I study the optimal Mirrlees taxation problem in a two-period endowment economy with heterogeneous income types and private information, where the government’s lack of commitment severely limits its ability to provide incentives for agents to reveal their types and thus hinders social redistribution. I show how agents’ involvement in observable yet imperfect financial markets enables the government to gain at least limited effective commitment, and therefore can be welfare improving. Agents borrow against their promised income and enter individual consumption commitments: While the financial market is competitive ex-ante, it is incomplete ex-post. Claims cannot be resold, instead any adjustment of contracts leads to a deadweight loss. Taking agents’ contractual positions into account changes the government’s ex-post incentives to renege on the promised tax schedule and misuse the information it has collected. The impending individual “default losses” add up to an effective commitment device for the government. Methodologically, even though the Revelation Principle does not apply, I show how disconnecting truth-telling from full revelation of information still allows to restrict attention to direct albeit not fully revealing mechanisms. Finding the optimal mechanism can then be represented as a two-step problem: First to optimally restrict revealed information and second to choose a tax schedule based on the restricted information. This structure explicitly links limited commitment to limited information revelation and forms of partial pooling.

1 Introduction

Ever since the seminal contribution of Mirrlees (1971), it is widely recognized that the problem of income taxation is one of eliciting private information. The optimal

1Please send comments to jenny.simon@eui.edu. I would like to thank Daron Acemoglu and Ivan Werning for invaluable guidance and support. I also thank Arpad Abraham, Maya Eden, Mikhail Golosov, Nathan Hendren, Luigi Iovino, James Poterba, Florian Scheuer, seminar participants at MIT, EUI, and IIES, as well as conference participants at the SED and EEA meetings 2011 and the AEA meetings 2012 for helpful comments. I gratefully acknowledge generous financial support from the Peters-Beer-Stiftung. All errors are my own.
tax-transfer schedule, so the underlying argument, needs to provide incentives for agents to reveal their true productivity types and contribute to social redistribution schemes\textsuperscript{2}. With the incorporation of the Mirrleesian model into dynamic\textsuperscript{3} settings, however, it became apparent that in order to achieve a satisfactory compromise in the inherent efficiency-equity trade-off, the government needs to be able to commit not to misuse information it collected or renege on promised incentive payments over time. A lack of such commitment is generally expected to lead to extremely inefficient outcomes, as demonstrated first by Roberts (1984) and more recently extended to a dynamic setting by Golosov et al. (2006).

In reality, however, tax codes do rely on private information, and governments seem to be able to provide incentives for its revelation, despite the lack of a commitment device. The question then is: Which characteristics of the economy, the evolution of agents’ skills, or the nature of interaction between agents and the government enable such effective commitment? This paper contributes one possible explanation: Agents’ involvement in observable yet imperfect financial markets.

Individuals typically do not constrain their consumption to equal net-of-tax income every period. Instead, access to financial markets allows them to allocate their resources over time. On the other hand, the markets that individual agents trade in are usually imperfect in the sense that adjustments to contracts are costly. Consequently, by using markets, agents enter individual commitments. Chetty and Szeidl (2007) report that nearly 65\% of the average US household’s budget is allocated to consumption commitments that cannot be adjusted costlessly. I argue that optimal redistributive policy ought to take agents’ involvement in such financial markets into account. When re-optimizing, the benevolent government takes agents’ contractual position into account. If an agent ends up with less net income than promised, he will have to adjust his consumption plan downward and possibly adjust his financial contracts. The costs of such adjustment (or “default”) can deter the government from reneging on past promises. I show that even though these consumption commitments are enforceable only at the individual level, the imminent default costs for each agent add up to an effective commitment device for the government.

As the main result, I derive a simple condition that links the degree of market imperfection (modeled as an individual default cost) to the level of incentive payments the government is able to commit to. Intuitively, this condition equalizes the marginal benefit from additional redistribution toward the low end of the type distribution to the marginal cost due to default. The larger the default costs are, the more separation and so the more incentive provision is possible. I characterize conditions under which this new degree of freedom leads to a strict welfare improvement.

Even though theoretically possible, this mechanism arguably may not be strong enough in reality to provide full commitment. The main result of the paper shows,

\textsuperscript{2}See also Dasgupta et al. (1979), Harris and Townsend (1981), and Holmström and Myerson (1983) for early contributions.

\textsuperscript{3}Golosov et al. (2006) provide an extensive overview of the New Dynamic Public Finance literature.
however, that even a small market imperfection still leads to a limited degree of
effective commitment. Moreover, I show that whenever ex-ante inequality exceeds
a certain level, the government will use its limited commitment ability optimally
by collecting only a limited amount of information. The optimal tax schedule will
partially pool some agents of the type distribution.

In the course of deriving these results, the paper also makes a theoretical contrib-
ution to the literature on mechanism design under limited commitment. Since I
study a finite horizon model where the government is not able to commit, the Reve-
lation Principle does not apply. Instead, I prove a Truth-telling theorem that still
allows attention to be restricted to direct albeit not fully revealing mechanisms. The
separation of truth-telling from full revelation allows me to significantly restrict the
set of mechanisms to search over for the optimum. This class of mechanisms still
provides incentives for agents to tell the truth directly to the mechanism designer,
yet allows for truthful messages to reveal only partial information. With this re-
sult, I show that finding the best implementable mechanism can be divided into
two problems: The determination of how much information should be revealed (i.e.
which types should pool) and the search for the optimal tax-transfer scheme based
on that limited information. This structure enables me to explicitly link limited
commitment to limited information revelation and forms of partial pooling.

Related Literature

There is a growing interest in characterizing optimal Mirrleesian taxes in setups
without commitment. Brett and Weymark (2011) consider a two-period setup with
savings and show that the government without commitment will always find it op-
timal to distort savings. Berliant and Ledyard (2005) consider optimal dynamic
income taxes in a setup where income cannot be transfered between periods, in
which they demonstrate an equivalence of dynamic and static optimal taxes. Both
papers find that some but limited separation of types is possible under some circum-
stances even when the government has no commitment.

On the other hand, mechanisms that can potentially provide the government with
effective commitment have also been studied. Acemoglu et al. (2008, 2010) consider
self-interested politicians who cannot commit not to misuse information and can
appropriate resources for their own benefit. They show that in an infinite horizon
setup such governments can effectively commit on the equilibrium path, essentially
because they want to maintain their rents agreed upon in the social contract. Such
an equilibrium can only exist when it is supported by the threat of agents reverting
to the worst outcome after a government deviates from promised policy (either by
not producing anything, or by replacing the government). In that sense, their find-
ings are parallel to reputation mechanisms - a channel completely abstracted from
in this paper. Gaube (2012) studies the taxation of annual income as a commitment
device. The paper presented here contributes another potential channel by which
characteristics of the economy can lead at least to limited effective commitment:
agents trading in imperfect markets.
The effect of agents being able to use financial markets to allocate their resources on optimal taxation has also received attention. Many authors have considered environments in which agents cannot only contract with a principal, but also in anonymous outside markets that make it harder to extract information from the agents truthfully. See for example Hammond (1987) for a general treatment or Golosov and Tsyvinsky (2007) for a more recent example from the dynamic public finance literature. The general conclusion is that when the government has an exogenous commitment device, letting agents use markets to allocate resources decreases the set of policy instruments available to the government and so hinders redistribution. The main argument of the paper presented here is that this conclusion does not necessarily hold when the government has no commitment.

Bisin and Rampini (2006) study a no-commitment setup similar to the one considered here, but focus on the allocative role of anonymous markets. They find that allowing agents access to such markets is beneficial in a world where the government has no commitment, because it allows them to allocate resources over time without revealing any information, thereby increasing efficiency. However, the government’s commitment problem is unchanged, no social redistribution can be implemented. In contrast, I analyze a market that does not act as a “tax haven” by enabling agents to hide information from the government. The crucial characteristic of private contracts I consider is that they constitute consumption commitments that cannot costlessly be changed. This increases the government’s commitment power, enabling it to implement some social insurance.

There are potentially many ways in which the presence and functioning of markets influences the government’s ability to implement redistributive policy. Some characteristics of markets have been considered in the literature: Scheuer (2010) explores the impact of incomplete credit markets on optimal entrepreneurial taxation. He finds that a market friction which gives rise to cross-subsidization between different types of potential entrepreneurs may induce inefficient entry at both ends of the skill distribution, which in turn promotes an additional corrective role for type-differential, redistributive taxation, even when the government originally has no redistributive objective. Unlike in Scheuer’s paper, I consider a market that is not incomplete in that sense, and instead is able to provide credit without cross-subsidization.

The theoretical contribution of this paper relates to a growing body of literature on mechanism design and lack of commitment. Bester and Strausz (2001) show that a version of the Revelation Principle applies in situations when the principle without commitment faces only one agent. Their argument, however, does not extend to the multi-agent case (Bester and Strausz (2000)). In an infinite horizon setup with a continuum of agents, Acemoglu et al. (2006) show that the Revelation Principle holds on the equilibrium path, supported by the threat of agents reverting to not revealing any information after a deviation by the government.

The paper is organized as follows: Section 2 sets up the economy and describes
how agents can trade in imperfect markets. Section 3 defines strategies of agents and the government as well as the timing of the policy game and the equilibrium. In section 4 I derive the theoretical results that allow me to analyze this game as a mechanism design problem. Section 5 then proceeds by applying these results to the model economy, the main result of the paper - how agents’ involvement in imperfect markets can serve as a commitment device for the government - is derived here. Conditions under which this mechanism leads to a strict welfare improvement as well as conditions under which the optimal tax schedule will involve partial pooling are derived in section 6. Finally, section 7 concludes with a discussion of the relevance and applicability of the results.

2 Endowment Economy

The model economy lasts for 2 periods (indexed t=1,2) and is inhabited by a continuum of agents of unit mass. Agents derive utility from a single consumption good according to

$$U = u(c_1) + u(c_2).$$

(A1) Utility is time-separable, and the per period utility function $u(\cdot)$ is strictly increasing, concave, and $\lim_{c \to 0} u'(c) = \infty$. Moreover, I assume that $u$ displays constant elasticity of intertemporal substitution. Further, for simplicity, I assume that agents do not discount between periods.

Agents receive heterogeneous income $y_t(\theta)$ at the beginning of each period. Their income types $\theta$ are perfectly persistent over time and are private information. Across the population, $\theta$ is continuously distributed over a support $\Theta = [\underline{\theta}, \bar{\theta}]$. $F(\theta)$ denotes its cdf, and is assumed to be twice continuously differentiable. Apart from income heterogeneity, agents are identical.

Per period income $y_t(\theta)$ is deterministic. I assume that

(A2) $y_1(\theta) < y_2(\theta) \quad \forall \theta$

(A3) $\frac{\partial y_t(\theta)}{\partial \theta} > 0 \quad \forall t, \theta$

That is, all agents have an increasing income stream over their lifetime. Moreover, lower type agents receive smaller endowments than higher types at all times.

2.1 First-best benchmark

Suppose there exists a technology to costlessly transfer resources over time and consider the problem of a benevolent social planner with a utilitarian objective and equal Pareto weights on all agents. He chooses an allocation $\{c_t(\theta)\}_{t, \Theta}$ that assigns a consumption level to each type $\theta \in \Theta$, for each period $t = 1, 2$

$$\max_{\{c_t(\theta)\}_{t, \Theta}} \int_{\Theta} (u(c_1(\theta)) + u(c_2(\theta))) dF(\theta)$$

s.t. $\int_{\Theta} (c_1(\theta) + c_2(\theta)) dF(\theta) \leq \int_{\Theta} (y_1(\theta) + y_2(\theta)) dF(\theta)$

(1)
Lemma 1 (First-Best Allocation)
At the first-best allocation there is full social redistribution and perfect smoothing of consumption over time. All agents consume a constant share of the economy’s total endowment in each period: \[ c_1(\theta) = c_2(\theta) = c = \frac{1}{2} \int_\Theta (y_1(\theta) + y_2(\theta)) \, dF(\theta) \]

Proof: The first order condition with respect to any agent’s consumption in either period satisfies
\[ u'(c_t(\theta)) - \lambda = 0 \quad \forall t, \theta \]
where \( \lambda \) is the Lagrange multiplier on the aggregate feasibility constraint. Thus, \[ c_t(\theta) = c_t'(\theta') \quad \forall t, t', \theta, \theta'. \]

2.2 Government with information constraints and lack of commitment

Suppose that instead of an omniscient planner a benevolent government with the same utilitarian objective and no additional revenue requirements is in charge. To implement the desired allocation, it would like to institute schedules of type specific taxes and transfers. In doing so, however, it faces several difficulties:

(A4) The government cannot observe an agent’s type.

When conditioning the allocation on income types, it must rely on information provided directly by the agents or indirectly through their actions in the economy.

(A5) The government is not able to commit to a second period transfer schedule ex-ante.

I assume that the government can always commit to the announced tax schedule within each period. The potential commitment problem that is the subject of this paper arises between periods. At the beginning of the second period, possibly contrary to earlier promises, the government might implement a tax schedule based on information it has learned about the agents in the mean time. This lack of commitment with respect to the tax schedule is known to all players in the economy.

Like the planner, the government has access to a costless transfer technology:

(A6) The government can save and borrow at \( R = 1 \) from exogenous funds.

(A7) The government can always commit to repaying its debt to the exogenous fund.

The only difference from the planned economy arises from the private information constraints. The resulting difficulties then are exacerbated by the government’s lack of commitment with respect to using acquired information.

2.3 Agents trading in imperfect markets

Agents in this economy are also able to allocate their own resources over time. They have access to a financial market with the following exogenous characteristics:
(A8) The market is perfectly competitive. It consists of many banks that have access to unlimited outside funding.

(A9) Banks can verify income types at no cost. However, they do not sell or use this information for any purpose other than tailoring type specific financial contracts to their clients.\(^4\)

Assumptions (A8) and (A9) together imply that agents can borrow and save at the gross rate \( R = 1 \). That is, they face exactly the same cost for transferring resources as the government.

In this financial market, agents and banks may sign contracts

\[
x = (b_1, b_2, h_1, h_2) \quad \text{with } h_t, b_t \geq 0
\]

where the bank agrees to provide a consumption stream of \( h_t \) units in period \( t \) for payments of \( b_t \) by the agent. Since banks know the agents’ types, these contracts may in principle be type-dependent. This format of financial contracts is very general, in particular it allows for agents to simultaneously make a payment to and receive a payment from the contracting bank. Accordingly, I refer to contracts with \( h_2 > 0 \) as gross contracts\(^5\). For future reference, let \( X \) denote the set of all contracts that fulfill the ex-ante feasibility requirements specified below in the agents’ problem of period 1. For notational consistency, let \( X \) include the element \( x_0 = (0, 0, 0, 0) \) which symbolizes that the agent has not signed a contract.

The financial market agents may trade in is slightly imperfect: Once a contract is signed, it is not possible for agents to re-sell parts of their claims. No secondary markets exist. It is possible to renegotiate a contract, if both parties agree. This process, however, is generally not costless.

(A10) Once a contract is signed, it can only be changed at cost \( D \geq 0 \).

This cost is a deadweight loss that arises from having to change a contract. It essentially renders agents’ financial contracts to be individual consumption commitments that cannot costlessly be changed. \( D \) can be thought of as a measure of the severity of market imperfection. If \( D = 0 \) we recover complete markets.

To gain some intuition for the market imperfection \( D \) is supposed to capture, consider a mortgage contract. Since agents simultaneously repay their mortgage and live in the house, this would be considered a gross contract. If at any point in time the agent ends up with less net income than they expected, he might have to refinance his mortgage or even default. This process is never without loss: A foreclosed

\(^4\)This assumption clearly restricts the model’s fit with reality. However, it considerably simplifies the analysis in that it avoids the added complication of any adverse selection problems in the financial market. I make this assumption to focus on the commitment mechanism for the government only. An alternative model without this assumption is provided in Simon (2011). All qualitative results hold there, too.

\(^5\)A mortgage is an example for a gross contract: The agent makes mortgage payments to the bank and receives a transfer in form of the house (or increased ownership) in return every period.
house often does not sell for the same amount as it was worth to the original owner. Administration of defaults or refinancing is costly. But also non-pecuniary losses may occur: When agents have to move out of the house they grew attached to, they may suffer further disutility. Instead, if markets were complete, the agent would simply sell partial user rights to the house and continue living in it for a fraction of his life time, so that he could avoid all of the above mentioned costs.

(A11) Banks have the power to enforce repayment of outstanding debt from the individual agent only.

The enforceability of contracts is never revoked. However, this does not mean that banks can force the government to bail out an agent. They only possess enforcement power over the party they directly contracted with, i.e. the individual agent. The government has the first take on agents’ income, only thereafter can banks enforce the repayment of outstanding debt from the individual agent. However, since agents cannot consume before repaying all their debt, the benevolent government will take the agents’ contractual position into account. To that end:

(A12) Contracts are observable to the government.

### 3 Strategies and equilibrium

The government’s lack of the ability to commit to a tax schedule over time turns the setup into a policy game between agents, choosing which information to report to the government and which contract to sign in the financial market, and the government, choosing the transfers to implement in each period. The focus of this paper is to characterize the equilibrium of this game that corresponds to the best implementable mechanism, i.e. the mechanism (or tax schedule) that maximizes ex-ante welfare in the economy. In this section, I formally define the game, the players’ strategies and the equilibrium.

Denote with $Z$ a general message space, and by $z$ a generic element of $Z$. The message space includes messages about an agent’s type, but may also include other elements. A tax schedule, or more formally a mechanism $\tau = (\tau_1, \tau_2)$ consists of two mappings $\tau_i : Z \times X \mapsto \mathbb{R}$. Each of these mappings specifies a tax or transfer payment (for period 1 and 2 respectively) for an agent who reported message $z$ and signed contract $x$. Let $T$ denote the set of mechanisms that satisfy the ex-ante feasibility constraint (4) specified below. Moreover, let $T'$ denote the set of feasible mappings $\tau_2$ given $\tau_1$, that satisfy the ex-interim feasibility constraint (6).

To analyze this game formally, consider the following timing of action:

$t=1$

a) The government announces a mechanism $\tilde{\tau} = (\tilde{\tau}_1, \tilde{\tau}_2), \tilde{\tau} \in T$.

Agents receive their first endowment and send a message $z \in Z$.

Agents may sign a contract $x \in X$ in the financial market.

b) $\tilde{\tau}_1$ is implemented according to $\tilde{\tau}$.

Payments $(b_1, h_1)$ of all financial contracts are made.

Agents consume.
t=2  a) The government decides whether to deviate from the mechanism $\hat{\tau}$. This
decision is denoted by $\xi \in \{0, 1\}$. If $\xi = 1$, the government chooses a new
tax schedule $\hat{\tau}'_2 \in T'$ for period 2.
Agents receive their second endowment.
Agents and banks (may) renegotiate their private contracts to $x'$.
b) $\hat{\tau}_2$ or $\hat{\tau}'_2$ is implemented.
Payments $(b'_2, h'_2)$ of all financial contracts are made.
Agents consume.

Each period begins with an active stage where the government announces a tax
schedule and agents may reveal information and sign or modify a contract in the
financial market$^6$. It is followed by an implementation stage that is automatic, no
further actions can be taken by any player of the game. I chose this setup to em-
phasize the commitment problem of the government: While it can commit to a tax
schedule within a period (i.e. the implementation in stage b) follows automatically),
it is not able to commit to a mechanism across periods. After agents have revealed
information in period 1, the government can renege on the promised tax schedule $\hat{\tau}$
and use the information acquired.

In summary, the government’s action $\gamma = (\hat{\tau}, \xi, \hat{\tau}'_2)$ consists of three elements: the
initial announcement of a mechanism (or tax schedule) $\hat{\tau}$, the decision whether to
deviate from $\hat{\tau}$ in period 2, $\xi$, and the amended tax schedule $\hat{\tau}'_2$ that is implemented
in case of deviation. Let $G$ denote the set of $\gamma$’s.

The action of an agent of type $\theta$, denoted $\sigma_{\theta} = (z, x, x')$, consists of three ele-
ments as well: First, it specifies the message $z \in Z$ sent to the government. Second,
it specifies the contract $x$ the agent signs with a bank in period 1. Third, it includes
the contract $x'$ the agent holds after a possible renegotiation with the bank in pe-
riod 2. For notational simplicity, a contract that is not actually changed will still be
called $x'$, so that in case an agent does not renegotiate, his contract will be $x' = x$.
The renegotiation cost $D$ that stands for market imperfection is only incurred when-
ever $x' \neq x$. In that sense, $x$ and $x'$ summarize the contractual position of an agent
in period 1 and period 2 respectively. Let $X'$ denote the set of contracts that are
feasible at the point of possible renegotiation (according to the agents’ problem in
period 2 specified below). Let $\Sigma_{\theta}$ denote the set of possible actions for an agent of
type $\theta$ and $\Sigma$ the collection of these sets for all possible types, $\Sigma = \bigcap_{\theta} \Sigma_{\theta}$. Before
the revelation of his type, the strategy $\sigma$ of an agent thus is a mapping $\sigma : \Theta \times G \mapsto \Sigma$.

I denote with $z(\sigma(\theta, \gamma))$ the message that results from strategy $\sigma$ for an agent with
realized type $\theta$ and for the government action $\gamma$.

**Definition 1 (Truth-telling)**

A strategy $\sigma^*$ is called **truthful** if

$$z(\sigma^*(\theta, \gamma)) = z[\theta] \quad \forall \theta \in \Theta, \gamma \in G$$

$^6$ For the sake of formal equilibrium analysis, one may think of the actions in stage a) of each period
to take place simultaneously.
where the notation $z[\theta]$ means that the information revealed by the message is true.

An agent playing a truthful strategy will not lie about his type. Note, however, that a truthful strategy does not necessarily reveal the exact type of an agent. One might think of an agent revealing that he is “at least type $\theta$” or his income falls into a certain interval. The continuum of agents playing such a strategy could for example result in partial pooling around certain cutoff types. One particular truthful strategy does reveal the exact type, and is defined as follows:

**Definition 2 (Truthful revelation)**

A strategy $\sigma^R$ is called truthfully revealing if it is truthful and there exists a one-to-one mapping from the message to the exact type:

$$z[\theta] \mapsto \theta$$

If an agent plays a truthfully revealing strategy, the government will be able to infer his exact income type from the message sent.

Lastly, let $\sigma = \{\sigma\}_\Theta$ denote a strategy profile for all individuals, and $\Sigma$ the set of all such strategy profiles. Analogously, $\sigma^e$ and $\sigma^R$ denote strategy profiles where all types play truth-telling or truthfully revealing strategies respectively. Then the government’s strategy is defined as a mapping $\Gamma : \Sigma \mapsto G$

**Definition 3 (Equilibrium)**

A (perfect Bayesian) equilibrium in the game between agents and the government is given by a strategy $\Gamma^e$ for the government, a strategy profile $\sigma^e$ for the agents, and a belief system $B$, such that $\sigma^e$ and $\Gamma^e$ are best responses to each other, given $B$, and beliefs are derived from Bayesian updating\(^7\).

**Definition 4 (Implementable mechanism)**

$\tau = (\tau_1, \tau_2)$ is called an implementable mechanism if there exists a strategy profile $\sigma$ for the agents and a strategy $\Gamma$ for the government, which constitute an equilibrium and induce an action profile $\gamma = (\tilde{\tau}, \xi, \tilde{\tau}'_2)$ for the government such that $\tilde{\tau} = \tau$, $\xi = 0$, and $\tilde{\tau}'_2 = \tau_2$. Then $\sigma$ and $\Gamma$ are said to support the implementable mechanism $\tau$.

The rest of the paper aims at characterizing the best implementable mechanism, i.e. the tax schedule that maximizes the ex-ante welfare of the continuum of agents.

### 4 Truth-telling and partial revelation

The Revelation Principle is often used to analyze problems of finding and implementing optimal mechanisms in problems with private information (see e.g. Mas-Colell et al. (1995)). However, as is well understood, the government’s commitment problem in a dynamic setting renders the principle inapplicable: When the authority exercising the mechanism can revise its design after the revelation of information, non-revealing mechanisms might in fact outperform direct revealing ones. The optimal mechanism might well lead to complete pooling of agents, as for example in

\(^7\)In the following analysis there will be no need to explicitly derive or condition on these beliefs.

Problems without commitment, however, are of increasing interest, so that there has been according interest in suitable extensions to the Revelation Principle. Bester and Strausz (2001) show that a version of the Principle holds when there is only one agent who faces the principle without commitment. With multiple agents, however, their related paper (2000) comes to the opposite conclusion. More recently, the Bester and Strausz (2007) have explored noisy communication in contracting problems with imperfect commitment.

In a setup more closely related to the one presented here, Acemoglu et al. (2006) prove a modified version of the Revelation Principle, where truthful revelation occurs along the equilibrium path. In their model, the government’s desire to renege on the promised mechanism is counteracted by the threat of agents to revert to the worst equilibrium once and for all. The government thus can effectively commit in equilibrium, truthful revelation is supported by non-truthful reporting off the equilibrium path. Their result, however, cannot be used in the finite horizon setup presented here.

Nonetheless, some progress can be made to characterize the best implementable mechanism in the game presented above. In this section, I will show that while full truthful revelation cannot necessarily be achieved, the best implementable mechanism can always be represented as a truth-telling mechanism. This result will allow me to present the problem of finding the optimal mechanism as one analogous to a standard Mirrlees problem, with the simple addition of choosing an optimal information revelation rule.

Since the government is a utilitarian Planner, finding the best implementable mechanism amounts to solving the following problem:

$$\max_{\tau} \int_{\Theta} U(\theta|\sigma, \tau) dF(\theta)$$

subject to:

$$\int_{\Theta} c_1(\theta|\sigma, \tau) + c_2(\theta|\sigma, \tau) dF(\theta) \leq \int_{\Theta} y_1(\theta) + y_2(\theta) dF(\theta)$$

(4)

$$\sigma$$ is a best response to $$\Gamma$$

(5)

$$\tau_2 \in \arg\max_{\tau_2} \int_{\Theta} u_2(\theta|\sigma, \tau, \tau_2) dF(\theta)$$

subject to:

$$\int_{\Theta} c_1(\theta|\sigma, \tau, \tau_2) + c_2(\theta|\sigma, \tau, \tau_2) dF(\theta) \leq \int_{\Theta} y_1(\theta) + y_2(\theta) dF(\theta)$$

(6)

It maximizes the unweighted sum of agents’ utility (3), subject to an aggregate feasibility constraint (4) and a set of incentive compatibility requirements for the agents (5). Moreover, it must satisfy an implementability constraint (6) to ensure that the government does not want to choose $$\xi = 1$$ and renege on the tax schedule at the
beginning of period 2.

To turn this into a practicable problem, it will be useful to make some progress on the set of incentive compatibility constraints for the agents. To that end, I define a special type of mechanism:

**Definition 5 (Direct Mechanism)**

A direct mechanism is a mechanism \( \tau = (\tau_1, \tau_2) \) where \( \tau_i : M \times X \rightarrow \mathbb{R} \) and \( M \) is a partition of the type space \( \Theta \).

A direct mechanism is one that is based on a restricted message space \( M \). In an equilibrium that induces a direct mechanism, agents report partial information about their type and their tax or transfer payment is based directly on that information. This definition differs from the usual one in that it allows for forms of partial pooling. Note, however, that the definition includes the cases of \( M = \Theta \), where agents report their true type directly to the government, as well as \( M = \{m\} \), a message space with only one element, where agents pool completely and no information is revealed.

**Theorem 1 (Truth-telling Principle)**

Suppose Assumptions (A1)-(A7) hold and that \( \Gamma \) and \( \sigma \) support an implementable mechanism. Then there exists another pair \( \Gamma^* \) and \( \sigma^* \) such that \( \Gamma^* \) induces a direct mechanism and \( \sigma^* \) induces truth-telling, and \( c_t(\theta|\Gamma, \sigma) = c_t(\theta|\Gamma^*, \sigma^*) \) \( \forall \theta, t \).

**Proof**: See Appendix A.1.

The theorem implies that in the search for the best implementable mechanism one can restrict attention to the set of direct mechanisms and truthful strategies. Thus, the problem reduces to:

\[
\begin{align*}
\max_{\{\tau(m)\}_M} & \int_{\Theta} U(\theta|\sigma^*, \tau(m(\theta)))dF(\theta) \\
\text{s.t.} & \int_{\Theta} c_1(\theta|\sigma^*, \tau(m(\theta))) + c_2(\theta|\sigma^*, \tau(m(\theta)))dF(\theta) \leq \int_{\Theta} y_1(\theta) + y_2(\theta)dF(\theta) \\
& m(\theta) \in \arg\max_{\hat{m}} U(\theta, \hat{m}|\tau(\hat{m}), \sigma) \quad \forall \theta \in \Theta, m, \hat{m} \in M \\
\tau_2 & \in \arg\max_{\tau_2^*} \int_{\Theta} w_2(\theta|\sigma, \tau, \tau_2^*)dF(\theta) \\
\text{s.t.} & \int_{\Theta} c_1(\theta|\sigma, \tau) + c_2(\theta|\sigma, \tau, \tau_2^*)dF(\theta) \leq \int_{\Theta} y_1(\theta) + y_2(\theta)dF(\theta)
\end{align*}
\]

The so restricted problem is easier to solve, and delivers the same equilibrium allocation as the solution to the more general problem would. Beside the technical equivalence, it is useful to think about the economic interpretation of direct mechanisms: In reality, when taxes and transfers are conditioned on private information, the government must decide how people report this information. For example, the first step to implementing an income tax is to design a tax return form that people
use to report their income. The government can choose an institutional design that
asks agents only for *coarse* information. The tax return form could, for example,
ask for an agent’s approximate income, or an income bracket.

This intuition is supported in a further transformation of the problem (7)-(10). Instead of
the government choosing and executing the mechanism, one can always employ a more
general mechanism design approach (as e.g. in Bester and Strausz (2001) and Skreta (2007, 2010))
where a fictitious mechanism designer is in charge of choosing strategy sets for the agents and for
the government. Accordingly, let $\mathcal{M} = (\sigma, \Gamma) \in \Sigma \times G$ denote a *fictitious mechanism*, i.e. the fictitious designer’s
choice of strategies for the agents and the government.

The great advantage of the fictitious planner is that it is always able to commit,
so that the Revelation Principle allows attention to be restricted to direct fully
revealing fictitious mechanisms. This allows me to write the problem of the fictitious
planner as maximizing the same objective as the government, (7), subject to aggre-
gate feasibility (8), and the implementability constraint (10), but replacing the set
of incentive compatibility constraints (9) by

$$\theta \in \arg \max_{\hat{\theta}} U(\theta, \hat{\theta} | \tau(m(\hat{\theta})), \sigma) \quad \forall \theta, \hat{\theta} \in \Theta \quad \text{(11)}$$

$$m : \Theta \mapsto M \quad \text{(12)}$$

These two constraints ensure that agents find it optimal to directly reveal their type
truthfully to the planner, while the government can base its actions (and in par-
ticular its choice of the tax schedule) only on parts of this information. The two
problems are equivalent, since the fictitious designer is able to guarantee that the
reported information is encoded by the *information revelation rule* $m$ before being
transmitted to the government.

Reformulating the general problem (3) through (6) into a problem of maximizing
(7), subject to (8), (10), (11), and (12) allows me to explicitly study situations
where the government has limited commitment in the sense that it can commit not
to exploit a limited amount of information. The fictitious planner’s choice of the
information revelation rule makes this explicit: The function $m$ could be such that
no information is revealed (i.e. $m$ is constant), full information is revealed (i.e. $m$
is the identity function), but could also allow for any form of partial information
revelation (i.e. $m$ is constant over some subset of $\Theta$ so that some agents are pooled
together).

It is without loss of generality to assume that $m$ is weakly increasing in $\theta^8$. Moreover,
I normalize $m$ such that

$$m(\hat{\theta}) = \hat{\theta} \quad \text{for} \quad \hat{\theta} = \min \{ \theta : m(\theta) = m \}^9$$

This only excludes the possibility that non-adjacent types are pooled together, which could never
be optimal due to the monotonicity inherent in the setup (Assumption (A1)-(A3)).

This just means that when some types are pooled together, the message sent to the government is
normalized to be equal to the lowest type in that group.
The main focus of the analysis in the next section will be on characteristics of the information revelation rule \( m \) as a proxy for the commitment power of the government, and on how they depend on the severity of imperfection in the financial market agents trade in.

5 Limited commitment due to market imperfection

In Section 4, I made progress on the structure of the problem of finding the best implementable mechanism by transforming the game between the government and the agents into a mechanism design problem of a fictitious designer. The results developed are applicable beyond the model of this paper. Building on them, this section is concerned with deriving some key characteristics of the best implementable tax schedule in the specific setup at hand. The aim is to show that if agents are able to trade in imperfect markets, the government might effectively gain commitment, the degree of which will depend on the severity of the market imperfection measured by \( D \).

When agents trade in financial markets, the benevolent government will take their contractual position into account. Thus, the planning problem of the fictitious designer to find the best implementable mechanism is to maximize

\[
\max_{\tau} \int_{\Theta} \sum_{t=1}^{2} u_{t}(y_{t}(\theta)) + \tau_{t}(m(\theta)) + h_{1}(m(\theta)) - b_{1}(m(\theta)) \, dF(\theta) \quad (13)
\]

subject to aggregate feasibility (restated in terms of the government’s budget)

\[
\int_{\Theta} \tau_{1}(m(\theta)) + \tau_{2}(m(\theta)) \, dF(\theta) \leq 0 \quad (14)
\]

and incentive compatibility

\[
\theta \in \arg \max_{\hat{\theta} \in \Theta} U(\theta, \hat{\theta}|\tau(m(\hat{\theta})), \sigma) \quad \forall \theta, \hat{\theta} \in \Theta \quad (15)
\]

where the information revelation rule (12) is promised. Lastly, the mechanism \( \tau \) must satisfy the implementability requirement (10) to ensure that the government does not choose to deviate from it at the beginning of the second period. Thus, the constraint needs to explicitly take into account what happens if the announced tax schedule is changed. Generally, the government has no commitment, so that choosing to renege on its promise after it learns new information about the agents is of no inherent consequence. It is only due to the agents’ involvement in the imperfect financial market that the government might suffer a loss from changing the tax schedule. In particular, if an agent cannot afford the payment \( b_{2} \) he agreed to provide to the bank in period 2, he is forced to renegotiate his financial contract - and incur a loss of \( D \).
It is without loss of generality to assume that if an agent asks for a renegotiation of his contract, the bank will only agree to a contract with $h'_2 = 0$. Yet, due to assumption (A11) it remains in power to collect any outstanding balance $d_1 \equiv h_1 - b_1$ from period 1 as well as the default (or renegotiation) cost $D$. It cannot, however, enforce a bailout, so that

$$b'_2 = \min\{d_1 + D, y_2 + \tau'_2\}$$

Further, since banks remain able to collect the renegotiation cost, an agent would obviously never choose to renegotiate a contract unless he is forced to by a new tax schedule that leaves him without the means to fulfill his commitments. Notice that assumption (A9) rules out the possibility that agents scheme against the bank and plan a default. While this is certainly an unrealistic assumption, it simplifies matters greatly. A model with the more realistic assumption of banks not being able to observe types is discussed in Simon (2011), the results presented here remain true.

With these features of financial contracts, the implementability constraint (10) becomes

$$\tau_2 \in \arg \max_{\tau_2} \int_{\Theta} u\left(y_2(\theta) + \tau'_2(m(\theta)) - h_2(m(\theta)) - b_2(m(\theta))\right) \mathbb{I}\{y_2(\theta) + \tau'_2(m(\theta)) \geq b_2(\theta)\} + u\left(y_2(\theta) + \tau'_2(m(\theta)) - d_1(m(\theta) + D)\right) \mathbb{I}\{y_2(\theta) + \tau'_2(m(\theta)) < b_2(\theta)\} dF(\theta)$$

s.t. $\int_{\Theta} \tau_1(m(\theta)) + \tau'_2(m(\theta)) dF(\theta) \leq 0$

Examining the problem leads to the main result of the paper:

**Proposition 1 (Effective Commitment)**

Suppose that $D > 0$. Then there exists an equilibrium in which all agents sign gross contracts with $b_2(\theta) = y_2(\theta) + \tau_2(\theta)$ and that supports a fully revealing implementable mechanism that sustains separation in the second period such that:

$$u'(c_2(\bar{\theta}))(c_2(\bar{\theta}) - c_2(\theta) - D) \leq u(c_2(\bar{\theta})) - u(c_2(\theta))$$

*Proof:* See Appendix A.2.

The proposition states that as long as markets are slightly imperfect, there exists a mechanism in which the government is able to abstain from using the information it acquired about agents and reneging on the promised tax schedule. Agents pledging their entire net income in private contracts that cannot costlessly be changed effectively provides the government with a commitment device.

The intuition for condition (17) is simple: it equates the marginal benefit from deviating from the promised allocation (as measured by the marginal utility of the lowest type who would be distributed toward times the amount of resources available for redistribution) with the marginal cost of such a deviation (the utility loss of the
highest type who would have to default). For a given functional form of utility, the higher the default costs, the larger the tolerable difference in consumption levels. Conversely, for given costs $D$, the more concave the utility function is, the higher would be the ex-post welfare gain from redistribution and so the less effective commitment is provided by the market imperfection. Note that when $D = 0$, the case of no commitment is recovered: As long as the utility function is strictly concave, condition (17) is then only satisfied if $h_2(\theta) = h_2(\bar{\theta})$. Thus, when markets are perfect in the sense that agents can simply resell their contracts without a loss, then no fully revealing mechanism with separation in the second period is implementable.

Corollary 1

*Aggregate welfare achieved under the best implementable mechanism is always weakly increasing in $D$.*

Proof: With being able to effectively commit to a fully separating allocation in period 2, the government gains an additional policy instrument: It is able to provide incentives for information revelation by promising a differential consumption allocation in period 2. As is straightforward to see from condition (1), the degree of separation the government can sustain is increasing in $D$. On the other hand, this policy instrument doesn’t have to be used, so that the government cannot do worse in implementing a tax schedule that maximizes welfare than if $D = 0$. □

Agents’ involvement in imperfect markets can improve the government’s ability to implement social redistribution. This is the core result of the paper.

As is stated in Proposition 1, for this channel to work, agents have to pledge all their income in period 2 in a financial contract. Since the effective commitment for the government stems from the threat of default losses should it renege on the promised tax schedule, it is quite obvious that when agents don’t sign such contracts, the mechanism won’t work. However, since all financial contracts are observable to the government, agents find it optimal to pledge their income to the bank to protect it from being take away by the government. One might argue that in reality agents cannot easily assess the government’s commitment power, and so won’t sign contracts simply to induce commitment for the government. Note, however, that it would be easy for the government to induce agents to sign contracts, by offering very non-smooth allocations. Moreover, in reality, a large fraction of household income is indeed pledged in contracts that cannot costlessly be changed (see Chetty and Szeidl (2007) for an extensive study).

6 Limited use of information

Corollary 1 only talks about a *weak* improvement. In fact, a small degree of imperfection (i.e. a small $D$) only allows for a small degree of separation in the second period, which might not be enough to provide incentives for high types to reveal themselves truthfully ex-ante: While Proposition 1 shows that with a positive cost $D$ the government is always able to sustain full separation in the second period, it
adds the qualification that the spread between the lowest and highest type’s consumption cannot be too high, depending on the renegotiation costs \( D \). However, promising a small advantage for high types might not be enough to satisfy their incentive constraints against pretending to be a low type. Thus, even when all conditions of proposition 1 are met, the government might not choose to offer a mechanism that fully separates agents in the second period, even though it could.

In this section, I show that instead of effectively committing to a fully separating allocation with a small spread in consumption levels, the government might choose to implement an only partially revealing mechanism. In that sense, the effective limited commitment translates into a limited information intake, with its form chosen such that the government’s temptation to misuse information ex-post is exactly offset by the potential default loss.

To that end, notice first that a finite cost \( D \) is enough to enable the implementation of the same allocation than if the government exogenously had full commitment (denoted with superscript \( FC \)):

**Corollary 2 (Limit Case: Full Commitment)**

When all agents pledge their complete income in the financial market and if \( D \geq c^FC_2(\tilde{\theta}) \), the government can implement the same allocation as if it had full commitment.

*Proof:* In case the default cost \( D \) is larger than what the highest type was promised to consume, the government would gain no resources for redistribution from letting him default, and thus would never attempt any redistribution ex-post. It can therefore implement the same allocation as if it had full commitment ex-ante. \( \square \)

The result of Corollary 2 should be understood as a limit result: If default costs are so high that they leave no value after renegotiation, it obviously serves as a device for full commitment. Such high costs are unrealistic, they could be interpreted as not offering default as an option. It is interesting, however, that a finite default cost would be enough to induce full commitment. The more relevant case, though, is one where the market imperfection is too small to offer full commitment:

**Corollary 3 (Limited Information Revelation)**

Suppose that \( D > 0 \). Then there exists an equilibrium in which all agents sign gross contracts with \( b_2(m(\theta)) = y_2(m(\theta)) + \tau_2(m(\theta)) \) and that supports a partially revealing implementable mechanism with an information revelation rule such that agents are pooled above but separated below a cutoff type \( \tilde{\theta} \):

\[
m(\theta) = \theta \quad \forall \theta \leq \tilde{\theta} \\
m(\theta) = \tilde{\theta} \quad \forall \theta > \tilde{\theta}
\]

The cutoff \( \tilde{\theta} \) and transfers \( \tau \) must be such that

\[
u'(c_2(\tilde{\theta}))(c_2(\tilde{\theta}) - c_2(\tilde{\theta}) - D) = u(c_2(\tilde{\theta})) - u(c_2(\tilde{\theta}))
\]

(18)
Proof: See appendix A.3.

Corollary 3 is not about the optimality of a specific implementable mechanism. Rather, it highlights that a commitment device with only limited effectiveness (like a small market imperfection) might still be of use. If the difference in consumption levels that the government can effectively commit to sustain is not enough to persuade the highest type of truthful revelation, it might still be enough to at least elicit a limited amount of information. High type agents, instead of having to report their exact income in exchange for little incentive payments, could be asked to only report that their income exceeds a certain threshold. That way, the government gains the ability to at least draw on some of their income for redistribution.

Patterns of partial information revelation are implemented in many real world tax codes. In the US, for example, social security relevant wages are capped, income in excess of the cap is not being considered for the direct federal redistribution scheme. One can also imagine other forms of partial pooling to be optimal, e.g. stepwise pooling into income brackets.

The optimal form of the information revelation rule in the best implementable mechanism will depend on the form of the utility function, on the distribution of types, but also on specifics of the income process. Suppose that

(A13) The functional form of preferences, the type distribution and the income process are such that the government without commitment cannot implement any separation.

This assumption essentially restricts attention to the more interesting cases when the lack of commitment indeed has severe consequences: When Assumption (A13) holds, the government’s lack of commitment leads to the inefficient outcome of no social redistribution at all. The interested reader is referred to Simon (2011) for the full analysis of a concrete example.

Corollary 4
Suppose assumption (A13) holds. Then aggregate welfare achieved under the best implementable mechanism is strictly higher when \( D > 0 \) compared to \( D = 0 \). Moreover, there exists a lower bound on the spread of total income \( \sum_i (y_i(\bar{\theta}) - y_i(\bar{\theta})) \) such that the best implementable mechanism implements some form of partial pooling whenever inequality exceeds that bound.

Proof: See Appendix A.4.

Arguably, the imperfection of markets might not be strong enough to provide a government with full commitment. However, limited commitment can always be achieved. Corollary 4 shows that if inequality is sufficiently large, this effective limited commitment will manifest itself in a limited information intake. Using this limited amount of information, the government can at least achieve some redistribution and so welfare is strictly improved.
7 Discussion

This paper uncovers a mechanism by which the presence of a financial market may effectively provide the government with a (limited) commitment device, thereby enabling the implementation of commitment-type policies. It thus helps reconcile the observation of policies that are suggestive of governments being able to commit, even though there is no apparent commitment device. Moreover, the model provides a rationale for why governments do not implement policies contingent on complete information: When they have no ex-ante commitment power, a reasonable default cost provides them with limited commitment, i.e. with the power to commit to not exploit a limited amount of information.

In the presence of private information, the ability of a government to implement social redistribution crucially depends on its power to commit to future policy. In reality, there is little reason to believe that governments possess some exogenous commitment device. Instead, commitment must stem from the environment the government operates in. The literature has focused on political economy constraints as mechanisms for commitment. In contrast to that, the presented paper highlights the fact that also the economic environment might enhance the commitment power of the government.

For the described mechanism to work, the market agents trade in must be imperfect, in the sense that costs result from renegotiating contracts. In other words, by using financial markets to allocate their resources over time, agents must enter individual consumption commitments. Empirical research by Chetty and Szeidl (2007) shows that this is indeed the case: As much as 65% of the average US household income are devoted to consumption commitments that cannot costlessly be changed. Those include not only mortgage or other loan payments. Also the consumption of durables, insurances or energy contracts can typically not be adjusted instantly or without cost. In that sense, the interpretation of market imperfection goes beyond direct default costs. Rather, it is a measure of incompleteness. If markets were perfect, agents could simply resell infinitesimally small fractions of their contracts on secondary markets, thereby avoiding these adjustment costs.

The model presented is very stylized. In particular, the central characteristic of markets on which the whole commitment mechanism hinges is modeled as an exogenous default cost that is fixed per head. In fact, the results remain unchanged as long as the costs are weakly increasing in the amount of default - and assumption that is more realistic.
References


### Appendix

#### A.1 Proof of Theorem 1

This proof proceeds in two steps. First, I show that for any profile of messages resulting from an equilibrium strategy profile $\sigma^e$ there exists another message profile that consists only of truthful messages but conveys the same information about the agents’ types. Second, I show that for any pair of equilibrium strategies $\Gamma$ and $\sigma$ that support an implementable mechanism, there exists another pair $\Gamma^*$ and $\sigma^*$ which are best responses to each other and that induce a direct mechanism with the same payoffs and truth-telling.

**Step 1:** This step aims to show that any form of information revealed about the agents in an equilibrium that supports an implementable mechanism can be achieved by truth-telling.

First note that it is without loss of generality to consider only deterministic strategies. Since there is no exogenous uncertainty in the model and agents have concave utility, the only reason they would consider randomizing messages would be in response to random tax assignments. However, due to the CRRA assumption, the government would not choose non-degenerate stochastic mechanisms (tax schedules). Since the objective function is concave, introducing risk could only improve matters if some incentive constraints were relaxed. Making payoffs for lower type agents riskier does indeed relax higher types’ incentive constraints. However, since
CRRA implies decreasing absolute risk aversion, the loss for the low types from facing such risk is always higher than the gain in terms of relaxing incentive constraints for higher types. See for example Fudenberg and Tirole (1991).

Let \( z_{\sigma, \Gamma} \) denote the message profile resulting from strategy profile \( \sigma \) of the agents, given a strategy \( \Gamma \) of the government. Then there exists a partition \( M \) of the type space \( \Theta \), such that agents playing a truth-telling strategy and choosing one element \( m \in M \) as their truthful message results in a message profile \( z^* \) such that there exists a one-to-one mapping between the message profiles. In other words, one can always construct a message space from which agents truthfully choose their report that reveals the exact same information to the planner. Since random messages have been ruled out, this is trivial to see. Any form of (partial) pooling that results from agents lying about their types can be replicated by partitioning the type space and using messages such as “at least type \( \theta \)” or “between \( \theta_1 \) and \( \theta_2 \)”. Even messages with no content about the type whatsoever can be replaced by the truthful message “at least type \( \theta \)”.

**Step 2:** Now consider equilibrium strategies \( \Gamma \) and \( \sigma \) that support an implementable mechanism \( \tau \). Then by definition \( \xi = 0 \), i.e. the government’s best response to the information learned from the message profile \( z_{\sigma, \Gamma} \) provides no incentive for it to deviate from the proposed tax schedule. Moreover, it must be true (by definition of an equilibrium) that the messages \( z(\sigma(\theta, \gamma)) \) resulting from the strategy profile \( \sigma \) and leading to the message profile \( z_{\sigma, \Gamma} \) are best responses to the mechanism \( \tau \), so that:

\[
U(z((\theta))|\theta, \sigma, \Gamma) \geq U(\hat{z}((\theta))|\theta, \sigma, \Gamma) \quad \forall z, \hat{z} \in Z
\]  

Now consider the alternative strategy \( \Gamma^* \) for the government that induces the action \( \gamma = (\tau^*, \xi = 0, \tau^*_2 = \tau^*_2) \), where \( \tau^* \) is a direct mechanism that maps from a message set \( \tilde{M} \) to a set of tax and transfer payments, and where further \( \tilde{M} \) is a partition of the type space \( \Theta \) such that agents’ truthful reporting would lead to a message profile \( z^* \) for which a one-to-one mapping to the message profile \( z_{\sigma, \Gamma} \) exists. Moreover, suppose that the induced tax schedule \( \tau^* \) assigns the same transfer payments, i.e

\[
\tau_t(z(\sigma(\theta, \gamma))) = \tau^*_t(z^*(\theta)) \quad \forall \theta, t
\]

Then, by construction, it must be true that

\[
U(z^*(\theta)|\sigma^*, \Gamma^*) = U(z((\theta))|\theta, \sigma, \Gamma) \geq U(\hat{z}(\theta)|\theta, \sigma, \Gamma) = U(z^*(\hat{\theta})|\sigma^*, \Gamma^*)
\]

for all \( \theta, \hat{\theta} \in \Theta \). This implies that truth-telling is indeed a best response to \( \Gamma^* \). Moreover, since the government payoff is unchanged, \( \Gamma^* \) must also be a best response to \( \sigma^* \), thus establishing that the pair \( (\Gamma^*, \sigma^*) \) are an equilibrium. □
A.2 Proof of Proposition 1

The proof of this proposition proceeds in two steps: First, I show that a fully revealing mechanism with separation in the second period is implementable if agents pledged their entire net income in financial contracts, $D > 0$, and condition (17) is satisfied. Second, I show that agents pledging the entire net income in the second period is a best response to such a mechanism.

**Step 1:** Suppose agents signed gross contracts that pledge all their income in period 2, i.e.

$$b_2(\theta) = y_2(\theta) + \tau_2(m(\theta)) \quad (22)$$

Suppose further that the government proposed a fully revealing mechanism, i.e. $m(\theta) = \theta$. Then, each agent’s consumption in the second period is

$$c_2(\theta) = h_2(m(\theta)) \quad (23)$$

i.e. agents planned to consume only what they arranged to receive from the bank.

For the mechanism $\tau$ to be implementable, it must be true that the government has no incentive to deviate from it in period 2:

$$\tau_2 \in \arg \max \int_{\Theta} \left[ u(h_2(m(\theta))) \mathbb{I}\{y_2(\theta) + \tau'_2(\theta) \geq b_2(\theta)\} + u(y_2(\theta) + \tau'_2(\theta) - (d_1(\theta) + D)) \mathbb{I}\{y_2(\theta) + \tau'_2(\theta) < b_2(\theta)\} \right] dF(\theta) \quad (24)$$

s.t. $\int_{\Theta} \tau_1(\theta) + \tau'_2(\theta)dF(\theta) \leq 0$

Since all agents have pledged their entire net income, every change in the promised tax schedule will lead to contracts being renegotiated and the default cost $D$ incurred. Therefore, this ex-interim problem can equivalently be expressed as the planner choosing a new consumption allocation $\{\hat{c}_2\}$ for the agents, taking the cost into account:

$$\max_{\{\hat{c}_2(\theta)\}} \int_{\Theta} u(\hat{c}_2(\theta))dF(\theta) \quad (25)$$

s.t. $\int_{\Theta} \hat{c}_2(\theta)dF(\theta) \leq \int_{\Theta} [h_2(\theta) - D\mathbb{I}\{\hat{c}_2(\theta) < h_2(\theta)\}]dF(\theta)$

This formulation nicely depicts the main point: The government is free to redistribute, but doing so is costly. The default cost $D$ enters only on the resource side of the feasibility constraint. From here it is straightforward to note that:

**Lemma 2**

*If there exists a solution to problem (25) such that $\hat{c}_2(\theta) \neq h_2(\theta)$ for at least one $\theta \in \Theta$, then it must be that $\hat{c}_2(\theta) < h_2(\theta)$.***
Proof: Ex-ante incentive compatibility implies that the promised allocation in \( t = 2 \) is at least weakly increasing in type, so that \( \frac{\partial h_2(\theta)}{\partial \theta} \geq 0 \). Thus, the gains from redistributing ex-post are highest when letting the highest type change his contract. The cost \( D \), on the other hand, is fixed per head. Therefore, any optimal deviation from the promised allocation will distribute away from the highest type. □

The lemma shows that a deviation from the promised tax schedule would always start at the top. Choosing the optimal deviation then becomes a problem of choosing the fraction of types at the top to distribute away from:

\[
\max_{\epsilon} \left[ F(\hat{\theta}) + 1 - F(\bar{\theta}) \right] u(h_2(\hat{\theta})) - \int_{\hat{\theta}}^{\bar{\theta}} u(h_2(\theta)) \, dF(\theta) - \int_{\theta-\epsilon}^{\theta} u(h_2(\theta)) \, dF(\theta)
\]

\[
\text{s.t. } h_2(\theta) = \frac{\int_{\theta-\epsilon}^{\theta} h_2(\theta) \, dF(\theta) + \int_{\theta}^{\hat{\theta}} (h_2(\theta) - D) \, dF(\theta)}{F(\hat{\theta}) + 1 - F(\theta)}
\]

\( \epsilon \geq 0 \)

The optimal \( \epsilon \) maximizes the gain from distributing away from the top types \( \theta \in [\bar{\theta} - \epsilon, \bar{\theta}] \) toward the lowest types \( \theta \in [\hat{\theta}, \bar{\theta}] \), where \( \hat{\theta} \) is chosen to make optimal use of the resources gained at the top (i.e. is endogenous to the choice of \( \epsilon \)). The graph illustrates this choice:

The promised mechanism is implementable, if the government would not even let the highest type default on his contract, i.e. if \( \epsilon = 0 \) is optimal.

The first order condition to this problem, disregarding constraint (28) for the moment, is:

\[
f(\bar{\theta} - \epsilon)u(h_2(\hat{\theta})) + [F(\hat{\theta}) + 1 - F(\bar{\theta} - \epsilon)]u'(\hat{\theta}) \frac{\partial h_2(\hat{\theta})}{\partial \epsilon} - u(h_2(\bar{\theta} - \epsilon))f(\bar{\theta} - \epsilon) = 0
\]

where \( f(\cdot) \) denotes the pdf of the type distribution \( F(\cdot) \) and
\[
\frac{\partial h_2(\hat{\theta})}{\partial \epsilon} = \frac{(h_2(\hat{\theta} - \epsilon) - D)}{[F(\hat{\theta}) + 1 - F(\hat{\theta} - \epsilon)]} - \frac{\int^\hat{\theta}_\epsilon h_2(\theta)dF(\theta) + (h_2(\hat{\theta}) - D)dF(\theta) - (1 - F(\hat{\theta} - \epsilon))h_2(\hat{\theta})}{[F(\hat{\theta}) + 1 - F(\hat{\theta} - \epsilon)]^2}.
\]

Note that since \( \hat{\theta} \) is chosen optimally depending on \( \epsilon \), by the Envelope Theorem the derivative of \( \hat{\theta} \) with respect to \( \epsilon \) need not be taken into account. Further, note that

\[
\int^\hat{\theta}_\epsilon h_2(\theta)dF(\theta) = [F(\hat{\theta}) + 1 - F(\hat{\theta} - \epsilon)]h_2(\hat{\theta}) - \int^\hat{\theta}_\epsilon h_2(\theta)dF(\theta) - (1 - F(\hat{\theta} - \epsilon))h_2(\hat{\theta})
\]

so that the first order condition (29) simplifies to

\[
u'(h_2(\hat{\theta}))(h_2(\hat{\theta} - \epsilon) - h_2(\hat{\theta}) - D) + u(h_2(\hat{\theta})) - u(h_2(\hat{\theta} - \epsilon)) = 0
\]

Thus, for \( \epsilon = 0 \) to be optimal, condition (32) evaluated at 0 must be satisfied:

\[
u'(h_2(\hat{\theta}))(h_2(\hat{\theta}) - h_2(\theta) - D) \leq u(h_2(\hat{\theta})) - u(h_2(\theta))
\]

It is a weak inequality instead of an equality, since the original problem also has the restriction (28). It is straightforward to show that the second order condition at \( \epsilon = 0 \) is negative, so that (33) is a necessary and sufficient condition for optimality. This concludes the first step of the proof. It establishes that if all agents pledge their entire net income in the second period to the bank, then there exists a fully revealing mechanism that is implementable.

**Step 2:** It remains to be shown that signing a gross contract is indeed a best response to the mechanism. Since contracts are observable, it is obvious that any unpledged income will be taken away and redistributed. The cost \( D \) that keeps the government from deviating from the promised tax schedule is only a threat when agents need to actually change their contracts. Thus, signing the required gross contracts is indeed a best response to any fully revealing and separating mechanism. 

**A.3 Proof of Corollary 3**

The proof follows almost immediately from the proof of Proposition 1. The key is to notice that Lemma 2 includes situations when agents are pooled at the top. The lemma establishes that the government will always prefer to distribute away from the highest identified types first. This holds true, even if types are pooled at the top. Suppose agents above some cutoff \( \tilde{\theta} \) are pooled together. Even if it is not
optimal to let all of them default on their contracts, it might still be profitable for the government to default only on a fraction $\pi$ of them. The reason is that neither the gained resources nor the gain in welfare from redistributing these resources are linear in $\pi$. The resources saved are optimally distributed toward the lowest types. Thus, the gain is the highest for the first redistributed dollars and decreases thereafter. The cost of changing contracts, however, is linear in the fraction $\pi$.

Suppose the government proposes a tax schedule that pools agents above a cutoff $\tilde{\theta}$ and fully separates them below, i.e. the information revelation rule of the mechanism is such that:

$$m(\theta) = \begin{cases} \theta & \forall \theta \leq \tilde{\theta} \\ \tilde{\theta} & \forall \theta > \tilde{\theta} \end{cases}$$

Then choosing the optimal tax schedule that uses this form of restricted information and satisfies agents’ incentive constraints to report their types according to this scheme is the following:

$$\max_{\pi} \left[ F(\hat{\theta}) + \pi(1 - F(\tilde{\theta})) \right] u(h_2(\theta)) - \int_\theta^\hat{\theta} u(h_2(\theta)) dF(\theta) - \pi(1 - F(\tilde{\theta})) u(h_2(\tilde{\theta}))$$

$$\text{s.t.} \quad h_2(\hat{\theta}) = \frac{\int_\theta^\hat{\theta} (h_2(\theta)) dF(\theta) + \pi(1 - F(\tilde{\theta}))(h_2(\hat{\theta}) - H)}{(F(\hat{\theta}) + \pi(1 - F(\tilde{\theta})))}$$

$$0 \leq \pi \leq 1$$

Following similar steps as in proof A.2 yields the analogous condition for the spread of consumption levels that the government knowingly can tolerate:

$$u'(c_2(\hat{\theta}))(c_2(\tilde{\theta}) - c_2(\hat{\theta}) - D) = u(c_2(\hat{\theta})) - u(c_2(\tilde{\theta}))$$

$\Box$.

### A.4 Proof of Corollary 4

Following from Corollary 1, the ability to provide incentives for revelation achieved by $D > 0$ is a weak improvement. However, under Assumption (A13) the government without commitment cannot implement any separation in types, so that with $D = 0$ no social redistribution can be implemented. Then, the ability to achieve separation is a strict improvement. However, from Proposition 1 we know that the incentives that can be offered are constrained, so that the government might still not be able to achieve a full revelation. Yet, following from Corollary 3, the government is always able to implement a partially revealing allocation that pools agents at the top. Then, since the government was not able to achieve any redistribution otherwise, it will find it optimal to use this ability and implement at least a partially
separating allocation. This constitutes a strict improvement over the case of \( D = 0 \).

Moreover, it is straightforward to see that for any \( D > 0 \), there exists a level of income inequality between the lowest and highest type such that the ability to offer a differential consumption allocation that satisfies condition (17) of Proposition 1 is not enough to incentivize agents for full truthful revelation of their types. In that case, the best implementable mechanism must have a partially pooling information revelation rule, since only so can a welfare improvement be achieved. \( \square \)