Price and Quantity Competition with Product Differentiation and Wage Bargaining

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October 2005

Abstract

We consider an oligopolistic model with product differentiation in which a firm’s costs are not given exogenously but are the result of a wage bargaining process between firms and unions. Contrary to the standard results of Singh and Vives (1984) that Cournot equilibrium profits always exceed those under Bertrand competition when goods are imperfect substitute. With a generalised model of Lopez and Naylor (2004), we show that Bertrand profits can be higher than Cournot profits for particular values of the parameters of the wage bargaining (i.e., unions’ preference over wage and bargaining power). This holds even if there are more than two firms in the economy. However, there is a threshold in the number of firms above which, independently on the values of the parameters of the model, the standard result continues to hold. Furthermore, when goods are complements, depending on the values of the bargaining process, we can have that equilibrium prices under Cournot are higher than prices under Bertrand competition.

JEL Classification: D43; J50; L13

Keywords: Product Differentiation; Wage Bargaining; Cournot; Bertrand

1 Introduction

There is an extensive literature comparing Cournot and Bertrand results in models with product differentiation. In their classical work Singh and Vives (1984), using the duopoly framework developed by Dixit (1979), show that Cournot profits are...
always larger than Bertrand profits when goods are imperfect substitutes.\(^1\) López and Naylor (2004) tested the robustness of that result by introducing endogenous costs into the Singh and Vives’ duopoly framework. Costs faced by the two firms are endogenised by assuming that the wages paid by each firm is the outcome of a strategic bargain with its labour union. They show that the standard result that Cournot equilibrium profits exceed those under Bertrand competition—when the differentiated duopoly game is played in imperfect substitutes—is reversible. Whether equilibrium profits are higher under Cournot or Bertrand competition is shown to depend upon the nature of the labour unions’ preferences and the distribution of bargaining power over the wage paid by each firm.

The key feature of this partial equilibrium model is that goods and labour market are imperfect. On one hand there are two vertically connected markets, with independent unions active in the input market and independent firms active in the final product market. Within this context one industry produces an input good that is used for production in the other industry. The equilibrium in the labour market is settled by firm-union wage bargaining, while the outcome of product market is modelled by Cournot/Bertrand oligopolies. This can be interpreted as a particular example of a more general situation of bargaining between an upstream supplier and a downstream retailer in the context of oligopoly in the retail market.

While López and Naylor analysis is limited to the case of a duopoly market, Häckner (2000) extends the analysis of Singh and Vives (1984) allowing for an arbitrary number of firms with vertical product differentiation. However, differently form López and Naylor, in Häckner (2000) costs are exogenously given and normalised to zero. In that setting, he shows that the results in Singh and Vives are sensitive to the duopoly assumption. In particular, he shows that with more than two firms in the market, prices may be higher under Bertrand competition than under Cournot competition if goods are complements and if quality differences between goods are large. Our aim is to generalise the duopoly analysis of Singh and Vives by introducing the bargaining process of López and Naylor into Häckner’s framework. This allows us to extend the Singh and Vives model both horizontally, through the increase in the number of firms, and vertically, by examining the consequences of introducing upstream suppliers to the downstream firms. Thus, our paper can be thought as a generalisation of López and Naylor analysis, by allowing for an arbitrary number of firms, and also that of Häckner (2000), by endogenising the costs faced by firms through a wage bargaining process.

Our framework is closely related to Naylor (2002), however, it differs from his model in two main aspects: first, Naylor (2002) considers a non-cooperative two-stage game in which an arbitrary number of firms produce an identical good, while we focus on a differentiated oligopoly. Second, he focuses only on Cournot compe-

\(^1\) See also Cheng (1985) for a geometrical representation of the results; Vives (1985) for the analysis to the case of general demand and cost structure; and Qui (1997) for the process R&D competition.
tition and on the relation between profits and number of firms, while we focus on profit/price differential between Cournot and Bertrand competition. Nevertheless, a remarkable result of Naylor’s analysis is that, with homogeneous products, contrary to the standard result of Cournot-Nash oligopoly model, industry profits can initially increase with the number of firms if input prices are determined by bargaining in a bilateral oligopoly and unions have a high bargaining power.

We examine whether the standard result on the ranking of Cournot and Bertrand equilibrium outcomes, in terms of profits and prices, under a differentiated oligopoly are robust to the inclusion of a decentralised wage-bargaining game played by each firm and a firm specific labour union over these parameters -the unions’ preferences over the input price, the distribution of bargaining power, the extent of product differentiation and the number of active firms in the market. We develop a model of a non-cooperative two-stage game. In Stage 1, the wage is determined as a result of decentralised bargaining between firm and its union. In the Stage 2, we consider both Cournot and Bertrand solutions to the non-cooperative product market game. In our analysis we assume that there is symmetry across all the union-firm wage bargains. Hence, in equilibrium, we retain the property of symmetric costs, typically assumed in the standard model (e.g., Horn and Wolinsky 1988). The paper is structured as follows. In Section 2 and 3, we consider the determinants of these profits and prices under the Cournot and Bertrand competition, respectively. Section 4 and 5 compare the outcomes of profits and prices, respectively. Finally, conclusions are in Section 6.

2 Cournot Equilibrium under Unionised Market

We consider a differentiated oligopoly market as in Häckner (2000), that is a generalisation of Singh and Vives (1984) and Qiu (1987). We analyse a two-stage non-cooperative game in which firms produce imperfect substitutes goods. As in López and Naylor (2004), in Stage 1, each firms independently bargains over wage with its labour union, while in Stage 2, each firm sets its output, given the wage outcome of the Stage 1 bargaining game, to maximise profits. Our equilibrium notion is the standard sub-game perfect and the model is solved through backward induction. The utility function of the representative consumer is given by:

\[ U(q) = \alpha \sum_{i=1}^{n} q_i - \frac{1}{2} \left( \sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{k \neq i}^{n} q_i q_k \right) + I \]

\( \gamma \in [-1, 1] \) is the substitution parameter, \( \alpha > 0 \). If \( \gamma = 0 \), each firm has monopolistic market power, while if \( \gamma = 1 \), the products are perfect substitutes. A negative \( \gamma \) implies that goods are complementary, \( \gamma \in [-1, 0] \). For profits differential between

\(^{2}\)Unlike Häckner (2000) that the term \( \alpha_i \) represents a quality distribution between firm \( i \) and its competitors, in this model the term \( \alpha \) is constant.
Cournot and Bertrand competition, we strictly focus only on the case of \( \gamma \in [0, 1] \) as in Lopez and Naylor (2004).\(^3\) We consider both substitutability and complementarity case for the prices differential. The utility function is quadratic in the consumption of \( q \)-goods while is linear in the consumption of the other goods, \( I \). The budget constraint is simply: \( \sum_{i=1}^{n} q_i p_i + I \leq M \), where \( M \) is the income level. The first-order conditions determining the optimal consumption of good \( i \) is

\[
\frac{\partial U}{\partial q_i} = \alpha - q_i - \gamma \sum_{k \neq i}^{n} q_k - p_i = 0
\]

(1)

The firm \( i \)'s inverse demand function can be solved for directly from Eq. (1):

\[
p_i (q_i, q_{-i}) = \alpha - q_i - \gamma \sum_{k \neq i}^{n} q_k.
\]

(2)

with \( p_i \) as the price of the product and \( q_{-i} = \sum_{k \neq i}^{n} q_k \). Labour is the only input used. Followed Lopez and Naylor (2004), we assume that all firms have the same technology and the same bargaining power over wages. Each firm produces one unit of the output by the use of one unit of labour. In symmetric equilibrium, each firm will have identical marginal costs, however, these will be the outcome of strategic bargaining between the two union-firm pairs. The profit function of a typical firm in the industry has the following form:

\[
\pi_i (q) = \left( \alpha - q_i - \gamma \sum_{k \neq i}^{n} q_k - w_i \right) q_i
\]

(3)

where \( w_i \) is the outcome of the wage bargain between union-firm \( i \) and \( q = \sum_{i=1}^{n} q_i \).

Firms set quantities to maximise profits, \( \pi_i \), taking the other firms’ quantities as given. Firm \( i \)'s reaction function is

\[
q_i (q_{-i}) = \frac{\alpha - w_i - \gamma \sum_{k \neq i}^{n} q_k}{2}.
\]

(4)

Eq. (4) can be interpreted as a firm’s output reaction function with respect both to the rival firm’s output in the relevant product market and to the firm’s own union wage. As \( \gamma > 0 \), by assumption, the best-reply functions are downward sloping. Under the assumption of Cournot competition in the product market, the game is played in strategic substitutes. Summing over all firms equations (4), and using the

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\(^3\)When goods are substitutes, the degree of substitutability could be interpreted in terms of horizontal product differentiation.
fact that $\sum_{i=1}^{n} q_i = q_i + \sum_{k \neq i}^{n} q_k$, we can solve for labour demand in equilibrium of firm $i$, given $w_i$ and $w_{-i}$:

$$q_i (w_i) = \frac{(2 - \gamma) \alpha - (\gamma (n - 2) + 2) w_i + \gamma w_{-i}}{(2 - \gamma) (\gamma (n - 1) + 2)}$$

(5)

where $q_i$ denotes a profit-maximising output of firm $i$, with $w_{-i} = \sum_{k \neq i}^{n} w_k$, which is decreasing with its own wage, while it is increasing with the other firm’s wage. The profit function under Cournot competition is then given by:

$$\pi_i = \left( \frac{(2 - \gamma) \alpha - (\gamma (n - 2) + 2) w_i + \gamma w_{-i}}{(2 - \gamma) (\gamma (n - 1) + 2)} \right)^2 = q_i^2$$

(6)

A firm’s profit falls with its own wage, but rises with the wage of the competitors in that industry, because the competition situation improves.

### 2.1 The Wage Bargaining

We now solve for Stage 1 of our game. Assume that, in Stage 1, firm $i$ bargains over the wage $w_i$, with its union. Union $i$’s utility function is given by

$$u_i = (w_i - \bar{w})^\theta q_i^{1-\theta},$$

(7)

The parameter $\theta$ captures the relative strength of the union’s preference for wages over employment and $0 \leq \theta \leq 1$ and $\bar{w}$ is the competitive or reservation wage level and is common to all unions. This assumption has a number of advantages. It is quite general and encompasses common functional forms such as rent utility function, arising when $\theta = \frac{1}{2}$ and wage bill utility function when $\theta = \frac{1}{2}$ and $\bar{w} = 0$. Under the assumption of a right-to-manage model of Nash-bargaining over wages, we have the Nash maximand as:

$$\Phi_i = \arg \max_{w_i} (u_i - \bar{u})^\beta (\pi_i - \bar{\pi})^{1-\beta}$$

(8)

where $\beta \in [0,1]$ denotes the exogeneous bargaining power of firm $i$, which we assume to be symmetric across firms within an industry; $\bar{u}$ and $\bar{\pi}$ represent the conflict payoffs to the union and the firm, respectively. We assume conflict payoffs to be exogeneous and set equal to zero. We rule out the special case in which $\beta = \theta = 1$.

Substituting Eqs. (5), (6) and (7) into Eq. (8), we obtain

$$\Phi_i = \arg \max_{w_i} (w_i - \bar{w})^{\beta\theta} (q_i^{2-\beta(1+\theta})$$

(9)

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4See Oswald (1985) for discussion.
From Eqs. (5) and (9), it follows that the first-order condition for Nash maximand yields

\[ w_i^C = \frac{\beta \theta [(2 - \gamma) \alpha + \gamma w_{-i}] + (\gamma (n - 2) + 2) (2 - \beta (1 + \theta)) \bar{w}}{(\gamma (n - 2) + 2) (2 - \beta)} \]  

(10)

This equation can be interpreted as union \( i \)'s wage reaction function with respect to the given wage set by all the other unions. From (10), the slope of union-firm pair \( i \)'s best-reply function is given by

\[ \frac{\partial w_i^C}{\partial w_{-i}} = \frac{\beta \theta \gamma}{(\gamma (n - 2) + 2) (2 - \beta)} \]  

(11)

The slope of the best-reply wage function is positive for \( \gamma > 0, \theta > 0, \beta > 0 \), confirming that the labour market game is played in strategic complements. In a symmetric sub-game perfect equilibrium we have that \( w_i = w \), with \( w_{-i} = (n - 1)w \), and hence, from (10), equilibrium wages are given by

\[ w^C = \frac{\beta \theta (2 - \gamma) \alpha + (\gamma (n - 2) + 2) (2 - \beta (1 + \theta)) \bar{w}}{[(2 - \gamma) (\gamma (n - 2) + 2)] - \beta \theta \gamma (n - 1)} \]  

(12)

From Eq. (12), it is straightforward to show that if \( \alpha \geq \bar{w} \), we have that \( \frac{\partial w^C}{\partial \beta} = \frac{2 \zeta [2 \gamma (\gamma - 3) + 2 - \gamma (\alpha - \bar{w})]}{[n - 1] (\gamma \beta (1 + \zeta) - 2 \gamma) + \beta \gamma (n - 2)]^2} \geq 0 \), \( \frac{\partial w^C}{\partial \gamma} = \frac{\beta \theta \zeta (\alpha - \bar{w}) (2 - \beta)}{[n - 1] (\gamma \beta (1 + \zeta) - 2 \gamma) + \beta \gamma (n - 2)]^2} \geq 0 \), and \( \frac{\partial w^C}{\partial \gamma} = \frac{2 \zeta (\alpha - \bar{w}) (2 - \beta)}{[n - 1] (\gamma \beta (1 + \zeta) - 2 \gamma) + \beta \gamma (n - 2)]^2} \leq 0 \). Furthermore, we have that \( w^C = \bar{w} \) if either \( \theta = 0 \) or \( \beta = 0 \). Therefore, either an increase in the firm’s bargaining power \((1 - \beta)\) or a decrease in the union’s preference over wage \((\theta)\) reduces the equilibrium wage rate, as expected. The degree of product substitutability also affects wages in a predictable way: less differentiation in the goods market induces wage moderation, \( \frac{\partial w^C}{\partial \gamma} < 0 \) for \( \gamma \in [0, 1] \). From Eq. (12), we can substitute back into the labour demand function Eq. (5) to obtain the levels of output and employment. Further substitution into (6), we conclude that sub-game perfect equilibrium profits under Cournot competition are given by

\[ \pi^C = \left( \frac{[(\gamma (n - 2) + 2) (2 - \beta (1 + \theta))] (\alpha - \bar{w})}{(\gamma (n - 1) + 2) [(2 - \beta) (\gamma (n - 2) + 2) - \beta \theta \gamma (n - 1)]} \right)^2 = q^2 \]  

(13)

Thus, firm \( i \)'s equilibrium profit depends on the union preference over the wage, bargaining power, product differentiation and the number of firms.

3 Bertrand Equilibrium under Unionised Market

In this section of the paper, we suppose that the product market game in Stage 2 is characterised by price-setting behaviour by firms. Summing over all firms, Eq. (1) can be written as
\[ n\alpha - \sum_{i=1}^{n} q_i - \gamma (n-1) \sum_{k\neq i}^{n} q_k - \sum_{i=1}^{n} p_i = 0 \quad (14) \]

Eq. (1) and (14) then yield firm \( i \)'s demand function,

\[ q_i (p_i, p_{-i}) = \frac{(1-\gamma) \alpha - (\gamma (n-2) + 1) p_i + \gamma p_{-i}}{(1-\gamma) (\gamma (n-1) + 1)} \quad (15) \]

where \( p_{-i} = \sum_{k \neq i}^{n} p_k \). Profit for the representative firm \( i \) can be written as:

\[ \pi_i (p_i, p_{-i}) = \left( \frac{(1-\gamma) \alpha - (\gamma (n-2) + 1) p_i + \gamma p_{-i}}{(1-\gamma) (\gamma (n-1) + 1)} \right) (p_i - w_i) \quad (16) \]

From (16), the first-order condition for profit-maximisation gives

\[ p_i (p_{-i}) = \left( 1-\gamma \right) \alpha + (\gamma (n-2) + 1) w_i + \gamma p_{-i} \frac{2 (\gamma (n-2) + 1)}{(1-\gamma) (\gamma (n-1) + 1)} \quad (17) \]

Summing Eq. (17) over all firms, and using the fact that \( \sum_{i=1}^{n} p_i = p_i + \sum_{k \neq i}^{n} p_k \), we obtain the price setting rule for each firm.

\[ p_i (w) = \frac{[1-\gamma] (\gamma (2n-3) + 2) \alpha + \gamma (\gamma (n-2) + 1) w_{-i}}{(\gamma (2n-3) + 2) (\gamma (n-3) + 2)} \quad (18) \]

and hence, for \( \gamma > 0 \), the Bertrand product market game is played in strategic complements. Hence, substituting (18) in (15) and (16) yields the equilibrium quantities and profits, given wages, under Bertrand competition:

\[ q_i = \frac{(\gamma (n-2) + 1) \left[ (1-\gamma) (\gamma (2n-3) + 2) \alpha + \gamma (\gamma (n-2) + 1) w_{-i} \right] - [(\gamma (n-2) + 1) (\gamma (n-2) + 2) - \gamma^2 (n-1)] w_i}{(1-\gamma) (\gamma (n-1) + 1) (\gamma (2n-3) + 2) (\gamma (n-3) + 2)} \quad (19) \]

\[ \pi_i = \frac{(1-\gamma) (\gamma (n-1) + 1) q_i^2}{(\gamma (n-2) + 1)} \quad (20) \]

As we expect, the equilibrium output and profit of an individual firm would be reduced by a wage increase within that firm, but would be increased by an increase in the competitors' average wage.
3.1 The Wage Bargaining

As for the case of Cournot competition, the wage is determined as a result of decentralised bargaining between firm and its union. Union preferences are given by (7) and the Nash maximand is represented by (8). Substituting (19), (20) and (7) into (8) the first-order condition derived from the Nash maximand is:

\[ w_i^B = \frac{\beta \theta \left( (1 - \gamma) \left( \gamma (2n - 3) + 2 \right) \alpha + \gamma \left( \gamma (n - 2) + 1 \right) w_{-i} \right)}{\left[ \left( \gamma (n - 2) + 1 \right) \left( \gamma (n - 2) + 2 \right) - \gamma^2 (n - 1) \right] (2 - \beta (1 + \theta)) \bar{w}} \]  

(21)

which defines the sub-game perfect best-reply function in wages of union-firm pair \( i \) under the assumption of a non-cooperative Bertrand-Nash equilibrium in the product market. It depends positively on rival firm’s wage. From (21), the slope of union-firm pair \( i \)’s best-reply function is given by

\[ \frac{\partial w_i^B}{\partial w_{-i}} = \frac{\beta \theta \gamma (\gamma (n - 2) + 1)}{\left[ \left( \gamma (n - 2) + 1 \right) \left( \gamma (n - 2) + 2 \right) - \gamma^2 (n - 1) \right] (2 - \beta)} \]  

(22)

The slope of the best-reply wage function is positive for \( \gamma > 0, \theta > 0, \beta > 0 \), confirming that the labour market game is played in strategic complements. It follows that, in symmetric equilibrium, with \( w_i = w \),

\[ w^B = \frac{\beta \theta \left( (1 - \gamma) \left( \gamma (2n - 3) + 2 \right) \alpha + \gamma \left( \gamma (n - 2) + 1 \right) w_{-i} \right)}{\left[ \left( \gamma (n - 2) + 1 \right) \left( \gamma (n - 2) + 2 \right) - \gamma^2 (n - 1) \right] (2 - \beta (1 + \theta)) \bar{w}} \]  

(23)

As for the Cournot case, it is straightforward to see that if \( \alpha \geq \bar{w} \), we have \( \frac{\partial w^B}{\partial \beta} \geq 0 \), \( \frac{\partial w^B}{\partial \theta} \geq 0 \), and \( \frac{\partial w^B}{\partial \gamma} \leq 0 \). Furthermore, \( w^B = \bar{w} \) if either \( \theta = 0 \) or \( \beta = 0 \). Substituting (23) into (20) gives the equilibrium profit under Bertrand competition:

\[ \pi^B = \frac{(\gamma (n - 2) + 1)}{\left[ \left( \gamma (n - 2) + 1 \right) \left( \gamma (n - 2) + 2 \right) - \gamma (n - 1) \left( \gamma (n - 1) + 1 \right) \bar{w} \right]^2} \]  

(24)

It is important to see in which different ways the parameters of the model \( (\beta, \gamma, \theta, n) \) affect the profit between the two competition equilibria.
4 The Cournot-Bertrand Profit Comparison

We now investigate how Cournot/Bertrand profits vary with the parameters of the model \((\beta, \gamma, \theta, n)\) by comparing the Cournot profit function, given by (13), and the Bertrand profit function given by (24). In order to analyse the Cournot-Bertrand profit differential, we define the following ratio under a symmetric equilibrium:

\[
\frac{\pi^C}{\pi^B} = \frac{(\gamma(n-2) + 2) (\gamma(n-1) + 1) (2 + \gamma(n-3))^2}{((\gamma(n-2) + 1)(\gamma(n-2) + 2) - \gamma^2(n-1))^2 (2 - \beta) - \theta \beta \gamma (\gamma(n-2) + 1)}
\]

\[
\frac{(\gamma(n-1) + 2)^2 ((2 - \beta)(\gamma(n-2) + 2) - \beta \theta \gamma(n-1))^2}{(\gamma(n-2) + 1)(1 - \gamma) [(\gamma(n-2) + 1)(\gamma(n-2) + 2) - \gamma^2(n-1)]^2}
\]

Eq. (25) establishes conditions under which the parameters of the model obtains the profit ratio of greater or less than 1. In other words, with imperfect substitutes product we evaluate whether there is a range of parameter values over which firms prefer the Bertrand-type of product market competition to the Cournot-type in the presence of unions. Unlike López and Naylor, when taking the profit ratio instead of profit differential Eq. (25) does not depend on the reservation wage \(\bar{w}\) and on the intercept of the demand function \(\alpha\). López and Naylor showed that, with \(n = 2\) firms, the profit ratio (25) is less than 1 for relatively high values of \(\theta\) and \(\beta\). This implies that in a differentiated duopoly with wage bargaining, in the sub-game perfect equilibrium, the Bertrand profits exceed Cournot when unions are both relatively powerful in the wage bargaining process and attach relatively high preference to the wages in their objective function.\(^5\) Here we test whether their results are robust to an arbitrary number of firms in the market. Following López and Naylor (2004) we evaluate profit ratio of (25) for \(\gamma = 0.5\). We first define the critical values of \(\beta\) and \(\theta\) that can make the profit ratio of (25) equal to 1. Initially, consider the special case that \(\beta = 1\) and \(\gamma = 0.5\), we find the values of \(\theta\) as a function of \(n\), that can make \(\frac{\pi^C}{\pi^B} = 1\). Then consider the case where \(\gamma = 0.5\) and \(\theta = 1\), we find the values of \(\beta\) as a function of \(n\), that can make \(\frac{\pi^C}{\pi^B} = 1\). Figure 1 illustrates the relationships between \(\theta, \beta\) and \(n\).

The initial values of \(\beta = 0.91\) and \(\theta = 0.84\) for \(n = 2\) are the ones considered by López and Naylor. The values of \(\beta\) and \(\theta\) described in Figure 1 may be thought as the critical values for a given level of \(n\) where \(\gamma = 0.5\), above which the profit ratio given by (25) may fall under one. Those critical values are increasing in the number of firms, implying that higher is the number of firms and higher are the values of \(\beta\) and \(\theta\) that can make the profits under Cournot equal to the profits under Bertrand. For an arbitrary number of firms, we are able to establish the result of the Cournot-Bertrand profits differential reported under Proposition 1:

\(^5\)López and Naylor (2004) use numerical evaluation to find their results. In particular, they use \(\gamma = 0.5, \theta > 0.84\) and \(\beta > 0.91\).
Proposition 1 In the sub-game perfect equilibrium, for sufficient high values of $\beta$ and $\theta$, the profits under Bertrand is higher than the profits under Cournot competition even if $n > 2$. However, there is a critical value of $n^* > 2$, such that, for any value of the parameters ($\beta$, $\theta$ and $\gamma$) the profits under Cournot are always higher than profits under Bertrand competition.

As stated in Proposition 1, López and Naylor (2004) result concerning the ranking of Cournot and Bertrand profits, when products are imperfect substitutes, hold only with relatively low number of firms. It follows that for large number of firms, the reversal result does not obtain for any value of $\beta$ and $\theta$. The results in Proposition 1 are displayed in Figure 2 for the relationship between profit ratio (25) and different set of values of $\beta$ and $\theta$, followed by López and Naylor assumption of $\gamma$, that is $\gamma = 0.5$. For a relatively low number of firms, the profit ratio (25) may be less than one (see Figure 2). There exists a threshold value of $n^*$ such that $n > n^*$ implies that $\frac{\pi^C}{\pi^B} > 1$. For example, for $\beta = 0.98$ and $\theta = 0.97$, the critical value of $n^*$ for which the $\frac{\pi^C}{\pi^B} \geq 1$ is equal to 8.

Figure 3 shows the relationship between $\frac{\pi^C_n}{\pi^B_n}$ and $\gamma$ for two different values of $n$. In particular, we evaluate the profit ratio (25) for $n = 3$ and $n = 10$. Figure 3 is obtained for $\beta = 0.98$ and $\theta = 0.97$, two values that can assure the profit ratio (25) to be less than 1 for some values of $n$. For sufficiently small number of firms (e.g., $n = 3$), the less differentiated the products are (higher $\gamma$), the higher is the possibility that the ratio $\frac{\pi^C}{\pi^B}$ is less than one. When $n = 3$, and for sufficient high values for $\beta$ and $\theta$, higher is $\gamma$ the higher are profits under Bertrand respect to the ones under Cournot. When $n = 10$, no values of the parameters can provide the reversal results, that is Bertrand’s profits is always lower than that of Cournot. Thus, there are two different elements that affects the Cournot-Bertrand profit differential in our model. On one
hand, there is the effect of the wage bargaining parameters ($\beta$ and $\theta$) and product differentiation ($\gamma$) on the level of competition. On the other hand there is the effect of the number of firms. For $n < n^*$, the effect of high values of $\beta, \theta$ and $\gamma$ is dominant, and Bertrand competition can lead to higher profits than Cournot competition. As $n$ increases, the effect of the number of firms on the competition level becomes more relevant and thus, Bertrand competition is tougher than Cournot competition, and so the profits under price setting behaviour become lower than under quantity setting.

When $n < n^*$ and unions are both power and place considerable weight on the wage argument in their utility function, if firms may commit themselves to offer a certain type of contract, they will choose the price contract. Unlike Singh and Vives (1984), we show that if the goods are substitutes, it may not be a dominant strategy for firm $i$ to choose the quantity contract.

5 The Cournot-Bertrand Price Comparison
In the previous section we showed that under particular values of the parameters $(\beta, \theta, n, \gamma)$ of the model, the profits under Cournot competition may be lower than profits under Bertrand competition even if goods are imperfect substitutes. In this section, we compare the equilibrium prices under the two kinds of competition when goods are substitutes, independent or complement according to whether $\gamma \gtrless 0$. Häckner (2000) generalised the model of Singh and Vives’ (1984) and showed that Bertrand prices can be higher than Cournot prices if goods are complements and quality differences across firms ($\alpha_i$) are large. We also find that prices under Bertrand competition can be higher than prices under Cournot equilibrium if goods are complements. However, the prices differential can be positive or negative, depending on the values of the parameters of the wage bargaining process ($\beta$ and $\theta$), given that the quality measure to be constant across firms ($\alpha$ constant). The symmetric equilibrium prices for each firm $i$ under Cournot competition can be found by substituting equation (4) into (2). This gives us:

$$p_i^C(\beta, \theta, n, \gamma) = q_i^C + w_i^C$$
\[ p^C = \left( \frac{[(\gamma (n-2) + 2) (2 - \beta (1 + \theta))] (\alpha - \bar{w})}{(\gamma (n-1) + 2) [(2 - \beta) (\gamma (n-2) + 2) - \beta \theta \gamma (n-1)]} \right) + \]
\[ + \frac{\beta \theta (2 - \gamma) \alpha + (\gamma (n-2) + 2) (2 - \beta (1 + \theta)) \bar{w}}{[(2 - \beta) (\gamma (n-2) + 2) - \beta \theta \gamma (n-1)]} \]  

Equation (18) and (23) then yield the equilibrium Bertrand prices for each firm \( i \), that is:

\[ p^B = \frac{(1 - \gamma) \alpha [1 + (\gamma (n-2) + 1) \beta \theta (\gamma (2n-3) + 2)]}{(\gamma (n-2) + 1) [(\gamma (n-2) + 1)(\gamma (n-2) + 2) - \gamma^2 (n-1)] (2 - \beta (1 + \theta)) \bar{w}} \]

\[ = \frac{(\gamma (n-3) + 2)}{[(\gamma (n-2) + 1)(\gamma (n-2) + 2) - \gamma^2 (n-1)] (2 - \beta)} \left\{ \frac{[\gamma (n-2) + 1]}{\gamma (n-2) + 1} \right\} - \beta \theta \gamma (n-2) + 1 \]  

The price differential between Cournot and Bertrand is thus given by:

\[ z^* = p^C_i - p^B_i \]  

The price differential between Cournot and Bertrand is thus given by:

We analyse under which configuration of the parameters the price differential can be negative or positive. The following proposition states the main results about the sign of equation (28):

**Proposition 2** Assume that \( n \geq 2 \). Then:

(i) When goods are substitutes, \( \gamma \in [0, 1] \), we have \( z^* \geq 0 \), implying that prices are higher under Cournot competition than under Bertrand competition regardless of the values of the parameters \( (\beta, \theta, n, \gamma) \);

(ii) When goods are independent \( \gamma = 0 \), \( z^* = 0 \) regardless of the values of the parameters \( (\beta, \theta, n, \gamma) \);

(iii) When goods are complements \( \gamma \in [-1, 0] \), it can be the case that \( z^* \geq 0 \) or \( z^* \leq 0 \). The case \( z^* \geq 0 \) arises only if:

(a) the difference given by \( |\beta - \theta| \) is sufficiently high and for \( n < n^* \);

(b) \( \beta \) and \( \theta \) are relatively low;

(c) otherwise, \( z^* \leq 0 \).

Proposition 2 suggests that when goods are imperfect substitutes, equilibrium prices under Cournot competition are always higher than prices under Bertrand competition and in the extreme situation of independent goods the difference is zero regardless of the values of the parameters \( (\beta, \theta, n, \gamma) \). The type of competition becomes less important, the less related the goods are. These results complement the existing finding in the duopoly model of Singh and Vives (1984).
Using the complementarity property, we obtain the following results. First, prices under Cournot are higher than those under Bertrand if the unions’ preferences over the input price ($\theta$) are relatively high (low) and the bargaining power of unions ($\beta$) are relatively low (high), or if the unions’ preferences over the wage paid ($\eta$) and the unions’ bargaining power ($\beta$) are both relatively low. For all other cases, the prices under Bertrand are higher than prices under Cournot when products are complements as in Häckner (2000).

The results of Proposition 2 are summarised in Figure 4 and 5. In Figure 4 we show the relationship between the price differential and the number of firms when products are imperfect substitutes.\(^6\)

Figure 5 shows the prices differential for complementarity products for different values of $\theta$ and $\beta$. For the second-order conditions to be satisfied, we assume that $\gamma \in [1/(1 - n), 0].\(^7\) It shows that $z^* \geq 0$ or $z^* \leq 0$ depends on the values of $\beta$, $\theta$ and $n$. When $\beta$ and $\theta$ are relatively low, then, independently of $n$, prices under Cournot are always higher than prices under Bertrand ($z^* \geq 0$). We obtain the reversal result ($z^* \leq 0$) if $\beta$ and $\theta$ become sufficiently high. In the case, when there is a sufficiently high

\(^6\)Figure 4 is obtained for $\gamma = 0.5$, $\alpha = 5$ and $\overline{\pi} = 1$. Different values of those parameters affect the shape of the relationship but not the sign. The same values of $\alpha$ and $\overline{\pi}$ are also used in Figure 5.

\(^7\)See Häckner (2000).
high difference between $\beta$ and $\theta$, then, depending on the number of firms in the market, we can have that $z^* \geq 0$ for relatively low number of $n$, while $z^* \leq 0$ when there is a sufficient high number of firms in the market. Hence, in that case, there exists a threshold value of $n^*$ such that $n > n^*$ implies that $p_i^C < p_i^B$. This is represented by the dotted line in Figure 5, that is drawn assuming a low value for $\beta$ and a high value for $\theta$.

6 Conclusions

In this paper, we have considered an oligopolistic model with $n$-firms and product differentiation in which firm’s costs are not given exogenously but are the result of a wage bargaining process between firms and unions. We established conditions under which the standard result on Cournot/Bertrand profit differential may be reversed depending on the unions’ preferences over the input price, the distribution of bargaining power, the extent of product differentiation and the number of active firms in the market. Using a generalised model of López and Naylor (2004), we showed that if unions are sufficiently powerful and care about wage more than employment, then Bertrand profits exceed Cournot profits in the sub-game perfect equilibrium when goods are imperfect substitutes even if there are more than two firms in the market.
Furthermore, if unions are sufficiently powerful and place considerable weight on the wage argument in their utility function, lower is the level of product differentiation and higher are Bertrand profits relative to Cournot profits if the number of firms is sufficiently low. However, there is a threshold number of firms \( (n^*) \) such that for \( n > n^* \), independently on the values of the parameters of the model, Cournot profits are always higher than Bertrand profits and thus the Singh and Vives’ result continues to hold.

For the price differential between Cournot and Bertrand equilibria, we found that if products are imperfect substitutes, then, regardless of the values of the parameters of the model, prices under Cournot competition are always higher than prices under Bertrand competition. Thus, for imperfect substitutes, the classical result of Singh and Vives (1984) continues to hold also in our framework. Furthermore, also when products are independent we obtained the same result as Singh and Vives (1984). While uniform ranking of prices between Bertrand and Cournot competition guarantees a definitive ranking for imperfect substitutes, it is not necessary for the complementarity. When goods are complement, a reversal result may arise. In particular, if the unions’ preferences over the wage paid and the unions’ bargaining power are sufficiently low we still have that Cournot equilibrium prices are higher than Bertrand’s ones. While, if the unions’ preferences over the input price are high (low) while the bargaining power of unions are relatively low (high), then for a sufficient low number of firms in the market, the prices under Cournot are higher than the prices under Bertrand. As the number of firms increases, the equilibrium prices under Bertrand competition become higher than the equilibrium prices under Cournot competition. For all other parameters configuration we have that prices under Bertrand are higher than prices under Cournot when products are complements as in Häckner (2000). There are a number of ways in which this work can be extended. For example, it would be interesting to consider how the results we obtained can change if we consider different alternatives to the wage bargaining process, such as efficient bargaining or if we allow for more general functional forms.

References


