for the marginal entrant gives
\[
p_K = \frac{1}{2 - \alpha \xi e^{-r(C-k)}} x_K + \frac{\alpha v_0 e^{-r(C-K)}}{2 - \alpha \xi e^{-r(C-K)}} - e^{-r(C-K)} R
\]
\[
= g_K x_K + f_K.
\]

Now we work backward. Consider the value at an arbitrary period beginning at time \( t < K \), with bargaining to cease at \( K \) and with period length \( \Delta \). The value to the country at time \( t \) will be
\[
W_t(x_t, \Delta, K) = \max_{x_{t+\Delta}} p_t(x_t - x_{t+\Delta}) + (x_{t+\Delta} + a)e^{-r\Delta} W_{t+\Delta}(x_{t+\Delta}, \Delta, K).
\]
The price at time \( t \) will make the marginal firm \( x_{t+\Delta} \) indifferent between entering and not entering, and hence must satisfy \( x_{t+\Delta} - p_t = e^{-r\Delta}(x_{t+\Delta} - p_{t+\Delta}) \). Substituting \( p_{t+\Delta} = f_{t+\Delta} + g_{t+\Delta} x_{t+\Delta} \), this gives
\[
p_t = (1 - e^{-r\Delta}(1 - g_{t+\Delta}))x_{t+\Delta} + e^{-r\Delta} f_{t+\Delta}.
\]
(22)

We can substitute this price into the value function and then take a derivative with respect to \( x_{t+\Delta} \) to obtain the first-order condition (abbreviating the arguments of \( W \))
\[
(1-e^{-r\Delta})(1-g_{t+\Delta})(x_t - 2x_{t+\Delta}) - e^{-r\Delta} f_{t+\Delta} + e^{-r\Delta} \left( W_{t+\Delta}(x_{t+\Delta}) + (x_{t+\Delta} + a) \frac{dW_{t+\Delta}(x_{t+\Delta})}{dx_{t+\Delta}} \right) = 0.
\]
The potential obstacle in making sense of this expression is that we know little about the value function, but the envelope theorem gives us what we need, indicating that
\[
\frac{dW_{t+\Delta}(x_{t+\Delta})}{dx_{t+\Delta}} = \frac{p_{t+\Delta} - W_{t+\Delta}(x_{t+\Delta})}{x_{t+\Delta} + a}.
\]
We can insert this into the necessary condition and solve (with a bit of algebra) uniquely for \( x_{t+\Delta} \) and \( p_t \), which yields the values in Proposition 2.

This gives us a characterization of the path of prices and marginal valuations, given that entry ceases at time \( K \) with marginal entrant \( x_{K+\Delta} \).
5.4 Proof of Lemma 4

To look at the limiting case as period length gets small, we write

\[
\frac{g_{t+\Delta} - g_t}{\Delta} = \frac{(2(1 - e^{-r\Delta}) + e^{-r\Delta}g_{t+\Delta})g_{t+\Delta} - (1 - e^{-r\Delta}(1 - g_{t+\Delta}))^2}{\Delta(2(1 - e^{-r\Delta}) + e^{-r\Delta}g_{t+\Delta})}
\]

\[
= \left(1 - e^{-r\Delta}\right) \left(\frac{e^{-r\Delta}g_{t+\Delta}^2 + (1 - e^{-r\Delta})(2g_{t+\Delta} - 1)}{2(1 - e^{-r\Delta}) + e^{-r\Delta}g_{t+\Delta}}\right).
\]

Now we can take the limit as \(\Delta \to 0\) to find that

\[\dot{g}_t = rg_t,\]

which gives the solution \(g_t = e^{-r(K-t)}g_K\).

Starting with \(f_t = \beta f_{t+\Delta}\), we have

\[
\frac{f_{t+\Delta} - f_t}{\Delta} = \frac{(1 - e^{-r\Delta})f_{t+\Delta}}{\Delta},
\]

which gives

\[f_t = e^{-r(K-t)}f_K.\]

Now, given our linear representation of prices, this gives

\[p_t = e^{-r(K-t)}(g_Kx_t + f_K).\]

For the continuous time problem, let \(p(x, K-t)\) denote the offer made by the country when there is \(K-t\) remaining until the end of the interval over which offers are accepted and the highest remaining valuation is \(x\). We conjecture that this pricing function has the form:

\[p(x, K-t) = e^{-r(K-t)}p(x, 0).\]  \hspace{1cm} (23)

To determine \(p(x, 0)\), we consider the country’s problem at time \(K\), assuming that a compulsory license will be issued at \(K+D\) if the firm has not accepted an offer by time \(K\). If the country offers a price \(p\) at time \(K\), the lowest valuation firm that enters at time \(K\) will be \(x_{K+D} = p + e^{-rD}R\), where \(D = \max[C - K, 0]\) is the delay between the last offer and the compulsory license being issued. The country’s optimization problem will then be formulated as choosing \(x_{K+D}\) to maximize the expected payoff at \(K\), which yields payoff given by \(W_K(x_K, D)\) from (17), with the optimal entry threshold being

\[x_{K+D} = \frac{x_K + \alpha \nu_0 e^{-rD}}{2 - \alpha \xi e^{-rD}}.\]  \hspace{1cm} (24)

The critical value of \(x_K\) at which there will be no entry at time \(K\) is the solution to \(x_{K+D} = x_K\), which yields

\[x_{K+D} = \tilde{x}_K(D) = \frac{\alpha \nu_0 e^{-rD}}{1 - \alpha \xi e^{-rD}}.\]  \hspace{1cm} (25)
In order for it to be credible that the country will make no offers after time $K$, the stock at $K$ can be no greater than $\tilde{x}_K(D)$.

The country’s optimal price at time $K$ will be

$$p(x,0) = \begin{cases} 
\gamma_0 x + \psi_0 & x_K \geq \tilde{x}_K \\
\geq \gamma_0 \tilde{x}_K + \psi_0 & x_K < \tilde{x}_K,
\end{cases}$$

where $\gamma_0 = \frac{1}{2-\alpha \xi e^{-rD}}$ and $\psi_0 = \left(\frac{\alpha v_0}{2-\alpha \xi e^{-rD}} - R\right) e^{-rD}$. This offer function is consistent with (23) for $x_K \geq \tilde{x}_K(D)$, so we want to hit the terminal stock $\tilde{x}_K(D)$ exactly at $K$.

Firms will choose the optimal entry time to maximize $(x - p(x,K-t)) e^{-rt}$, which yields the necessary condition $r(x - p(x,K-t)) + \frac{dp}{dx} \dot{x} = 0$. Assuming that $p(x,K-t)$ satisfies (23) and (26), we obtain the differential equation for the evolution of $x$ to be $\dot{x}(t) + \frac{x}{\gamma_0} x(t) = 0$. Solving this differential equation with the condition that $x(0) = b$ yields

$$x_t = b \exp \left[ \frac{e^{TK}}{\gamma_0} (e^{-rt} - 1) \right].$$

This solution has the same form as that for the case where the firm sells up to a deadline $C$ after which no further agreements can be made. The difference is that in order for the country’s decision not to make any offers after time $K$ to be credible, it must be the case that $x_K = \tilde{x}_K(D)$. Therefore, we have to choose the value of $K$ such that this condition is satisfied,

$$K(D) = \log \left( 1 - \gamma_0(D) \log \left( \frac{\tilde{x}(D)}{b} \right) \right) / r.$$

A bargaining pause equilibrium with delay $D$ will exist for a given $b$ if $D + K(D) = C$. The function $K$ is decreasing in $\gamma_0 \log \left( \frac{\tilde{x}}{b} \right)$. We also have that $\gamma_0$ and $\tilde{x}_K$ are decreasing in $D$, so $D + K(D)$ is an increasing function of $D$. Therefore, a bargaining pause equilibrium will exist if $K(0) < C$, which yields the condition (15).

If $K(0) > C$, then there will be a licensing delay equilibrium in which a compulsory license is issued at $\tilde{K}(0)$.

To establish Lemma 4.3, observe that the payoff to a take-it-or-leave-it offer at time $K$ with a delay of $D$ until a compulsory license can be issued is given by $W_{K,CL}(b,D)$ from (17). Using the fact that the country is indifferent between receiving $p(x(t),K-t)e^{-rt}$ and $p(x(t),0)e^{-rK}$ from (23), we can write the payoff from bargaining as

$$e^{-rK} \left[ \int_{\tilde{x}(D)}^b p(x,0)dx + e^{-rD}W_{CL}(\tilde{x}(D)) \right].$$

Evaluating this expression using (26) yields the equivalence.
6 Online Appendix: Proof of Proposition 1

We first establish the following result, which characterizes the equilibrium with a finite terminal period $N$.

**Lemma A1** Let Assumption 1.1 hold, let there be no compulsory licensing, and let the firm and country bargain for periods $\{0, 1, \ldots, N\}$. Then there exists a unique perfect Bayesian equilibrium, with value function $V$ satisfying, for $n \in \{0, 1, \ldots, N\}$,

$$V_n^N(x_n) = \frac{g_n x_n^2}{2(x_n + a)},$$

where, for $n \in \{0, 1, \ldots, N - 1\}$

$$g_n = \frac{(1 - \beta (1 - g_{n+1}))^2}{2(1 - \beta) + \beta g_{n+1}},$$

and where we have the boundary condition

$$g_N = \frac{1}{2}.$$ 

The optimal path of firm cutoff valuations satisfies, for $n \in \{0, \ldots, N - 1\}$,

$$x_{n+1} = \frac{(1 - \beta (1 - g_{n+1})) x_n}{2(1 - \beta) + \beta g_{n+1}}$$

with $x_0 = b$ and $x_{N+1} = \frac{x_N}{2}$. The optimal path of prices satisfies, for $n \in \{0, \ldots, N\}$

$$p_n = g_n x_n.$$

**Proof** We present a recursive argument. First, we consider the final period. Suppose period $N$ is reached with the upper bound on the support of firm types $x_N$. Then we will have $x_{N+1} = p_N$ (this is the last period, and so the firm will enter if and only its valuation exceeds the price), and hence the country’s payoff is

$$\max_{x_{N+1}} \frac{x_N - x_{N+1}}{x_N + a} x_{N+1}.$$ 

Taking the first order condition and solving gives the unique solution

$$x_{N+1} = \frac{x_N}{2}$$

$$p_N = \frac{x_N}{2} = g_N x_N$$

$$V_N^N = \frac{x_N^2}{4(x_N + a)} = \frac{g_N x_N^2}{2(x_N + a)}$$

$$g_N = \frac{1}{2}.$$
Next, consider the penultimate period. Suppose the period $N - 1$ is reached with cutoff $x_{N - 1}$. The country determines $x_N$ by choosing $p_{N - 1}$, with the relationship being (using our previous solution for $p_N$)

$$x_N - p_{N - 1} = \beta(x_N - p_N) = \beta \frac{x_N}{2},$$

which we can solve for

$$p_{N - 1} = x_N \left(1 - \frac{\beta}{2}\right) = x_N \left(\frac{2 - \beta}{2}\right).$$

The expression to be maximized is then

$$\max_{x_N} x_N - x_{N - 1} x_{N - 1} + \frac{a x_N}{x_{N - 1} + a} x_N^2 + \frac{\beta x_N}{x_{N - 1} + a} \frac{x_N^2}{4(x_N + a)},$$

with a first order condition (taking a derivative with respect to $x_N$) of

$$(x_{N - 1} - 2x_N) \left(\frac{2 - \beta}{2}\right) + \frac{\beta}{2} x_N = 0,$$

which has the unique solution

$$x_{N - 1} = x_N \left(\frac{4 - 3\beta}{2 - \beta}\right).$$

We can then calculate

\begin{align*}
    x_N &= \frac{2 - \beta}{4 - 3\beta} x_{N - 1} \\
p_{N - 1} &= \frac{(2 - \beta)^2}{2(4 - 3\beta)} x_{N - 1} = g_{N - 1} x_{N - 1} \\
V_{N - 1}^N &= \frac{g_{N - 1} x_{N - 1}^2}{2(x_{N - 1} + a)} \\
g_{N - 1} &= \frac{(2 - \beta)^2}{2(4 - 3\beta)}
\end{align*}

We obtain the expression for $V_{N - 1}^N$ by inserting (35) into (34) and simplifying.

Now we turn to the general recursion step. Suppose period $n \in \{0, 1, \ldots, N - 1\}$ has been reached with cutoff $x_n$. The country chooses $x_{n + 1}$ by choosing a price $p_n$, satisfying the indifference relationship

$$x_{n + 1} - p_n = \beta(x_{n + 1} - p_{n + 1}) = \beta x_{n + 1} (1 - g_{n + 1}),$$

where the final equality follows from the hypothesis that $p_{n + 1} = g_{n + 1} x_{n + 1}$. We can solve for

$$p_n = x_{n + 1} (1 - \beta (1 - g_{n + 1})).$$

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The expression to be maximized is then
\[
\max_{x_{n+1}} \frac{x_n - x_{n+1}}{x_n + a}x_{n+1}(1 - \beta(1 - g_{n+1})) + \beta \frac{x_{n+1}^2 g_{n+1}}{x_n + a} \frac{x_n + 1}{2(x_{n+1} + a)},
\]
here using the hypothesis that \(V_{n+1} = g_{n+1}x_{n+1}^2/2(a + x_{n+1})\). The first order condition is
\[
(x_n - 2x_{n+1})(1 - \beta(1 - g_{n+1})) + \beta g_{n+1}x_{n+1},
\]
giving
\[
x_{n+1} = x_n \frac{(1 - \beta(1 - q_{n+1}))}{2(1 - \beta) + \beta g_{n+1}}.
\]
We can then calculate
\[
V_n^N = \frac{g_n x_n^2}{2(x_n + a)}
\]
\[
g_n = \frac{(1 - \beta(1 - g_{n+1}))^2}{2(1 - \beta) + \beta g_{n+1}}.
\]
Together with (30)–(33), this gives the result.

To complete the proof of Proposition 1, we take the limit of the equilibrium in Lemma A1 as \(N \to \infty\). We first show that

**Lemma A2** The equilibrium value \(g_0\) is decreasing in \(N\), with \(\lim_{N \to \infty} g_0 = \left(\sqrt{\frac{1 - \beta + \beta - 1}{\beta}}\right)\).

**Proof** Let \(f(h) = \frac{(1 - \beta + \beta h)^2}{2(1 - \beta) + \beta h}\), which has the properties that \(f(0) = \frac{(1 - \beta)}{2}\), \(f\left(\frac{1}{2}\right) < \frac{1}{2}\) and \(f'(h) = \frac{\beta(3(1 - \beta)^2 + 4(1 - \beta)h + (\beta h)^2)}{4(1 - \beta)^2 + 4(1 - \beta)h + (\beta h)^2} \in (0, 1)\) for \(\beta \in (0, 1)\). For \(h \in [0, \frac{1}{2}]\), \(f\) has a fixed point \(\left(\sqrt{\frac{1 - \beta + \beta - 1}{\beta}}\right)\). If \(h_0 = \frac{1}{2}\), then the sequence \(h_{s+1} = f(h_s)\) will be a strictly decreasing sequence with \(\lim_{s \to \infty} h_{s+1} = \left(\sqrt{\frac{1 - \beta + \beta - 1}{\beta}}\right)\).
This establishes the result, since \(g_0 = h_N\).

Substituting the result for \(g_0\) into (29) then yields the solution for \(V_0^\infty(x)\), completing the proof of Proposition 1.

**References**


