Communicating Quality: A Unified Model of Disclosure and Signaling

Andrew F. Daughety
Jennifer F. Reinganum

Department of Economics
and Law School
Vanderbilt University
Nashville, TN 37235

andrew.f.daughety@vanderbilt.edu
jennifer.f.reinganum@vanderbilt.edu

January 2007
Communicating Quality: A Unified Model of Disclosure and Signaling

Andrew F. Daughety
Jennifer F. Reinganum

ABSTRACT
Firms communicate product quality attributes to consumers through a variety of channels, such as pricing, advertising, releases of research reports and test results, or warranties and returns policies. The conceptualization of the economics of such communication is that it takes on one of two alternative forms when quality is exogenous: 1) disclosure of quality through a credible direct claim; 2) signaling of quality via producer actions that influence buyers’ beliefs about quality. In general, these two literatures have ignored one-another. In this paper we argue that disclosure and signaling are two sides of a coin and that firms should be viewed as choosing which means of communication they will employ. Moreover, we show that integration of these two alternatives leads to a number of new implications about disclosure, signaling, firm preferences over type, and the social efficiency of the channel of communication employed.
1. Introduction

For many products, consumers are unable to observe quality directly prior to purchase, but firms know the quality of the product they provide.\(^1\) Firms communicate product quality attributes to consumers through a variety of channels, such as pricing, advertising, releases of research reports and test results, or warranties and returns policies. The conceptualization of the economics of such communication is that it takes on one of two alternative forms when quality is exogenous:\(^2\)

1) *disclosure* of quality through a credible direct claim; 2) *signaling* of quality via producer actions that influence buyers’ beliefs about quality. Examples of the first form of communication would involve disclosure via the use of an independent (possibly costly) auditing process with public announcements of what quality was found to exist, or advertising in the presence of truth-in-advertising laws with high penalties for misrepresentation. Examples of the second form of communication include posting prices that consumers might use to infer quality, or advertising in unregulated environments where lost future sales due to misrepresentation can provide incentives for truthfulness.

Both approaches have generated extensive literatures that deal with firms selling products whose quality is determined exogenously, known to the firms themselves, but not observable to

---

\(^1\) In this paper we refer to the level of quality of the firm’s product as the firm’s “type.” Thus, in a two-type model a firm either produces a “high-quality” or a “low-quality” product. In this paper we consider a continuum of quality levels, and thus type is a continuous variable.

\(^2\) A third distinct approach to unobservable quality involves the idea of a “quality-guaranteeing price” (beginning with Klein and Leffler, 1981; see Bester, 1998, for a recent example and further references). In this literature, quality is endogenously-chosen by the firm after it posts its price. A firm that dilutes its quality will lose future sales, and a firm that charges too low a price will have an incentive to subsequently dilute quality. Understanding this, consumers will not buy at such prices. Instead, there is a threshold price that is high enough to make it optimal for the firm subsequently to provide high quality rather than to cash in now and forego future sales. Consumers will only be willing to buy at prices at or above this quality-guaranteeing price.
consumers prior to purchase. Remarkably, there has been little communication between these two approaches and yet they are intimately related. In this paper we argue that disclosure and signaling are two sides of a coin and that firms should be viewed as choosing which means of communication they will employ. Moreover, we show that integration of these two alternatives leads to a number of new implications about disclosure, signaling, firm preferences over type, and the social efficiency of the channel of communication employed.

Why have these approaches remained so distinct? The disclosure literature invariably assumes that marginal cost is independent of quality, which renders separation via signaling impossible. Thus, for these models, non-disclosure is consistent with all non-disclosing types charging the same price; these types pool.³ On the other hand, few signaling models of product quality assume that products of different quality are equally costly to produce.⁴ Most signaling models assume that higher-quality products are more costly to produce,⁵ though some are agnostic on the issue, allowing higher-quality products to be either more or less costly.⁶ This difference in costs typically allows the price chosen by the firm to reveal its product’s quality.

We argue that the alternative to disclosure should not be viewed as “non-disclosure,” but

³ For examples of models which involve a single firm (or can be viewed as such due to a lack of strategic interaction), see Viscusi (1978), Grossman (1981), Milgrom (1981), Jovanovic (1982), Matthews and Postlewaite (1985), Milgrom and Roberts (1986b), and Polinsky and Shavell (2006). For examples involving multiple firms, see Board (2003), Cheong and Kim (2004), Levin, Peck and Ye (2005), and Hotz and Xiao (2005).

⁴ But see Hertzendorf and Overgaard (2001a,b) for models that do make this assumption.


⁶ Examples include Milgrom and Roberts (1986a) and Daughety and Reinganum (1995, 2005a).
rather as revealing type via other channels such as price; this modification of perspective alters a number of previously-developed results. We focus on the case wherein higher quality is associated with higher marginal costs of production, and we analyze a continuum-type model. We show that, in equilibrium, a firm with the lowest-quality product will not disclose for any positive disclosure cost, since it obtains its full-information profits in a separating signaling equilibrium. A firm with a higher-quality product will typically need to distort its price in order to signal its true quality (and this distortion increases with an increase in the true quality of the product) and thus, in a separating equilibrium, its signaling equilibrium profits will be less than those under full information. We then show that there is a level of the cost of disclosure that induces a sufficiently high-quality firm to choose disclosure over signaling. Therefore, for this intermediate level of disclosure cost, some types reveal quality via disclosure while other types reveal quality via signaling. We show that overall profits, as a function of type, are “U-shaped” in that both the lowest-quality and highest-quality types of firm have higher profits than those for intermediate types. Moreover, if disclosure costs are “moderate” (to be made precise below), then the highest-quality types make the highest profits; this resolves a dilemma in much of the price-quality signaling literature wherein higher profits are associated with lower quality, suggesting that firms would prefer to produce lower-quality products.

A welfare analysis of voluntary disclosure therefore focuses not on how much information

---

7 In a companion paper focused on safety and legal liability considerations (Daughety and Reinganum, 2006), we examine a two-type model in which marginal costs may be increasing or decreasing with safety, depending upon the liability regime.

8 See, for example, Bagwell and Riordan (1991), Bagwell (1993), and Daughety and Reinganum (1995, 2005b).
is ultimately revealed, *but whether it is revealed through the socially-optimal channel.* When marginal cost is increasing in quality, in a separating signaling equilibrium the high-quality firm charges a higher price and sells less output than it would under full information (*a fortiori*, this is less than the socially-optimal output). Since the firm considers its own profit increase from disclosure, but not the value of the additional output to consumers, there will be a range of disclosure costs for which the firm inefficiently chooses to signal rather than disclose. In this parameter regime, a mandatory disclosure rule may be surplus-increasing as it results in a price reduction and an increase in output produced. However, a more nuanced approach is possible. We describe a simple, decentralized subsidy policy that induces socially-efficient disclosure (given equilibrium pricing).

*Related Literature*

In this subsection, we briefly describe how this paper relates to its closest antecedents. Several previous papers develop models in which price signals product quality (see footnote 5 above); such a model will appear as part of our analysis, but it will be augmented with a disclosure decision. Bagwell and Riordan (1991) provide a two-type monopoly model wherein high quality is more costly to produce. The signaling portion of our current paper is based on the model in Daughety and Reinganum (1995), which linked liability considerations to the relationship between a firm’s full marginal cost (i.e., production plus liability costs) and safety. Daughety and Reinganum

---

9 More distant literature includes papers using as signaling instruments: 1) advertising, such as Kihlstrom and Riordan (1984), Milgrom and Roberts (1986a), Hertendorf and Overgaard (2001b), and Fluet and Garella (2002); and 2) warranties, such as Spence (1977), Gal-Or (1989) and Lutz (1989).

10 Daughety and Reinganum (2005b, 2007) employ a two-type model to analyze price-quality signaling in oligopolies under the assumption that marginal cost is increasing in quality.
One exception is Levin, Peck and Ye (2005), who consider both alternatives: (1) the disclosure decision is made before the firm learns its product quality; and (2) the disclosure decision is made after the firm learns its product quality. In the current paper, we model the disclosure decision as occurring after the firm has learned its type, consistent with the usual timing in the disclosure literature.11

Some of the earliest product-quality disclosure literature (e.g., Grossman, 1981; and Milgrom, 1981) assumed that disclosure was costless. In this case, consumers can induce complete disclosure by all types (known as "unraveling") by employing skeptical beliefs: any non-disclosing firm is assumed to have the lowest possible quality type. Other models (e.g., Viscusi, 1978; Jovanovic, 1982; and Levin, Peck and Ye, 2005) have assumed positive disclosure costs. These models find that high-quality types engage in disclosure, as do we. However, these models often find that there is excessive disclosure, while we find that voluntary disclosure is insufficient. We attribute this to the typical assumption in the disclosure literature that consumers have unit demand; in this case, it turns out that disclosure is essentially redistributive. In our model with downward-sloping demand, the equilibrium level of output increases as a firm switches from signaling to disclosure. Since the social gain from this increase in output exceeds the private gain, firms engage in too little disclosure.

The only published paper of which we are aware that involves both disclosure and signaling

---

11 One exception is Levin, Peck and Ye (2005), who consider both alternatives: (1) the disclosure decision is made before the firm learns its product quality; and (2) the disclosure decision is made after the firm learns its product quality.
of quality is Fishman and Hagerty (2003). However, in their model disclosure and signaling are not substitutes (as they are in our model), but rather are complements due to an externality between different types of consumers. Thus, signaling does not accompany non-disclosure in their model (because they maintain the crucial assumption that marginal costs are the same for high- and low-quality products); it can only accompany disclosure. Our model is very different from that of Fishman and Hagerty (2003) in that we assume only one type of consumer (all of our consumers become informed about quality when the firm discloses it); moreover, our consumers have downward-sloping demand. Our firm has a marginal cost that is increasing in quality. Thus, if a firm does not disclose its quality directly, it reveals it through its price: disclosure and signaling are substitutes.

12 Since completing this paper, we have become aware of a working paper by Caldieraro, Shin and Stivers (2007) that assumes one high-quality and one low-quality firm (firms know each others’ qualities; consumers cannot observe quality directly). After firms individually choose whether or not to disclose, only some consumers observe the disclosure, leading to both disclosure and signaling by the firms. Even though disclosure is free, consumers cannot induce “unraveling” as they cannot employ skeptical beliefs.

13 A related paper from the auction literature is Cai, Riley and Ye (forthcoming). A seller with private information regarding her own valuation auctions an item to multiple potential buyers, whose valuations are positively-correlated with that of the seller. The seller’s reserve price signals her value to the buyers but signaling involves distortion. In the working paper version, the authors briefly discuss an application involving costly external value certification. In their model, higher seller types make higher profits under signaling (the opposite is true in our model), but they too find that higher-valuation sellers will certify while lower-valuation sellers will signal.

14 They assume two different types of (unit demand) consumer; one type becomes “informed” about quality when a disclosure is made, while the other remains “uninformed” about actual quality, but is aware that a disclosure has been made. Suppose that most consumers are capable of becoming “informed.” Then a firm which makes a disclosure and charges a high price will only do so if it is a high-quality firm, for if it were a low-quality firm charging a high price, it would alienate all of the informed consumers. So an uninformed consumer who knows that a disclosure was made (but not its content) can infer from a high price that the firm has a high-quality product.
Plan of the Paper

In Section 2, we describe the notation and provide the results of the analysis when firms can choose between disclosure via a (costly) credible statement and signaling via pricing. We also show that the firm with access to both disclosure and signaling has “U-shaped” profits with respect to its type, and discuss the implications of this result. In Section 3 we characterize socially-optimal disclosure, assuming the firm retains control of the pricing decision; in general, the socially efficient amount of disclosure does not occur in equilibrium. Section 4 details a decentralized subsidy scheme that induces socially efficient use of communication channels by the firm, but leaves the firm’s payoff unchanged. Section 5 summarizes our results and suggests some possible extensions.

2. Equilibrium Disclosure and Signaling

Model setup

Quality

We assume that a single firm produces a product whose quality, \( \theta \), is unobservable to consumers prior to purchase. Quality here is the probability that the consumer is completely satisfied with the product. We capture this by assuming that \( \theta \in [\theta^*, \tilde{\theta}] \), with \( 0 < \theta^* < \tilde{\theta} < 1 \); thus, the lowest type is \( \theta^* \) while the highest type is \( \tilde{\theta} \). Further, assume that the consumer’s prior belief about \( \theta \) is that it is distributed according to a continuously differentiable distribution function, \( G(\bullet) \), with positive density, \( g(\bullet) \), on the interval \([\theta^*, \tilde{\theta}]\). Thus, any type of product may be completely satisfactory in a given instance of its consumption, but any type of product may also disappoint in a given instance; the key attribute is that higher-quality products are more likely to be satisfactory. This “consumer satisfaction” interpretation of quality is useful in that “satisfaction” is non-verifiable,
and thus the firm cannot offer a warranty of the form: “your money will be refunded if you are not completely satisfied with the product,” as this would introduce moral hazard on the part of the consumer, who would always claim, \textit{ex post} of consuming the product, that she was disappointed.\footnote{We trace this interpretation of quality, and the argument that no such warranty can be offered, to Milgrom and Roberts (1986a); this interpretation is also used in Shieh (1993).}

\textit{Consumers}

All types of product provide utility to the consumer, but a unit that is completely satisfactory provides greater utility. In particular, we assume that the consumer’s utility is quadratic in the quantity consumed of the product of interest, with the coefficient on the quadratic term denoted $\beta$, and the coefficient on the linear term denoted $\alpha$ in the case of a satisfactory unit and $\alpha - \delta$ in the case of a disappointing unit. The consumer is unable to observe directly the product’s quality before purchase. Let the \textit{perceived quality} of the good be denoted $\tilde{\theta}$. If the firm discloses the quality before purchase, then $\tilde{\theta} = \theta$; on the other hand, if the product’s quality is not disclosed, then these perceptions will be determined as part of a perfect Bayesian equilibrium wherein the firm’s strategy is its price.

The consumer’s utility function is quasi-linear in all other goods; thus, if the price of the product is $p$, the consumer’s income is $I$, and she consumes $q$ units of perceived quality $\tilde{\theta}$, then her utility is given by:

$$U(p, q, \tilde{\theta}) = (\alpha - (1 - \tilde{\theta})\delta)q - \beta q^2/2 + I - pq.$$  

Thus, the consumer’s demand for the product of perceived quality $\tilde{\theta}$ is given by:

$$q(p, \tilde{\theta}) = (\alpha - (1 - \tilde{\theta})\delta - p)/\beta.$$
The Firm

The firm of type $\theta$ manufactures units of the product at a constant unit cost of $k\theta$, with $k > 0$, so that marginal cost is increasing in $\theta$. The gross profits for the firm depend on its true price-cost margin and consumer demand, which depends on perceived quality:

$$\pi(p, \theta, \tilde{\theta}) = (p - k\theta) \frac{(\alpha - (1 - \tilde{\theta})\delta - p)}{\beta}.$$  

The firm of type $\theta$ can affect its perceived quality in two ways, through its disclosure policy and via its price. We model the choice of disclosure policy and price as being simultaneous, once the firm has learned its true type. Further, we assume that disclosure requires an expenditure of the amount $D > 0$ and that if the firm elects to disclose its quality then, in keeping with the disclosure literature, the disclosure is truthful so that $\tilde{\theta} = \theta$. As an example of this expenditure, $D$ might reflect the cost of obtaining, from an independent third party, a verification of the firm’s type. Note that we are assuming that it is the firm’s type which is verifiable at a cost, not the consumer’s satisfaction with an individual unit of the product. For example, this verification could be achieved by testing a sample of units. If the firm elects not to disclose its quality, consumers will base their perceptions of quality on the posted price.

Finally, the following parameter restrictions will be maintained throughout the paper.

**Assumption 1.** $\alpha - (1 - \theta)\delta - k\theta$ is increasing in $\theta$.

**Assumption 2.** $\alpha - (1 - \theta)\delta > k\bar{\theta}$.

Assumption 1 implies that higher-quality products are socially preferred to lower-quality products (even though the high-quality product is more costly to produce). This reduces to the assumption that $\delta > k$: the marginal gain in reduced consumer dissatisfaction exceeds the marginal cost of its provision. Assumption 2 implies that there is a price in the interval $(k\bar{\theta}, \alpha - (1 - \theta)\delta)$ at which any
firm type will have positive demand and a positive price-cost margin, thereby guaranteeing that each
type can make positive profits whether they are correctly-perceived or mis-perceived as the worst
possible type. Another implication of Assumption 2 is that, while consumers might (in principle)
over-pay for a unit, no product type generates negative surplus overall.

**Analysis of Equilibrium Pricing**

We first characterize the equilibrium pricing behavior that accompanies a decision to
disclose quality directly. Next, we characterize equilibrium pricing behavior when the disclosure
cost $D$ is prohibitively high; this involves solving a relatively straightforward signaling model in
which price reveals quality. Finally, we lower the disclosure cost to determine which types, if any,
will defect from signaling to the outside option of direct disclosure.

Note that any firm type that discloses can (and will) charge its full-information monopoly
price (that is, the price it would charge if consumers could observe quality directly). Let $P_f(\theta)$
denote the full-information monopoly price for a firm producing a product of type $\theta$, and let $\Pi_f(\theta)$
denote the corresponding full-information monopoly profits; then $P_f(\theta) = (\alpha - (1 - \theta)\delta + k\theta)/2$ and
$\Pi_f(\theta) = (\alpha - (1 - \theta)\delta - k\theta)^2/4\beta$. Since disclosure is costly, but the pricing game accompanying
disclosure is one of full information, the equilibrium price and payoff for a disclosing firm of type
$\theta$ are simply $P_f(\theta)$ and $\Pi_f(\theta) - D$ for $\theta \in [\underline{\theta}, \bar{\theta}]$.

Now suppose that the disclosure cost $D$ is prohibitively high, so it is common knowledge that
no firm will choose disclosure. Then consumers will try to infer product quality from the price that
is being charged. We characterize a separating perfect Bayesian equilibrium in which price serves
11

We focus on separating equilibria in this paper. In two-type models of this sort, refinement using the Intuitive Criterion eliminates pooling equilibria. Extending this result to continuum type-space models is beyond the scope of this paper.

Let $B(p)$ be the belief function that relates the firm’s price to the consumer’s perceived quality; thus, if the firm charges the price $p$, then it is inferred to have quality $B(p) \in [\theta, \bar{\theta}]$. A firm charging price $p$, with true quality $\theta$ and perceived quality $\tilde{\theta} = B(p)$, obtains a profit of:

$$\pi(p, \theta, B(p)) = (p - k\theta)(\alpha - (1 - B(p))\delta - p)/\beta.$$ 

In addition to incentive compatibility constraints that ensure separation, a separating perfect Bayesian equilibrium requires that consumers infer correctly the firm’s type from its price; that is, the beliefs must be consistent with equilibrium play. This is formalized in the following definition.

**Definition 1.** Suppose that $D$ is prohibitively high, so no firm type discloses. A separating perfect Bayesian equilibrium in prices consists of a price function, $P^*(\theta)$, and beliefs, $B^*(p)$, such that for all $\theta \in [\underline{\theta}, \bar{\theta}]$:

(i) $\pi(P^*(\theta), \theta, \theta) \geq \max_p \pi(p, \theta, B^*(p));$  

(ii) $B^*(P^*(\theta)) = \theta.$  

Consistency

In a separating equilibrium, the equilibrium price function $P^*(\theta)$ must be either everywhere increasing, or everywhere decreasing, in $\theta$. Let $\underline{P}$ and $\bar{P}$ denote the minimum and maximum prices in the separating equilibrium. In what follows, we will assume that the beliefs $B(p)$ are continuous on $[\underline{P}, \bar{P}]$ and twice differentiable on $(\underline{P}, \bar{P})$ and weakly monotonic everywhere; we will refer to such beliefs as *regular* beliefs. Under these assumptions, the function $\pi(p, \theta, B(p))$ is twice differentiable in $p$, and we will be able to represent the equilibrium price function as the solution of a differential
Recall that $B_N(p)δ - 1 < 0$. The second-order condition is that 
\[
\frac{d^2π}{dp^2} = \frac{π_p}{π_{pp}} - (B_N(p)δ - 1) = 0.
\] 
(1)

We note for future reference that since the price-cost margin, $(p - kθ)$, and the quantity demanded, $(α - (1 - B(p))δ - p)$, are positive numbers, equation (1) implies that $B'(p)δ - 1 < 0$. Since $B'(p) = 1/p'(θ)$, then $B'(p)δ - 1 < 0$ is equivalent to $p'(θ) > δ$. Substituting $B'(p) = 1/p'(θ)$ and $B(p(θ)) = θ$ (consistency of beliefs), equation (1) can be re-written as the following ordinary differential equation:
\[
\frac{dp}{dθ} = δ(p - θk)/(2p - (α - (1 - θ)δ) - θk).
\] 
(2)

Notice that the numerator is positive, while the denominator is positive if and only if $p > P(θ) = (α - (1 - θ)δ + kθ)/2$. Thus, the solution of this ordinary differential equation is either an increasing function of $θ$ that lies above the full-information price function, or a decreasing function of $θ$ that lies below the full-information price function. It is straightforward to show that the second-order condition selects the first version: higher quality is associated with a higher price (and thus, $B(p)$ is increasing in $p$), and quality is signaled with a price that is upward-distorted relative to the full-information price.\(^{17}\) The relevant boundary condition for this problem is that there is no need for the lowest type to distort in a separating equilibrium, so that at $θ$, $P(θ) = P'(θ)$.

The solution $P(θ)$ to equation (1) is described (implicitly) by:\(^{18}\)

\(^{17}\) Recall that $B'(p)δ - 1 < 0$. The second-order condition is that $d^2π/dp^2 = π_{pp} = 2(B'(p)δ - 1) + (p - kθ)(B''(p)δ < 0$. Moreover, since equation (1) is a differential equation, it must hold for all $(p, θ)$. Thus, $π_{pp}dp + π_{pθ}dθ = 0$, which implies that $dp/dθ = -π_{pθ}/π_{pp} = k(B'(p)δ - 1)/π_{pp}$. Since both the numerator and denominator are negative, we conclude that $dp/dθ > 0$.

\(^{18}\) This solution is found by reducing equation (2) first to a homogeneous equation and then to a separable equation via changes in variables, solving the resulting ordinary differential equation,
and then using the earlier substitutions to recover the solution to equation (2) (for these general procedures, see, for example, Hildebrand, 1962, p. 37).

There liability was of interest, but the same general differential equation arises, and the structure of the equilibrium signaling function is found in exactly the same manner as is discussed therein.

\[(\alpha - (1 - \theta)\delta - p)^{(\delta - k)}(2p - k\theta + k(\alpha - \delta)/(\delta - k))^{\delta} = K, \tag{3}\]

where \(K\) is a constant found by using the boundary condition in equation (3) above. As was discussed in a related version of this problem\textsuperscript{19} in Daughety and Reinganum (1995), the solution to the implicit representation (3) is a hyperbola in \((p, \theta)\)-space; this holds because equation (3) is multiplicatively separable into two linear functions of \(p\) and \(\theta\). Therefore the relevant portion is strictly increasing and concave. Since this is the unique solution to the differential equation, we have the following proposition; Figure 1 illustrates the signaling and full-information price functions.

**Proposition 1.** There is a unique separating perfect Bayesian equilibrium with regular beliefs.

(i) The lowest-type firm always charges its full-information price: \(P^*(\theta) = P^f(\theta)\).

(ii) \(P^*(\theta), \theta \in [\underline{\theta}, \bar{\theta}]\) is the solution to the implicit equation (3) using the boundary condition from part (i). \(P^*(\theta) > P^f(\theta)\) for all \(\theta \in (\underline{\theta}, \bar{\theta})\) and \(P^*(\theta)\) is strictly increasing and concave on this interval.

(iii) \(B^*(p) = (P^*)^{-1}(p)\) for \(p \in [P^*(\underline{\theta}), P^*(\bar{\theta})]\), \(B^*(p) = \underline{\theta}\) for \(p < P^*(\theta)\), and \(B^*(p) = \bar{\theta}\) for \(p > P^*(\bar{\theta})\).

To see how firm profits in the signaling equilibrium depend upon \(\theta\), substitute the solution from equation (3) into the profit function, to obtain the reduced-form profits:

\[\text{and then using the earlier substitutions to recover the solution to equation (2) (for these general procedures, see, for example, Hildebrand, 1962, p. 37).}\]
Differentiating with respect to $\theta$, employing the envelope theorem and the earlier-stated requirement that beliefs are correct in equilibrium, provides the following:

$$d\Pi'(\theta)/d\theta = -k(\alpha - (1 - \theta)\delta - P'(\theta))/\beta,$$

for all $\theta \in [\underline{\theta}, \bar{\theta}]$. (4)

This derivative is strictly negative since $(\alpha - (1 - \theta)\delta - P'(\theta))/\beta$ is simply the quantity demanded at price $P'(\theta)$ and, by Assumption 2, there is always a price at which any firm type can obtain positive demand (and therefore it does so at its equilibrium price). This means that (in the signaling equilibrium) profits are declining in quality; this is due to the need to distort price.

**Proposition 2.** Properties of Signaling and Full-Information Profits as a Function of Quality.

i) $\Pi'(\theta)$, the reduced-form profit function in the signaling equilibrium, is strictly decreasing in $\theta$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$;

![Figure 1: Pricing Under Signaling and Under Full Information](image-url)
ii) $\Pi'(\theta)$, the full-information profit function, is strictly increasing in $\theta$ for all $\theta \in [\theta, \tilde{\theta}]$;

iii) since $P'(\theta) = P_f'(\theta)$, then $\Pi'(\theta) = \Pi_f'(\theta)$, and the difference in profits, $\Pi_f'(\theta) - \Pi_s'(\theta)$, is strictly increasing in $\theta$ for all $\theta \in [\theta, \tilde{\theta}]$.

*Private Incentives for Voluntarily Disclosure.*

Proposition 2 implies that there exists a range of disclosure costs, D, such that some types will signal and others will disclose. That is, for D such that $0 < D < \Pi_f'(\tilde{\theta}) - \Pi_s'(\tilde{\theta})$, there is a marginal voluntarily-disclosing type $\theta^V \in (\theta, \tilde{\theta})$ with the property that:

$$\Pi_f'(\theta) - D (>, =, <) \Pi_s'(\theta) \text{ as } \theta (>, =, <) \theta^V.$$  

Those types below $\theta^V$ choose to signal, while those types above $\theta^V$ choose to disclose. The marginal type, $\theta^V$, is indifferent between signaling and disclosing; we take this type as disclosing so as to be consistent with the traditional disclosure literature wherein the lowest type discloses when $D = 0$.

This produces a pricing function as shown in Figure 2 below. When $D > \Pi_f'(\tilde{\theta}) - \Pi_s'(\tilde{\theta})$, then signaling is the least-costly means of communicating quality, while if $D = 0$, then all types can credibly disclose and are able to avoid using distortionary pricing as a means for signaling quality. On the other hand, when $0 < D < \Pi_f'(\tilde{\theta}) - \Pi_s'(\tilde{\theta})$, then disclosure is “from above;” that is, the higher quality types, who have much to gain from switching from signaling to disclosing, pay the cost $D$ and credibly disclose their types. This allows them to lower their prices from the (distorted) monopoly signaling prices to the (undistorted) monopoly full-information prices, thereby increasing their profits. As the gap between the full-information and signaling profits declines, we find the marginal type that is disclosing voluntarily; all types below this marginal type would obtain full information profits (net of disclosure costs) that are below what they can achieve via signaling, so they choose to communicate quality via a distorted price. Notice that non-disclosure accompanied
by a price $p > P^*(\theta^V)$ is an out-of-equilibrium event; we assume that consumers treat such a price as coming from the type $B^*(p) > \theta^V$ (as described in Proposition 1). That is, the consumer responds as if type $B^*(p)$, who should have disclosed in equilibrium, “trembled” and signaled instead. The incentive compatibility constraints guarantee that the resulting profit along the signaling price line would be no greater than what the non-disclosing type would have obtained if it had correctly signaled its type, which (for types above $\theta^V$) is lower than the full-information profit minus the disclosure cost. Thus, the beliefs specified in Proposition 1 are enough to deter this sort of defection.

Denote the overall equilibrium profits incorporating the disclosure-signaling choice as $\Pi(\theta)$, so that $\Pi(\theta) = \Pi(\theta)$ for $\theta \in [\bar{\theta}, \theta^V)$ and $\Pi(\theta) = \Pi(\theta) - D$ for $\theta \in [\theta^V, \bar{\theta}]$. By construction, $\Pi(\theta)$ is continuous and twice differentiable everywhere except at $\theta^V$. From equation (4) and from the earlier
discussion that δ < dP/θ, it follows that δ^2Π'(θ)/θ^2 = - k(δ - dP/θ)β > 0. It is also straightforward to show that Π'(θ) is strictly increasing and convex in θ. Thus, Figure 3 illustrates the case of greatest interest, wherein 0 < D < Π'(θ) - Π'(θ).

Figure 3: Payoff Relationships with Voluntary Disclosure

The equilibrium profit function, Π(θ), is “U-shaped” (with a kink at the bottom) but a little more can be observed. Let D^mod = Π'(θ) - Π'(θ), something that is very easy to compute; (0, D^mod) defines a set of “moderate” disclosure costs. If 0 < D < D^mod (that is, if the disclosure cost is moderate in magnitude), then Π'(θ) > Π(θ), so that the “U-shape” of Π(θ) involves the right end being higher than the left end; it is better to be a high-quality producer than a low-quality producer, when disclosure is incorporated in the equilibrium. In this case, the firm most prefers to be a high-quality producer, next prefers to be a low-quality producer, and least-prefers to be a producer of
middle-quality products. Alternatively, for \( D^{\text{mod}} < D < \Pi'(\tilde{\theta}) - \Pi'(\bar{\theta}) \), some types disclose, but now \( \Pi(\bar{\theta}) < \Pi(\tilde{\theta}) \), so it is better to be a low-quality producer than a high-quality producer.

Thus, the opportunity to disclose at a cost that is moderate (\( D \) such that \( 0 < D < D^{\text{mod}} \)) resolves a problem endemic to many price-quality signaling models: since it is the higher-quality types that must distort their strategies the most so as to separate, it is not unusual that the high-quality types have reduced-form profits that are less than those of the low-quality firms. Here, disclosure at moderate cost changes this picture as it results in the highest-quality types achieving payoffs higher than those of the lowest-quality types.

In this paper, the firm’s type space \([\theta, \bar{\theta}]\) is given exogenously, but previous work allows us to speculate intelligently about what would happen in an extension wherein the firm first engages in R&D in order to influence its type and then either discloses or signals as discussed above. In Daughety and Reinganum (1995) we model the R&D process as sequential sampling from a distribution defined over \([\theta, \bar{\theta}]\). That is, a firm pays a sampling cost and draws a quality level from \([\theta, \bar{\theta}]\); it then decides whether to stop sampling and produce this quality of product, or pay another sampling cost and take another draw. The firm’s problem is one of optimal stopping, and there will be a subset of \([\theta, \bar{\theta}]\) such that the firm will stop if it draws a quality level from within this set, and will otherwise sample again. Assuming the consumer knows the firm’s sampling cost then, in equilibrium, the consumer correctly conjectures the stopping set. Therefore the firm’s ultimate “type space” is determined endogenously as a subset of \([\theta, \bar{\theta}]\).

We can speculate about the results of doing this same exercise in the current model, which allows the firm to choose between signaling and disclosure. When disclosure costs are very high, then only signaling will occur; since signaling profits are decreasing in quality, firms will stop
sampling when they obtain a sufficiently low level of quality; the endogenously-determined type space will be of the form \([\theta, \theta_L]\), with \(\theta_L < \bar{\theta}\). When disclosure costs are negligible, then almost all firm types will engage in disclosure; since full-information profits are increasing in quality, firms will stop sampling when they obtain a sufficiently high level of quality; the endogenously-determined type space will be of the form \([\theta_H, \bar{\theta}]\), with \(\theta_H > \bar{\theta}\). Finally, when disclosure costs are neither very high nor very low, then some firms will signal and others will disclose, resulting in overall firm profits that are U-shaped. In this case, there will be an intermediate range of disclosure costs for which the firm’s optimal stopping set will be the union of two disconnected intervals; the firm will stop sampling either when it draws a sufficiently low quality level or a sufficiently high quality level, but it will reject intermediate levels of quality in favor of sampling again. Here the endogenously-determined type space will be of the form \([\theta, \theta_L] \cup [\theta_H, \bar{\theta}]\), with \(\theta < \theta_L < \theta_H < \bar{\theta}\).

3. Disclosure, Signaling, and Efficiency

Since the firm only considers its profit when making a decision to disclose, the decision to disclose or to signal is socially inefficient. To see this we construct total surplus and therefore require consumer’s surplus to be added to profits. Let \(CS_i'(\theta) = (\alpha - (1 - \theta)\delta - P'(\theta))^2/2\beta, i = f, s\), be the consumer’s surplus enjoyed from a transaction at price \(P'(\theta)\); here, as usual, the superscript \(s\) indicates signaling and the superscript \(f\) indicates full-information. The following proposition is straightforward to verify.

**Proposition 3.** Properties of Consumer’s Surplus as a Function of Quality.

(i) \(CS_i'(\theta) = CS_i'(\theta)\);

(ii) \(CS_i'(\theta)\) is strictly increasing in \(\theta\), for \(\theta \leq \theta \leq \bar{\theta}\);
(iii) \( CS'(\theta) \) is strictly decreasing in \( \theta \), for \( \theta \leq \theta \leq \hat{\theta} \);

(iv) \( CS'(\theta) - CS'(\theta) \) is strictly increasing in \( \theta \), for \( \theta \leq \theta \leq \hat{\theta} \).

Item (i) follows directly from the fact that the lowest-quality type always signals type without distorting its price. Item (ii) is straightforward to derive via employing the full-information price function. Item (iii) follows by differentiating \( CS'(\theta) \) and recalling that \( \delta < dP/\theta; \) item (iv) follows immediately from items (ii) and (iii).

Now let \( W(\hat{\theta}) \) be the total surplus associated with any policy wherein types in \( [\theta, \hat{\theta}) \) employ signaling while types in \( [\hat{\theta}, \theta] \) pay the disclosure cost \( D \), disclose type, and use the appropriate full-information price; we assume that \( D \in (0, \Pi'(\hat{\theta}) - \Pi'(\hat{\theta})) \), so at least some types would disclose voluntarily. Then total surplus is:

\[
W(\hat{\theta}) = \int_A (CS'(\theta) + \Pi'(\theta))dG(\theta) + \int_B (CS'(\theta) + \Pi'(\theta) - D)dG(\theta),
\]

where \( A = [\theta, \hat{\theta}) \) and \( B = [\hat{\theta}, \theta] \). Notice that:

\[
W'(\hat{\theta}) = [CS'(\hat{\theta}) + \Pi'(\hat{\theta}) - CS'(\hat{\theta}) - \Pi'(\hat{\theta}) + D]g(\hat{\theta}).
\]

Observe that the term in brackets is positive at \( \theta \) (since it reduces to simply \( D \)) and negative at \( \theta^v \) (where it reduces to \( CS'(\theta^v) - CS'(\theta^v) < 0 \)). Moreover, using Propositions 2 and 3, the term in brackets is decreasing in \( \hat{\theta} \). Thus there is a unique \( \hat{\theta} \) in the interval \( (\theta, \theta^v) \), which we denote as \( \theta^w \), which maximizes \( W(\hat{\theta}) \) over \( \hat{\theta} \in [\theta, \hat{\theta}] \). In other words, the equilibrium level of disclosure is socially inefficient, as social optimality would entail disclosure by all types in the interval \( [\theta^w, \theta] \), but the firm only discloses for types in the interval \( [\theta^v, \hat{\theta}] \), where \( \theta^w < \theta^v \).

---

\(^{20}\) For \( D \in [\Pi'(\hat{\theta}) - \Pi'(\hat{\theta}), CS'(\hat{\theta}) + \Pi'(\hat{\theta}) - CS'(\hat{\theta}) - \Pi'(\hat{\theta})] \) no type would disclose voluntarily but social optimality requires that some types do disclose. Considering this case adds complexity (boundary considerations) without adding insight, so we abstract from it.
Mandatory Disclosure

Figure 2 makes the effect of mandatory disclosure clear: all types disclose, but now those below $\theta^V$ all bear the cost of disclosure while some (those in $[0, \theta^W]$) should not. Of course, mandatory disclosure yields a benefit to consumers, since the price falls to the full-information line from the price-signaling line.

4. Subsidy-augmented Disclosure

As a more-nuanced alternative to mandatory disclosure, we now consider a subsidy scheme that induces voluntary disclosure only by those types for whom disclosure is socially efficient. Furthermore, we will see that this subsidy scheme leaves the firm’s overall profits unchanged and therefore results in the same U-shaped overall profits for the firm as indicated in the discussion above, with the highest-quality firms making the highest profits if disclosure costs are moderate.

Consider the following subsidy function:

$$s(\theta; D) = \Pi_s(\theta) - (\Pi_f(\theta) - D)$$

for $\theta$ in $[\theta^W, \theta^V]$ and zero elsewhere.

From the discussion in Section 2, it is straightforward to find the following properties of $s(\theta; D)$.

**Proposition 4.** Properties of the Subsidy Function.

(i) $s(\theta^W; D) = CS_s(\theta^W) - CS_f(\theta^W) > 0$;

(ii) $s(\theta^V; D) = 0$;

(iii) $s(\theta; D)$ is decreasing in $\theta$ for $\theta^W \leq \theta < \theta^V$;

(iv) $s(\theta; D)$ is increasing in $D$ for $\theta^W \leq \theta < \theta^V$.

Item (i) is due to equation (6) where $W'(\theta^W) = 0$; item (ii) is from the definition of $\theta^V$; item (iii) follows from Proposition 2(iii) and item (iv) is straightforward.
Now consider a social planner who chooses to make this subsidy available to any type that discloses. Those types at $\theta^V$ and above do not actually receive any subsidy, but they voluntarily disclose quality as discussed earlier. Furthermore, types below $\theta^W$ do not qualify for the subsidy, so they continue to signal type via their prices. Finally, the subsidy function makes all types in the interval $[\theta^W, \theta^V]$ indifferent between disclosing and signaling, which we heretofore assumed resulted in disclosure. Thus, under the subsidy function, disclosure and signaling cause all types to be revealed, some via one channel and some via the other channel, but now the efficient channel is always employed voluntarily. This is summarized in Proposition 5.

**Proposition 5.** Equilibrium Subsidy Payments and Resulting Overall Profits.

(i) The subsidy received by type $\theta$ is:

a) $0$ for $\theta \in [\theta, \theta^W)$;

b) $\Pi^s(\theta) - \Pi^f(\theta) + D$ for $\theta \in [\theta^W, \theta^V)$;

c) $0$ for $\theta \in [\theta^V, \bar{\theta}]$;

(ii) Using the subsidy function, the overall equilibrium profits, $\Pi(\theta)$, and the disclosure-signaling choices are as follows:

a) $\Pi(\theta) = \Pi^s(\theta)$ (signaling) for $\theta \in [\theta, \theta^W)$;

b) $\Pi(\theta) = \Pi^f(\theta)$ (disclosure) for $\theta \in [\theta^W, \theta^V)$;

c) $\Pi(\theta) = \Pi^f(\theta) - D$ (disclosure) for $\theta \in (\theta^V, \bar{\theta}]$.

All the gains in surplus from inducing additional disclosure accrue to the consumers. The expected payments under the subsidy policy are:

$$\int_{B1} s(\theta; D) dG(\theta),$$

where $B1 = [\theta^W, \theta^V]$, which we assume comes from general tax revenues for the economy. The
foregoing subsidy function allows for a decentralized scheme, wherein the planner simply announces the policy and any firm that seeks the subsidy must credibly disclose its type.

5. Summary and Conclusions

In this paper we model the firm as being able to choose to either signal quality (via price) or disclose quality (via paying a cost that guarantees credible disclosure; e.g., by employing an outside auditor). If the disclosure cost is sufficiently high the firm will always signal the quality of the product via the price it sets; in the unique separating equilibrium, consumers use the price to infer the quality of the product and buy accordingly. If the cost of disclosure were zero, then all types of the firm would choose to disclose, and the firm would post its type-specific full-information price. Again, consumers would react to the price, knowing the quality of the product, and buy accordingly. We show that full-information profits are increasing in quality, but signaling profits are decreasing in quality, and the lowest possible type’s profits are the same in the two informational settings. Thus, the gap between full-information and signaling profits is increasing in quality. Therefore, for disclosure costs that are positive but not prohibitively high, there is a marginal type of firm that is just indifferent between disclosing and signaling; all types below choose to signal and all types above choose to disclose. We then show that the overall profits of the firm (as a function of quality) are first decreasing (due to signaling) and then increasing (due to disclosure), so that these profits are “U-shaped.” If disclosure costs are moderate then this means that profits for the firm are highest for the highest-quality type, lower for the lowest-quality type and lower yet for types “in the middle.” This, in turn, affects a firm’s incentives to invest so as to influence its quality (see Section 2 above).
Information about quality is always revealed, but this may involve the use by the firm of an inefficient means of revelation. Since the firm’s choice between signaling and disclosure is based on its profits under signaling versus the full-information profits net of the disclosure cost, some types will inefficiently choose to signal when social welfare would be maximized by those types disclosing instead, as the ensuing reduction in price and expansion in output increase overall surplus.

Finally, we provide a subsidy scheme that addresses this inefficiency, with the subsidy generated from the overall tax base. The subsidy function specifies a payment based on the firm having elected to pay the disclosure cost and disclose type; it is made available to all types of firm but is positive only for those who would otherwise inefficiently choose to signal. This subsidy scheme results in each type of firm choosing the socially-efficient channel through which to communicate quality to consumers and maintains the same overall profits as would arise without the subsidy.

Our model allows only one instrument through which the firm could signal quality: price. As indicated in Section 1, a number of other signaling instruments have been investigated, sometimes in lieu of price but sometimes in conjunction with price (see, for example, Milgrom and Roberts, 1986a, wherein advertising augments price as a signal of quality). Expanding the model to allow for a richer strategy space in this sense would likely lead to a relaxation of Assumption 2 and may readily improve the profits of a firm engaged in signaling, thereby increasing the portion of the overall parameter space wherein a separating equilibrium can exist. Note also that strategies that might augment price and result in increased signaling profits will result in an increase in the marginal disclosing type ($\theta^V$).

Our model considers a monopolist; the extension to multiple firms is important but quite
complex. The complexity arises due to the fact that the incentive compatibility constraints for each firm also are best response functions with respect to the expected price of the firm’s rival (see Daughety and Reinganum, 2005 and 2007, for examples in the pure signaling context with two types for each firm). This means that the price-signaling function also must satisfy a fixed-point property (so that best responses yield an equilibrium), which makes the analysis considerably more difficult. In the case at hand, one would further need to allow each firm to choose whether to signal or to disclose, and this would also influence the overall pricing equilibrium. The payoff to this exercise would be a better understanding of equilibrium pricing and profits when both signaling and disclosure are possible firm strategies in an oligopoly setting.
References


Daughety, Andrew F. and Jennifer F. Reinganum. “Imperfect Competition and Quality Signaling,” Working Paper No. 05-W20, Department of Economics, Vanderbilt University, June 2005b
(last revised January 2007).


