Bidding Rings and the Winner’s Curse*

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June 29, 2004

Abstract

This paper extends the theory of legal cartels to affiliated private value and common value environments. We show that efficient collusion is always possible in private value environments, but may not be in common value environments. In the latter case, fear of the winner’s curse can cause bidders not to bid, which leads to inefficient trade. Buyers with high signals may be better off if no one colludes. This result provides a possible explanation for the low incidence of joint bidding, especially on marginal tracts, in the U.S. federal government offshore oil and gas lease auctions.

1 Introduction

Collusion in an auction market occurs when a group of bidders take actions to limit competition among themselves. Colluding bidders are often called a ring, which can include all of the bidders or some subset. There is evidence of collusion in many auction markets. Examples include highway construction contracts (Porter and Zona [20], school milk delivery (Pesendorfer [18], Porter and Zona [21]), and timber auctions (Baldwin et al.[3]). Collusion

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is not too surprising since noncooperative behavior is not jointly optimal for bidders. They are collectively better off colluding and transferring gains from trade from the seller to the ring. The problems that a ring faces in dividing the collusive surplus are detection by authorities or by the seller, internal enforcement, entry, and private information about the gains from trade. Legal rings do not have to worry as much about detection or enforcement. An important obstacle that they face is providing incentives to elicit each member’s private information about the gains to trade. This raises the following question: can a legal ring collude efficiently and still offer its members expected payoffs that exceed what they can earn if the ring does not operate?

The above question has been studied using the tools of mechanism design for auctions of private value assets with independent signals. Private value assets are assets where each buyer’s valuation depends only upon her own information. The main conclusion of this literature is that bidding rings can collude efficiently and make their members better off. Graham and Marshall [7] analyze collusion in second-price sealed bid and English auctions. They show that a second-price knockout auction tournament operated by an outside agent hired by the ring can implement efficient collusion by any subset of ex ante identical bidders. Their mechanism satisfies ex ante budget balance but not ex post budget balance. Mailath and Zemsky [14] study second-price auctions with heterogenous bidders and establish that efficient collusion by any subset of bidders is possible. McAfee and McMillan [15] study first-price sealed bid auctions and show that, if the ring includes all bidders, then efficient collusion with ex post budget balancing is possible but it requires transfers to be paid from the member with the highest valuation to those with lower valuations. They assume that bidders commit to the ring before they obtain their private information so that the relevant participation constraints are ex ante.

Our primary objective in this paper is to study ring formation in first-price sealed-bid auctions of common value assets. The motivation for our study is to explain the incidence
of joint bidding in U.S. federal auctions of oil and gas leases in the Outer Continental Shelf (OCS) off the coasts of Louisiana and Texas during the period 1954 to 1970, inclusive, when joint bidding ventures were legal for all firms.\footnote{In late 1975, concerns over bidding collusion caused Congress to pass legislation prohibiting joint bids involving two or more of the eight largest private oil firms (Exxon, Gulf, Mobil, Shell, Standard Oil of California, Standard Oil of Indiana, Texaco, and British Petroleum) on federal leases on the OCS.} Common value assets are assets where each buyer’s valuation depends upon the information of all of the buyers. The canonical example of such assets are oil and gas leases.\footnote{See Hendricks, Pinkse and Porter [8] for evidence that is consistent with the claim that oil and gas leases are common value assets.} The value of the oil and gas deposit on a tract is common to all bidders, even though they may have different information about the size of the deposit and different development costs. Conventional wisdom suggests that rings are more likely to form in common value auctions because buyers are more likely to achieve a consensus on the value of the asset and because they need to know the information of their rivals to determine whether the asset is worth acquiring. By contrast, in private value auctions, each buyer has a different valuation and knows whether or not the asset is worth acquiring. Indeed, McAfee and McMillan [15] argue that the reason they focus on the private values case is that the optimal ring mechanism in the pure common value case is too simple. Efficiency is attained regardless of which member gets the right to bid in the seller’s auction, so an all-inclusive ring can use some exogenous method to pick which of its members should win the right and ask each bidder to report his information. Bidders have no incentive to misrepresent their information, and the winner can determine on the basis of the pooled information whether the asset is worth acquiring.

Our main theoretical finding is that, contrary to conventional wisdom, efficient collusion is always possible in private value environments but not always possible in common value environments. The ring may not be able to guarantee payoffs for members with high signals that exceed what they can earn from competitive bidding. The reason is closely related to the phenomenon known as the “winner’s curse”.\footnote{Bulow and Klemperer [4] document a number of similar counterintuitive results that can arise when} Fear of the winner’s curse causes buyers
to bid less aggressively, which can lead to inefficient trade. No one may bid even though at least one of the buyers would be willing to do so if he knew all of the private signals. By contrast, each member of a ring knows that, if selected, he will learn the private signals of the other members prior to the acquisition decision and hence will purchase the asset if and only if his valuation conditional on all of the private signals exceeds investment costs. The efficiency of the ring relative to competitive bidding works to the advantage of buyers with low signals but against a buyer with a high signal. A high signal buyer will pay a little less to the seller but much more to the other bidders. In contrast, in private value auctions, ownership of the asset is always transferred efficiently whether bidders collude or not. This is because each buyer’s willingness to pay depends only upon his own private signal, and he bids if and only if his valuation conditional on his signal exceeds acquisition and investments costs. Our result provides an explanation for why collusion appears to be less frequent in common value auctions than private value auctions.

The remainder of this paper is organized as follows. In the next section we document the surprisingly low incidence of joint bidding in U.S. auctions of oil and gas leases in the OCS. The theoretical model is presented in Section 3. We focus on a simple comparison: the buyers’ expected payoffs if none collude versus their expected payoffs if all collude. We show that the first-price knockout auction is an ex post efficient and budget-balanced mechanism that yields payoffs that satisfy the interim participation constraints in private value auctions but not always in common value auctions. In Section 4 we use a parametric example to identify the conditions under which the all-inclusive ring does not satisfy interim participation constraints. In Section 5 we characterize the set of all incentive compatible, efficient collusive mechanisms for common value environments with independent signals and show that the first-price knockout auction is an optimal collusive mechanism. Section

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bidders bid noncooperatively for common value assets. They show that fear of the winner’s curse can cause increases in supply, allocation by rationing and exclusion of potential buyers to increase the prices of common value assets.
6 provides concluding remarks.

2 Application

The US government holds the mineral rights to offshore lands more than three miles from the coast, out to the 200 mile limit. Beginning in 1954, the federal government has transferred production rights on its lands to the private sector by a succession of lease sales in which hundreds of leases have been auctioned. Our sample consists of the nine sales of wildcat tracts off the coasts of Texas and Louisiana during the period 1954 to 1970 inclusive, in which a total of 1,260 tracts received bids. A wildcat lease sale is initiated when the Department of Interior (DOI) announces that certain offshore areas are available for exploration, and nominations are invited as to which tracts should be offered for sale. A tract is typically a block of 5,000 or 5,760 acres, or half a block. The number of tracts available in a sale is usually well over one hundred and tracts are often scattered over several different areas. They are sold simultaneously using a first-price, sealed bid auction. The announced reserve price for tracts in our sample is $15 per acre. Post-sale drilling costs were approximately one to two million dollars per tract. A participating buyer or consortium of buyers submits a separate bid on each tract that it has an interest in acquiring. A bid is a dollar figure, known as a bonus. At the sale date, DOI announces the value of the bids that have been submitted on each tract and the identities of the bidders. The firm or consortium that submits the highest bid on a tract is usually awarded the tract at a price equal to its bid. In practice, the government could and did reject bids above the stated minimum price. The rejection rate was less than 10% on wildcat tracts and usually occurred on marginal tracts receiving only one bid (Porter [19]).

Prior to the wildcat sale, firms acquire geophysical and geological information about the tracts. They are not permitted to drill exploratory wells. A geophysical company is often hired to “shoot” a seismic survey of a large, roughly 50 block area. The cost of the shoot is
approximately $12 million and it is often shared by several oil companies. Alternatively, the geophysical company may finance its own survey, anticipating that it can sell the report to the oil firms at a future date. In either case, the oil companies jointly underwrite the cost of the shoot. After receiving the data from the shoot, each firm identifies key geological features that it believes are evidence of the presence of hydrocarbons. At this point, each firm typically rejects at least half of the tracts in the 50 block area. Since the interpretation of seismic data varies considerably across firms, they frequently select different tracts. Each firm conducts in-depth evaluations of the tracts that it views as promising to determine whether they are worth bidding for and, if so, how much to bid. In this second stage, the oil firms often purchase more data and shoot “infill” or “cross-diagonal” lines on selected blocks to build a better picture of the substrata. Indeed, the major oil companies often reserve boat time at the time of joint shoot, anticipating their need to do follow up shoots. The cost of the information upgrade on the area is between $500,000 to $1 million. In addition, the firm must pay for the in-house expertise required to interpret the geophysical data. The rejection rate in the second stage is much lower. Each firm typically submits bids on 80% of the tracts that it has scrutinized more closely.

All firms were allowed to bid jointly prior to 1975, and the joint bidding agreements were enforced by legally binding contracts. Most of the joint bidding agreements are salespecific, that is, firms who bid jointly in one sale frequently did not do so in other sales. The agreements were typically struck after the firms had invested in the area-wide seismic

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4 The procedure is described by Mobil Oil Corporation in its testimony submitted on February 19, 1976 to the House of Representatives, Subcommittee on Monopolies and Commercial Law of the Committee on the Judiciary.

“The bidding groups are formed under a bidding agreement, a formal written document executed by all parties prior to any discussions relating to bonus values. This agreement establishes procedures for arriving at a joint bid and provides for the protection of each individual company in the event agreement cannot be reached. The agreement can either cover the entire sale area or, more commonly, be limited to a specific area of mutual interest (AMI) to the companies involved.”
studies. Hendricks and Porter [9] document that equal division is the predominant sharing
rule among the major bidders when they submit joint bids, although not when a major (or
more than one major) bids with smaller partners. Smaller partners tend to have smaller
shares. The agreements did allow firms to adjust their shares on individual tracts, possibly
to zero, if they could not agree upon a bid. To avoid strategic drop outs, the agreement
typically required each participant to offer their partners equal ownership shares if it wins
the tract with a bid that is not sanctioned by the ring. This provision eliminated the
incentive participants may otherwise have had to pretend disinterest in a tract and then
outbid the ring. As a result, members either shared in the bids submitted by the ring on
tracts within the AMI or they did not bid.

The above description establishes several important facts that are relevant to our model.
First, the joint venture agreements are negotiated shortly before the sale date and after the
firms have acquired their private information about the tracts. This timing makes it more
difficult for firms to enter in response to the joint venture agreement. Second, the joint
venture agreements cover blocks of tracts in a sale, typically 25 to 50 tracts. Third, if the
firms decide to participate in the joint venture, they are legally committed to jointly evaluate
tracts in the AMI and to coordinate their bids according to the mechanism specified in the
agreement. And fourth, the allocation decisions are made on a tract-by-tract basis.

Table 1 provides summary statistics on bidding behavior of the twelve most active
bidders (Big12) in our sample. The tracts are classified by the number of potential Big12
bidders, NPot12, which ranges from 0 to 12. This number is constructed for each tract
based on its location and who bid in the area around tract t, and when. For tracts that
were drilled, location is identified by the longitudinal and latitudinal coordinates of the well.
Tracts that were not drilled are assigned coordinates by interpolation from nearby tracts
that were drilled. On average a tract covers 0.0463 degrees of longitude and 0.0405 degrees
of latitude. A neighborhood for a tract consists of all tracts whose registered locations are
within 0.1158 (2.5 times 0.0463) degrees of longitude and 0.1012 (2.5 times 0.0405) degrees of latitude of the tract and that were offered for sale at the same time as or before the tract. Ignoring irregular tract sizes and boundary effects, the maximum possible size of a neighborhood is 25 tracts or 125,000 acres. The number of potential bidders on a tract is simply the number of Big12 firms that bid on the tract or in its neighborhood, either solo or jointly. The rationale is that if a Big12 firm is interested in the area, then it will probably bid on at least one tract.

For each value of $NPot12$, the second column of Table 1 gives the number of tracts, and the third gives the mean high bid. The mean high bid increases from $434$ thousand on tracts where none of the Big12 firms are potential bidders to $21.8$ million on tracts where every Big12 firm is a potential bidder. The mean high bid in the sample is $6.2$ million per tract. (Bids are expressed in 1982 dollars.) Ex ante expectations, as measured by the high bid, are positively correlated with the number of potential bidders.

The last three columns report the average number of bids per tract, the proportion of those bids submitted by Big12 firms, and the proportion of Big12 bids that are joint. Big12 firms submit 73.8% of all bids, and this proportion is roughly constant in $NPot12$. Since the number of bids per tract is increasing in $NPot12$, the average number of bids by non-Big12 firms increases with the level of Big12 competition. On average, there is less than one non-Big12 bid per tract. The proportion of Big12 bids that were joint is less than 10% on marginal tracts, and is roughly constant at approximately 20% for tracts with more than four Big12 potential bidders.

We find the low incidence of joint bidding by the major bidders, especially on marginal tracts, to be puzzling. The gains from reduced competition are high. The average winning bid on marginal tracts was over 1 million dollars, which is well above the minimum price of approximately $75,000. The gains from pooling information are also high. The cost of drilling an exploratory well is approximately 1.5 million dollars. Fifty per cent of the
tracts were not bid. Furthermore, the risk of drilling a dry well is high, since only 39% of the tracts bid were productive. Thus, information pooling may substantially increase the collusive surplus, especially on marginal tracts.

We have so far ignored the role of the government decision to reject the high bid. It is conceivable that firms do not submit joint bids on tracts with few potential bidders. They may be concerned that their bid will be rejected, if the government reacts to the absence of competition. As Porter [19] notes, wildcat bids were much more likely to be rejected when there were relatively few bids, and when these bids were low. However, only three high joint bids were rejected in our sample, out of 167 high joint bids, or 1.8%, as opposed to 7.9% of high solo bids. When one conditions on the level of the high bid, the rejection rule appears to favor joint bids, if anything. (Of course, this does not prove that, had more joint bids been submitted, they would have been accepted with the same frequency.)

Another explanation for the low incidence of joint bids is that joint bids are not an accurate measure of the incidence of collusive bidding. Indeed, in his Congressional testimony in 1976, Darius Gaskins [6] argued that the collusive effects of joint ventures should not be measured solely in terms of joint bids. In negotiating over which tracts to bid jointly, the members of the joint venture could learn more about each others’ bidding intentions on other tracts and coordinate their solo bids on these tracts. In a previous version of this paper [11], we examined this hypothesis and found no evidence that joint bidding in an area affected the participants’ contemporaneous bidding behavior outside that area.

3 The Model

The seller sells the representative tract using a first-price sealed bid auction with a pre-announced reserve price. Let $r$ denote the sum of the reserve price and post-sale investment. There are $n$ buyers, labelled $i = 1,..,n$. In terms of our application, a buyer is any firm that invested in a survey of the tract. The number and identities of the buyers are assumed
to be common knowledge.

We denote buyer \( i \)'s private signal on the representative tract by \( S_i \). The signals are real-valued and their support normalized to be the unit interval. Let \( V \) denote the unknown component that is common to both buyers’ valuations.

**Assumption 1:** \((V, S_1, \ldots, S_n)\) are affiliated and symmetric in \((S_1, \ldots, S_n)\).

Let \( F \) denote the cumulative distribution function of \((V, S_1, \ldots, S_n)\) with support \([\underline{v}, \overline{v}] \times [0, 1]^n\). It is assumed to have a density \( f \). Let \( F(s_{-i} | s_i) \) denote the conditional distribution of \( s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n) \), the signals of buyer \( i \)'s rivals, given \( S_i = s_i \). The value of the tract to buyer \( i \) is given by \( u(V, S_i) \) where \( u \) is non-negative, continuous, and increasing in both arguments. The buyer utilities depend upon the common component in the same manner and each buyer’s utility is also allowed to depend upon its own private information.

Laffont and Vuong [13] refer to this model as the Affiliated Values (AV) model. It was first introduced by Wilson [22] and is a special case of the general symmetric model of Milgrom and Weber [16]. In the AV model, the signals of the other buyers affect the expected utility of buyer \( i \) through their affiliation with \( V \) and \( S_i \), but they do not enter as an argument of the utility function.

The affiliated values model captures most of the special cases that have been considered in the literature. It includes the case of pure common values, in which each buyer’s valuation depends only upon the common factor (i.e., \( u(V, S_i) = V \)). It also includes the case of private values, in which a buyer’s valuation depends only its own signal (i.e., \( u(V, S_i) = S_i \)). If, in addition, the signals are independently distributed, then the model is called Independent Private Values (IPV); otherwise it is called affiliated private values (APV). Finally, it includes a class of models that have recently received attention, in which the common factor can be expressed as a (deterministic) function of the buyer signals, \( V = g(S_i, S_{-i}) \), where \( g \) is symmetric, increasing, and continuous. Define \( v(S_i, S_{-i}) = u(g(S_i, S_{-i}), S_i) \). Then the
restrictions on \( u \) and \( g \) imply that

\[
s_i \geq s_j \implies v(s_i, s_j, s_{-i,j}) \geq v(s_j, s_i, s_{-i,j})
\]

for all \( i, j \neq i \) and where \( s_{-i,j} \) denotes the signals of all buyers other than \( i \) and \( j \). If equality holds for all possible signals, then the model is one of pure common values. More generally, each buyer’s valuation can be expressed in terms of a common component and a private component. For example, Bulow and Klemperer [4] assume that \( V = \sum_{i=1}^{n} S_i \) and \( v(S_i, S_{-i}) = (1 + \alpha)S_i + \sum_{j \neq i} S_j \), where \( \alpha > 0 \).

We model collusion as a problem in mechanism design. Buyers can make binding commitments to the ring, and side payments are feasible. The collusive mechanism determines which members get the exclusive right to bid in the seller’s auction, and any transfers among members. We are primarily interested in comparing the buyers’ payoffs in two circumstances: when there is a coalition of all buyers and when individual buyers behave non-cooperatively.\(^5\) If buyers do not collude, they bid individually and competitively in the seller’s first-price sealed bid auction. The equilibrium payoffs of this auction determines the buyers’ participation constraints. If the buyers collude, the ring must decide whether or not to acquire the tract at cost \( r \) and how to divide the collusive surplus. A ring mechanism is \textit{ex post efficient} if (i) the buyer with the highest signal is given the exclusive right to purchase the tract and (ii) he does so if and only if the expected value of the tract conditional on the signals of all buyers exceeds \( r \). Condition (ii) is the distinguishing feature of common value environments. In private value environments, each buyer knows whether or not he is willing to pay at least \( r \) for the asset. In common value environments, buyers need to share their information to determine whether or not their willingness to pay exceeds \( r \). The pooling of information is especially valuable if \( r \) is large. Note that condition (i) is not necessary

\(^5\)One way that this situation can occur is if each buyer has veto power: if any buyer refuses to join the ring, then the ring breaks down.
in the case of pure common values. If all buyers value the tract equally conditional on the
same information, then any allocation satisfying condition (ii) is ex post efficient.

3.1 Participation Constraints

In our application, firms typically do not form joint bidding ventures until after they have
acquired their private signals. Thus, the relevant participation constraints are interim:
for every possible realization of the signal, a buyer must expect to be at least as well off
bidding jointly as he is bidding separately. In that case, a key issue in specifying the
payoffs to bidding competitively (or collusively) is whether buyers infer anything about
each other’s private information from their participation decision. Cramton and Palfrey [5]
refer to the possibility of learning as the leakage problem.6 We follow the literature and
assume that leakage does not occur. Furthermore, this assumption is not implausible for our
application since the joint venture decision is taken with respect to a block of tracts and not
on individual tracts. If buyer \(i\) refuses to join the ring, then the other buyers may infer that
buyer \(i\) has obtained favorable information about one or more tracts in the area, but they
do not know which tracts and, since most are not worth bidding for, the inference is likely
to have very little impact on bidding behavior on individual tracts. The situation would
be quite different if the decision to collude is taken on a tract by tract basis. In that case,
refusal to bid jointly on a tract could cause beliefs about that tract, and therefore bidding
behavior, to change. Indeed, we suspect that this is the reason why the geographical unit
of the joint venture agreement is not an individual tract but a large block of tracts.

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6In an earlier version of this paper, Hendricks and Porter [10] considered an extensive form game in which
firms simultaneously voted “yes” or “no” to an equal-sharing joint venture. Firms that voted “yes” formed a
ring and firms that voted “no” bid competitively against each other and the ring. More generally, Cramton
and Palfrey [5] consider a two-stage game in which firms simultaneously vote for or against a proposed
mechanism and have veto power. If the mechanism is unanimously ratified, an all-inclusive ring forms and
mechanism is implemented; otherwise the ring does not form and the firms bid competitively against each
other. One difficulty with these game forms is the assumption that firms can commit not to renegotiate the
outcome of their vote to an alternative mechanism in which all firms gain. For example, if two firms say
“no” to an equal-sharing joint venture, each may infer that the other has a high signal and, conditional on
this information, be better off colluding.
We can now specify the participation constraints. Without loss of generality, we set \( n \) equal to two, a restriction that simplifies the notation considerably. Define

\[
w(s, t) = E[u(V, S_i)|S_i = s, S_j = t]
\]
as buyer \( i \)'s expected value of the tract conditional on the event that his signal is equal to \( s \) and buyer \( j \)'s signal is equal to \( t \). We assume that \( r \) is less than \( w(1, 1) \), the highest possible valuation. It will also be convenient to normalize payoffs so that \( w(0, 0) = 0 \).

Suppose both buyers use a symmetric bidding strategy \( B(s) \) with boundary condition \( B(a) = r \) where

\[
a = \inf\{s \mid \int_0^s w(s, t) \frac{f(t|s)}{F(s|s)} dt \geq r\}.
\]

Here \( a \) is the cutoff signal below which the buyer does not believe the tract is worth \( r \) conditional on winning. It is straightforward to show that equilibrium profits to a buyer in the seller's auction are given by (Milgrom and Weber [16]):

\[
\pi^{NC}(s) = \int_0^s w(s, t) f(t|s) dt - B(s) F(s|s)
\]

where

\[
B(s) = rL(a|s) + \int_a^s w(t, t) dL(t|s).
\]

and

\[
L(t|s) = \exp(-\int_t^s \frac{f(x|x)}{F(x|x)} dx).
\]

Note that \( \pi^{NC} \) is equal to zero for \( s < a \) and increasing for \( s > a \).

We shall refer to the first-price sealed bid auction in which buyers bid individually and noncooperatively as the \textit{status quo mechanism}. 

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3.2 The First-Price Knockout Auction

The standard approach to mechanism design is to exploit the revelation principle and study collusive direct revelation mechanisms. However, as is well known, characterizing the set of incentive compatible collusive mechanisms is quite complicated when signals are affiliated. An alternative approach is to consider a specific indirect mechanism and investigate the conditions under which it generates an ex post efficient allocation that satisfies ex post budget balance and individual rationality. The mechanism that we consider is an augmented form of the first-price knockout auction described in McAfee and McMillan [15]. Each member submits a sealed bid, the member with the highest bid is awarded the exclusive right to acquire the tract at cost \( r \) from the seller, and pays his bid to the “losing” buyer. Ties are resolved by randomization. The losing buyer reports his signal to the winning buyer. The winning buyer updates his beliefs about the value of the tract and purchases it from the seller at price \( r \) if and only if the expected value of the tract conditional on his signal and the reported signal of the losing buyer exceeds \( r \).

In the ring, a bidder learns his rival’s signal before he has to decide whether or not to pay \( r \) to the seller whereas, in the status quo mechanism, he learns his rival’s signal after he pays his bid to the seller, if at all. Let \( b \) denote the cutoff signal below which a buyer does not bid in the knockout auction. It is defined as \( w(b, b) = r \). The interpretation of \( b \) is that it is the lowest signal at which a buyer can win the knockout auction (i.e., \( t < b \)) and be certain that the tract is not worth purchasing conditional on all of the available information. At any higher signal, a buyer is willing to pay a positive amount for the right to purchase the tract at price \( r \) since there is some chance that, after winning and learning the other buyer’s signal, his valuation exceeds \( r \). It follows from Assumption 1 that \( b \) is unique.

When valuations are private, the buyer’s purchasing decision is contingent only on his own valuation. He bids in either auction if and only if his valuation exceeds \( r \), which implies
that $a = b$. The difference in the timing of the revelation of a rival’s signal does matter when values are affiliated, assuming the reserve price is positive. In that case, applying Assumption 1, we obtain

$$w(b, b) = r = \int_0^a w(a, t) \frac{f(t|a)}{F(a|a)} dt < w(a, a) \implies b < a.$$ 

Buyers who draw signals between $b$ and $a$ are willing to bid a positive amount in the knockout auction but are not willing to bid in the status quo mechanism. As we shall see, the more aggressive bidding by buyers with low signals in the knockout auction can cause buyers with high signals to prefer the status quo mechanism.

Suppose that in the knockout auction both buyers use a symmetric, increasing bid strategy $B^K(s)$ with boundary condition $B^K(b) = 0$. It is straightforward to show that equilibrium profits are

$$\pi^K(s) = \int_0^s \max\{w(s, t) - r, 0\} dF(t|s) - B^K(s)F(s|s) + \int_s^1 B^K(t) dF(t|s) \quad (2)$$

where

$$B^K(s) = \frac{1}{2} \int_b^s \max\{w(t, t) - r, 0\} dL^K(t|s)$$

and

$$L^K(t|s) = \exp(-\int_t^s \frac{2f(x|x)}{F(x|x)} dx).$$

It is easily checked that $B^K$ is strictly increasing on the interval $[b, 1]$. For $s < b$, we define $B^K(s) = 0$. The expected payoff to a ring member is strictly positive and constant for $s$ less than $b$, and strictly increasing in $s$ above $b$. The payoffs in equation (2) consist of three terms. The first two terms reflect payoffs in the event the member wins the knockout auction, and the third expected payments when the rival wins the knockout.
Since the loser’s report does not affect his payment, he has no reason not to tell the truth. In fact, the only circumstance in which he needs to report his signal is when it is less than $b$. Otherwise, the winning buyer can infer his signal from his bid. Symmetry and monotonicity implies that the mechanism selects the buyer with the highest signal provided it exceeds $b$. Ties occur if both buyers submit a bid of zero, but in that case it does not matter who is selected, since neither buyer wants to purchase the tract. The selected buyer purchases the tract if and only if $w(s, t)$ exceeds $r$. The transfers among the buyers sum to zero by definition. We have therefore established the following result.

**Lemma 1** The first-price knockout auction with information sharing is an ex post efficient mechanism that satisfies ex post budget balance.

It is worth emphasizing the role of symmetry in the above lemma. The first-price knockout selects the buyer with the highest signal, but efficiency requires that the ring select the buyer with the highest valuation conditional on all of the private signals. In symmetric models, these two criteria are equivalent, and hence information sharing creates no incentive problems.

Do the payoffs of the knockout auction satisfy the interim participation constraints? In order to compare the payoffs for buyers with signals above $a$, we need the following technical lemma. Subscripts denote partial derivatives.

**Lemma 2** $A(s, t) \equiv \frac{f_2(t|s)}{f(t|s)} - \frac{F_2(s|s)}{F(s|s)} \geq 0$ for all $s \leq t$.

Lemma 2 is an implication of affiliation and its proof is relegated to the appendix. Note that, if signals are independent, then $F_2(s|t) = f_2(t|s) = 0$, which in turn implies that $A(s, t) = 0$.

Bidders with signals below $a$ earn a positive payoff in the knockout auction and zero in the status quo mechanism. Clearly, they are better off in the coalition. Thus, if $\pi^{NC}(s)$ exceeds $\pi^K(s)$ at higher signals, then the slope of $\pi^K(s)$ must exceed the slope of $\pi^{NC}(s)$ at
Lemma 3 For any $s > a$.

\[
\left(\frac{d\pi^K(s)}{ds} - \frac{d\pi^{NC}(s)}{ds}\right) - \frac{F_2(s|s)}{F(s|s)}[\pi^K(s) - \pi^{NC}(s)] = - \int_0^s \frac{\partial}{\partial s} \min[r, w(s, t)] f(t|s) dt \\
- \int_0^s \min[r, w(s, t)] A(s, t) f(t|s) dt \\
+ \int_s^1 B^K(t) A(s, t) f(t|s) dt.
\] (3)

Lemma 3 identifies two competing effects on the relative slopes of the equilibrium profit functions: the information sharing effect and the affiliation of signals effect. The information-sharing effect is absent when values are private or the reserve price is zero. In the former case, the buyer’s decision to purchase the tract does not depend upon the signal of the other buyer because $w(s, t) = s$. As a result, $r = a$, which in turn implies that the first term on the right-hand side of equation (3) is zero. The second term also vanishes since

\[
\int_0^s r A(s, t) f(t|s) dt = r \int_0^s \left( \frac{f_2(t|s)}{f(t|s)} - \frac{F_2(s|s)}{F(s|s)} \right) f(t|s) dt \\
= r(\int_0^s f_2(t|s) dt - F_2(s|s)) = 0.
\]

In the latter case, the buyer is always willing to purchase the tract because $w(s, t)$ is positive and $r$ is zero. Once again, the value of the first two terms on the right-hand side of equation (3) are zero. The third term is positive by Lemma 2. Hence, the two profit functions cannot cross, which implies that $\pi^K(s)$ exceeds $\pi^{NC}(s)$ for all $s$. We have therefore established the following proposition.

Proposition 4 Suppose values are private or the reserve price is not binding. Then the
first-price knockout auction is an ex post efficient mechanism that satisfies ex post budget balance and interim participation constraints.

McAfee and McMillan [15] have shown that buyers can collude efficiently and earn higher payoffs when values are private and independently distributed. Proposition 4 extends both of these results to affiliated private values. Note that, in the IPV case, \( \pi^K(s) \) exceeds \( \pi^{NC}(s) \) by a positive constant (i.e., slopes are equal)\(^7\).

The presence of the information sharing effect works against the formation of an all-inclusive ring. To see this, first define \( \theta(s) \) as

\[
   w(s, \theta(s)) = r.
\]

Since \( w \) is increasing in both arguments, and \( s > a > b \), we have that \( \theta(s) < b \). As a result, the first term on the right-hand side of equation (3) can be expressed as

\[
\begin{align*}
\int_0^s \frac{\partial}{\partial s} \{\min[r, w(s,t)]\} f(t|s)dt &= \int_0^{\theta(s)} \frac{\partial}{\partial s} \{\min[r, w(s,t)]\} f(t|s)dt \\
&\quad + \int_{\theta(s)}^s \frac{\partial}{\partial s} \{\min[r, w(s,t)]\} f(t|s)dt \\
&= \int_0^{\theta(s)} w_1(s,t) f(t|s)dt \geq 0,
\end{align*}
\]

where \( w_1 \) is the partial derivative of \( w \) with respect to the first argument. The term is a measure of the value of a rival’s information to a bidder with signal \( s \). When the other buyer’s signal lies between 0 and \( \theta(s) \), the efficient decision is not to purchase the tract from the seller. This outcome is implemented by the ring but not by the status quo mechanism.

Now suppose that signals are independent. In this case, \( F_2(s|s) = A(s,t) = 0 \) and the difference in the slopes of the two profit functions is simply equal to the expression given above. Since the value of learning a rival’s signal is lower for buyers with higher signals

\(^7\)Mailath and Zemsky [14] obtain a similar result for second-price auctions.
(i.e., $\theta(s)$ falls with an increase in $s$), $\pi^{NC}$ increases more rapidly with $s$ than $\pi^K$. As a result, $\pi^K$ can intersect $\pi^{NC}$ at a signal above $a$, violating the participation constraints at higher signals. We provide an example below in which this occurs.

When signals are affiliated, the second term on the right-hand side of equation (3) cannot be signed. It depends on the interaction of the information sharing and affiliation effects. However, $F_2$ is negative, which implies that affiliated signals tends to narrow the difference in slopes of the two profit functions. Thus, the affiliation effect favor the formation of a ring. The intuition is that bidding in the status quo mechanism is relatively more competitive when signals are affiliated.

Why may buyers with high signals prefer to bid competitively for common value tracts? The reason is the winner’s curse. Fear of the winner’s curse causes buyers to bid cautiously in the status quo mechanism, and buyers with low signals do not participate. The latter leads to inefficient trade, since no one may bid even though at least one of the buyers would be willing to do so if he knew all of the private signals. (The converse is also true - a buyer may purchase the tract in the status quo mechanism when he would not do so if informed of his rival’s signal.) In contrast, the ring is efficient. The winning bidder in the knockout auction learns the private signals of the other members, and therefore purchases the tract if and only if his valuation conditional on all of the private signals exceeds investment costs. The efficiency of the ring works to the advantage of buyers with low signals but against a buyer with a high signal. A high signal buyer ends up paying less to the seller but more to the other buyers.

Under the conditions of Proposition 4, the status quo mechanism is efficient and hence there is no trade-off between efficient collusion and individual rationality. In common value auctions with a binding reserve price, the status quo mechanism is inefficient and efficient collusion may be incompatible with individual rationality. However, a ring may be able to guarantee its members payoffs that exceed what they can earn in the status quo mechanism.
if it is willing to sacrifice efficiency.

**Proposition 5** *The first-price knockout auction without information sharing is a mechanism that satisfies ex post budget balance and interim participation constraints.*

The proof of Proposition 5 is given in Appendix A. Note that, as in the case of Proposition 4, the ring uses a mechanism that implements the same trades as the status quo mechanism. However, it is difficult to imagine oil and gas firms enforcing a joint venture contract that prohibits them from sharing information on individual tracts in the area covered by the agreement when it is ex post optimal for them to do so.

## 4 An Illustrative Example

When does the equilibrium payoff from competitive bidding exceed the expected payoff to collusion? Is an all-inclusive ring more likely to form in auctions with more buyers? We address these questions using the Wallet Game described by Bulow and Klemperer ([4],[12]).

Suppose there are three potential buyers and their preferences are given by

\[ u(V, S_i) = V = S_1 + S_2 + S_3. \]

For simplicity, we use equal weights, and so a model of pure common values. The signals of the buyer are assumed to be independent random variables with distribution \( F(s) = s^q \), where \( q > 0 \). The parameter \( q \) determines the shape of the distribution. A higher value of \( q \) shifts probability mass away from lower signals to higher signals. Higher values of \( q \) also means more optimistic priors since

\[ E(S_i) = \frac{q}{1 + q} \equiv \mu. \]

Note that if \( q = 1 \), signals are uniformly distributed on the unit interval. We use the reserve
price $r$ to parameterize the importance of the information sharing effect.

### 4.1 Two Buyers

We first consider the case in which two of the three potential buyers have searched the area and obtained their signals. To facilitate a comparison with three buyers, we hold the expected value of the tract constant by integrating over $S_3$ and replacing its value with $\mu$. If $r > \mu$, the critical cutoff signal for participation in the competitive auction is given by

$$a_2 = \frac{1 + q}{1 + 2q} [r - \mu],$$

and in the knockout auction, it is given by

$$b_2 = \frac{r - \mu}{2}.$$

Here we subscript the cutoff values by the number of buyers. If $r < \mu$, then $a_2 = b_2 = 0$, since every type is willing to bid in the seller’s auction, and information sharing has no value. Note that the set of types that bid in the knockout but not in the status quo mechanism, $a_2 - b_2$, decreases with $q$ and increases with $r$.

In the next section, we show that the participation constraints are satisfied for all types if they are is satisfied for the highest type. The equilibrium payoff to the highest type in the status quo mechanism is

$$\pi^{NC}(1) = \frac{1 - a_2^{1+q}}{1+q}.$$

His payoff in the knockout auction is

$$\pi^K(1) = \left(\frac{1}{1 + 2q}\right) \left[\frac{2q^2 + 2q + 1}{(1 + q)} - \left(\frac{r - \mu}{2}\right)^{1+2q}\right] - \frac{r - \mu}{2} + \frac{\max\{r - \mu - 1, 0\}^{1+q}}{1 + q}.$$

Details on the derivations of these equations are given in Appendix B.
Figure 1 compares the two payoffs for the highest type in \((r, q)\) space. The solid curve is the locus of points where the highest type’s ring profits are equal to his profits in the status quo mechanism. The region below this curve represents the area where highest type earns more from the status quo mechanism than from the ring. For fixed \(r\), a higher value of \(q\) means that information sharing becomes less valuable, causing the highest buyer to bid relatively more aggressively in the status quo mechanism than the knockout auction. Thus, high values of \(q\) favor the ring over the status quo mechanism. For fixed \(q\), an increase in \(r\) makes information sharing more valuable. This has two effects on the highest type’s payoffs. It enhances his strategic advantage in the competitive auction, since the winner’s curse is stronger and scares off more types. But it also makes learning the other buyer’s signal more valuable. The first effect dominates for low values of \(r\) (i.e., \(r < 1\)) and the second dominates for high values of \(r\) (i.e., \(r > 1\)). The trade-off between these two effects accounts for the non-monotonic relationship between \(q\) and \(r\).

In our application, rings frequently use the equal-sharing mechanism, which is efficient and incentive compatible in our example. The dashed line in Figure 1 corresponds to points where the highest type’s profits from the equal-sharing ring are equal to his profits in the status quo mechanism. Clearly, the region where the equal-sharing mechanism is not enforceable is larger than and contains the region where the first-price knockout is not enforceable. Thus, the ring is more likely to form when it allocates the exclusive right to bid in the seller’s auction with a first-price knockout auction than when an equal-sharing agreement is employed.

4.2 Three Buyers

The critical value for participation in the status quo mechanism is given by

\[
a_3 = \frac{1 + q}{1 + 3q^r},
\]
and the critical value for participation in the all-inclusive ring is

\[ b_3 = \frac{r}{3}. \]

It is easy to show that the set of types that bid in the knockout but not in the status quo mechanism is larger with three than two buyers (i.e., \( a_3 - b_3 > a_2 - b_2 \)). The equilibrium payoff to the highest type in the status quo mechanism is

\[ \pi^{NC}(1) = \frac{1}{1 + 2q}(1 - a_3^{1+2q}). \]

His payoff in the knockout auction does not have a closed form solution and requires numerical integration.

Figure 2 compares the highest type’s payoffs in the status quo mechanism and in the first-price knockout ring mechanism. The region below the dashed and solid curves represent the areas where the all-inclusive ring is not enforceable when there are, respectively, two and three buyers. Neither area is a subset of the other, although the area in the three buyer case is smaller. The increase in \( n \) has two effects. First, the winner’s curse is strengthened, which enhances the high type’s strategic advantage in the status quo mechanism but makes information pooling more valuable. When \( r \) is low, the strategic advantage is more important and the additional buyer reduces the likelihood that the ring is enforceable. When \( r \) is high, information pooling is more important, and the additional buyer makes it more likely for the ring to be enforceable. Second, the level of competition increases. Since the competitive effect is stronger in the status quo mechanism than in the ring mechanism, the area in which the ring is not enforceable is reduced.

We conjecture that the competitive effect dominates as the number of buyers gets large and that, in the limit, the all-inclusive ring satisfies the participation constraints. The problem with studying this issue in the context of a common value model with independent
signals is that the value of the tract goes to infinity with the number of signals. The expected value of the tract needs to be held constant as the number of buyers gets large. However, it is not difficult to specify affiliated environments in which even buyers with the highest signal will not want to participate in the status quo mechanism as the number of buyers gets large (i.e., \( a_n \) converges to 1). In these cases, the ring satisfies individual rationality since all buyers make positive profits from collusion. More generally, one wants to show that \( \pi^{NC} \) converges to zero faster than \( \pi^K \), which is likely to be true since aggregate expected profit converges to zero in the status quo mechanism and to a positive constant in the knockout auction.

In summary, the all-inclusive ring is not enforceable when priors are pessimistic (i.e., \( q \) is low), acquisition costs are substantial (i.e., \( r \) is not too low or too high), and the number of buyers is small.

5 Optimal Ring Mechanisms

The preceding analysis demonstrates that, if information-sharing is important, a buyer with the highest signal may prefer the status quo mechanism to an all-inclusive ring which uses a first-price knockout auction to allocate the option to purchase the tract at price \( r \). However, this result is not very interesting if there exist other efficient ring mechanisms that satisfy interim rationality. To study this issue, we exploit the revelation principle and study collusive direct revelation mechanisms in common value auctions with independent signals. We identify and characterize conditions under which ex post efficiency, budget balance, and individual rationality are not compatible for any indirect mechanism.

In a collusive direct revelation mechanism, the ring’s representative in the seller’s auction, and side-payments between the buyers, are determined as functions of the buyers’ reported signals. The mechanism is a pair \( \{Q, P\} \) where \( Q : [0,1]^2 \to [0,1]^2 \) and \( P : [0,1]^2 \to R^2 \). Let \( x_i \) denote the report by buyer \( i \). Given reports \( (x_1, x_2) \), the probabil-
ity that buyer \(i\) obtains the right to bid in the seller’s auction is \(Q_i(x_i, x_j)\) and its expected side-payment is \(P_i(x_i, x_j)\). Clearly,

\[
Q_1(x_1, x_2) + Q_2(x_1, x_2) \leq 1
\]

for all \((x_1, x_2) \in [0, 1]^2\). We assume that transfers are feasible if they satisfy

\[
P_1(x_1, x_2) + P_2(x_1, x_2) = 0
\]  

(4)

for every pair of reported signals \((x_1, x_2)\). This requires the ring to balance its budget ex post. A weaker requirement is ex ante budget balance which only requires that transfers between buyers sum to zero on average.

Suppose buyer \(j\) reports truthfully. Then the payoff to buyer \(i\) with signal \(s_i\) and report \(x_i\) is

\[
\pi_i(s_i, x_i) = E_{s_j}[Q_i(x_i, s_j) \max\{w(s_i, s_j) - r, 0\} + P_i(x_i, s_j)].
\]  

(5)

Denote \(\pi_i(s_i, s_i)\) by \(\pi_i(s_i)\). A ring mechanism \(\{Q, P\}\) is incentive compatible if for all \(s_i, x_i \in [0, 1], i = 1, 2,\)

\[
\pi_i(s_i) \geq \pi_i(s_i, x_i).
\]

The following standard lemma characterizes the set of incentive compatible mechanisms.

**Lemma 6** A ring mechanism \(\{Q, P\}\) is incentive compatible if and only if for any \(s_i, x_i \in [0, 1],\)

\[
\frac{d\pi_i(s_i)}{ds_i} = E_{s_j}[Q_i(s_i, s_j) \frac{\partial}{\partial s_i} \max\{w(s_i, s_j) - r, 0\}],
\]  

(6)

and

\[
E_{s_j}[(\partial Q_i(x_i, s_j)/\partial x_i) \max\{w(s_i, s_j) - r, 0]\} \geq 0.
\]  

(7)

Efficiency implies that the buyer with highest valuation is awarded the exclusive right

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to acquire the tract at price \( r \) and does so if and only if the expected value of the tract conditional on \((s_1, s_2)\) exceeds \( r \). More formally, a direct mechanism is ex post efficient if

\[
Q_i(s_i, s_j) = \begin{cases} 
1 & \text{if } s_i > s_j > \theta(s_i) \\
0 & \text{otherwise}
\end{cases}
\]

Combining the incentive compatibility and budget balance with efficiency yields the following characterization.

**Proposition 7** Suppose signals are independently distributed and \( w(s, t) > w(t, s) \) for all \( s > t \). Then the payoff to buyer \( i \) with signal \( s \) in any ex post efficient, incentive compatible mechanism that satisfies ex ante budget balance is given by

\[
\pi^C_i(s) = \pi_{i0} + \int_{\theta(s)}^{s} [w(s, t) - r]dF(t) - \int_{b}^{s} [w(t, t) - r]dF(t)
\]

for \( s > b \) and it is equal to \( \pi_{i0} \) otherwise, where

\[
\pi_{10} + \pi_{20} = 2 \int_{b}^{1} [w(t, t) - r][1 - F(t)]dF(t).
\]

Ex post efficiency, incentive compatibility, and ex ante budget balance uniquely determine the payoff of each member of the ring up to a constant. In an anonymous mechanism, the buyers are treated symmetrically, which implies that \( \pi_{10} = \pi_{20} \). Any indirect, anonymous ring mechanism that is ex post efficient and satisfies the stronger restriction of ex post budget balance generates identical expected payoffs. It then follows from Lemma 1 that these payoffs can implemented by the first-price knockout auction.

**Corollary 8** Suppose signals are independently distributed and \( w(s, t) > w(t, s) \) for all \( s > t \). Then any efficient, incentive compatible, anonymous ring mechanism can be implemented by a first-price knockout auction with information sharing.
The corollary extends McAfee and McMillan’s result for an independent private values model to affiliated value models with independent signals. Of course, the first-price knockout auction is not the only implementable mechanism. A second-price knockout auction also works.

**Proposition 9** Suppose signals are independently distributed and \( w(s, t) > w(t, s) \) for all \( s > t \). Any ex post efficient, incentive compatible, budget balancing ring mechanism satisfies the interim participation constraints if and only if \( \pi_i^C(1) > \pi_i^{NC}(1) \) for \( i = 1, 2 \).

Proposition 9 establishes a useful necessary and sufficient condition for efficiency, incentive compatibility, and budget balance to conflict with the interim participation constraints. The example in the preceding section exploits this condition to illustrate the conditions under which competitive bidding yields higher profits for high types.

In a pure common value environment with independent signals, efficiency does not require that the buyer with the highest signal win the tract. Ex post efficiency is attained regardless of which buyer wins the tract, as long as the buyers report their private signals. In this case, a weak ring, which McAfee and McMillan define as a ring that cannot make transfer payments, can be efficient. The mechanism that awards the right to purchase the tract randomly to one member, and all other members report their private signals, is efficient and incentive compatible. The equal sharing mechanism in which members report their signals, and share costs and revenues equally is another efficient, incentive compatible mechanism. Note, however, that these mechanisms do not generate the same payoffs as the first-price knockout auction. The indeterminacy of the allocation rule implies that efficiency, incentive compatibility, budget balance and anonymity do not uniquely determine the payoffs to ring members.
Proposition 10 Suppose signals are independently distributed, $w(s, t) = w(t, s)$, and

$$w_1(s, t) \frac{F(s)}{f(s)} \geq w_1(t, s) \frac{F(t)}{f(t)}$$

if and only if $s \geq t$. Then any ex post efficient, incentive compatible, budget balancing ring mechanism fails to satisfy the interim participation constraints if $\pi^K_i(1) < \pi^{NC}_i(1)$ for $i = 1, 2$.

Proposition 10 establishes a useful sufficient condition for checking whether an indirect mechanism such as the equal-sharing mechanism conflicts with interim rationality. If the highest type obtains a higher payoff from bidding competitively than from colluding when the ring uses a first-price knockout auction, then it also prefers the status quo mechanism to a ring that uses the equal-sharing mechanism, or any other efficient, incentive compatible, budget balancing mechanism.

The payoffs are also not uniquely determined in the affiliated values model when signals are affiliated. For instance, the first-price and second-price knockout auctions generate different payoffs. In these cases, the interim participation constraints may be compatible with the above three conditions but the characterization of incentive compatibility is quite complicated. In any case, even if the conditions are compatible, the indirect mechanism that implements those payoffs is not likely to be simple.

Finally, it is worth noting that the analysis of this section also applies to second-price auctions. It can be shown that the buyer’s equilibrium payoffs in the second-price and first-price auctions are the same when signals are independently distributed.

6 Conclusion

We have shown that the trading inefficiency caused by the “winner’s curse” can be an important obstacle to collusion in auctions of common value assets with a binding reserve
price or ex post investment. The theory predicts that buyers are unlikely to collude when investment costs are substantial, the number of buyers is small, and priors are relatively pessimistic. These conditions are satisfied in the auctions of federal offshore oil and gas leases on marginal tracts. Thus, our theory provides an explanation for the low incidence of joint bids on these tracts, where the relative gains from colluding were large, since there was less competition and information sharing was valuable.

In our application to offshore oil and gas auctions, most of the Big12 joint ventures consisted of only two firms. An analysis of partial rings in common value environments is an interesting subject for future research. McAfee and McMillan study this issue in a simple model in which each buyer’s private value is an independent Bernoulli random variable. They show that, in equilibrium, the noncolluding bidder is better off than ring members, and that a ring of at least three bidders always forms. Thus, a ring always forms, but the reduced competition effect can explain partial rings. However, the situation is considerably more complicated when values are affiliated, since the noncolluding bidder is at an informational disadvantage when his rivals form a ring, and this effect could offset the benefit from the reduction in competition.

Our results are for legal cartels. However, the analysis may also prove useful for understanding self-enforcing cartels. Athey and Bagwell [1, 2] study optimal collusion in markets where firms receive privately-observed, i.i.d. cost shocks. The firms can communicate with each other to determine who has the lowest cost but they cannot make side-payments to each other. Their modeling approach is to recast the repeated, hidden information game as a static mechanism, similar to that analyzed in the legal cartel literature. They show that, if firms are sufficiently patient, they can use “market share favors” to implement efficient collusion. Our results suggest that it may be more difficult to collude if cost shocks contain a common component.
References


Studies, 44, 511-518.
Appendix A

Proof of Lemma 2:

Affiliation implies that $f_2(t|s)/f(t|s)$ is increasing in $t$ and that $f(t|s)/F(t|s)$ is increasing in $s$. It follows that, for any $s \leq t$,

$$\frac{f_2(t|s)}{f(t|s)} \geq \frac{f_2(s|s)}{f(s|s)} \geq \frac{F_2(s|s)}{F(s|s)}$$

where the second inequality follows from

$$\frac{\partial}{\partial s} \left( \frac{f(t|s)}{F(f|s)} \right) = \frac{f_2(t|s)}{f(t|s)} \frac{f(t|s)}{F(t|s)} - \frac{f(t|s)F_2(t|s)}{F^2(t|s)} \geq 0.$$

Q.E.D.

Proof of Lemma 3:

In a first-price auction with a reserve price $r$, the equilibrium payoff to a buyer with signal $s > a$ is

$$\pi^{NC}(s) = \int_0^s w(s, t)f(t|s)dt - B(s)F(s|s).$$

Differentiating with respect to $s$ and using the envelope theorem, we obtain

$$\frac{d\pi^{NC}(s)}{ds} = \int_0^s \left( w_1(s, t) + w(s, t) \frac{f_2(t|s)}{f(t|s)} \right) f(t|s)dt - B(s)F_2(s|s).$$

where $w_1(s, t), F_2(x|s)$, and $f_2(x|s)$ represent partial derivatives with respect to $s$. Since

$$B(s) = \int_0^s \frac{w(s, t)f(t|s)ds}{F(s|s)} - \frac{\pi^{NC}(s)}{F(s|s)},$$

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it follows that

\[
\frac{d\pi^{NC}(s)}{ds} = \frac{F_2(s|s)}{F(s|s)} \pi^{NC}(s) + \int_{0}^{s} [w_1(s, t) + w(s, t)A(s, t)] f(t|s) dt,
\]

where

\[
A(s, t) = \frac{f_2(t|s)}{f(t|s)} - \frac{F_2(s|s)}{F(s|s)}.
\]

Equilibrium profits to a buyer with signal \(s > b\) in a first-price knockout auction with information pooling is

\[
\pi^K(s) = \int_{0}^{s} \max\{w(s, t) - r, 0\} f(t|s) dt - B^K(s)F(s|s) + \int_{s}^{1} B^K(t) f(t|s) dt.
\]

Differentiation with respect to \(s\) and using the envelope theorem, we obtain

\[
\frac{d\pi^K(s)}{ds} = \int_{0}^{s} \left( \frac{\partial}{\partial s} \{\max\{w(s, t) - r, 0\}\} + \max\{w(s, t) - r, 0\}\frac{f_2(t|s)}{f(t|s)} \right) f(t|s) dt
\]

\[-B^K(s)F_2(s|s) + \int_{s}^{1} B^K(t) f_2(t|s) f(t|s) dt.
\]

Using the definition of \(\pi^K(s)\) yields

\[
\frac{d\pi^K(s)}{ds} = \frac{F_2(s|s)}{F(s|s)} \pi^K(s) + \int_{0}^{s} \frac{\partial}{\partial s} \{\max\{w(s, t) - r, 0\}\} f(t|s) dt
\]

\[+ \int_{0}^{s} \max\{w(s, t) - r, 0\} A(s, t) f(t|s) dt + \int_{s}^{1} B^K(t) A(s, t) dF(t|s).
\]

The difference in slopes of the two curves at any \(s > a\) is
\[
\frac{d\pi^K (s)}{ds} - \frac{d\pi^{NC} (s)}{ds} = \frac{F_2(s|s)}{F(s|s)} [\pi^K (s) - \pi^{NC} (s)] - \int_0^s \frac{\partial}{\partial s} \{\min [r, w(s, t)]\} f(t|s) dt \\
- \int_0^s \min [r, w(s, t)] A(s, t) f(t|s) dt + \int_s^1 B^K (t) A(s, t) f(t|s) dt.
\]

Q.E.D.

**Proof of Proposition 5:**

Equilibrium profits to a buyer with signal \( s > a \) in a first-price knockout auction without information pooling is

\[
\pi(s) = (\bar{w}(s) - r - B(s)) F(s|s) + \int_s^1 B(t) f(t|s) dt,
\]

where

\[
\bar{w}(s) = E[w(s, t)|t < s].
\]

Differentiation with respect to \( s \) and using the envelope theorem, we obtain

\[
\frac{d\pi(s)}{ds} = \bar{w}'(s) F(s|s) + (\bar{w}(s) - r - B(s)) F_2(s|s) + \int_s^1 B(t) \frac{f_2(t|s)}{f(t|s)} f(t|s) dt.
\]

Using the definition of \( \pi(s) \) yields

\[
\frac{d\pi(s)}{ds} = \frac{F_2(s|s)}{F(s|s)} \pi(s) + \bar{w}'(s) F(s|s) + \int_s^1 B(t) A(s, t) f(t|s) dt.
\]

Noting that

\[
\bar{w}(s) F(s|s) = \int_0^s w(s, t) f(t|s) dt,
\]
it follows that

\[ w'(s)F(s|s) = (w(s, s) - \bar{w}(s))f(s|s) \]
\[ + \int_0^s w_1(s, t)f(t|s)dt + \int_0^s w(s, t)A(s, t)f(t|s)dt. \]

Thus, the difference in slopes of the two curves at any \( s > a \) is

\[ \frac{d\pi(s)}{ds} - \frac{d\pi^{NC}(s)}{ds} = \frac{F_2(s|s)[\pi(s) - \pi^{NC}(s)]}{F(s|s)} + (w(s, s) - \bar{w}(s))f(s|s) + \int_s^1 B(t)A(s, t)f(t|s)dt, \]

where the second term on the right-hand side is positive since \( w(s, t) \) is increasing in \( t \) and, due to Lemma 2, the third term is positive. It then follows that

\[ \frac{d\pi(s)}{ds} - \frac{d\pi^{NC}(s)}{ds} > 0 \]

whenever \( \pi(s) = \pi^{NC}(s) \). Q.E.D.

**Proof of Proposition 7:**

Efficiency implies that \( Q_1(s_1, s_2) = 1 \) if \( s_1 > b \) and \( s_1 > s_2 > \theta(s_1) \) and equal to 0 otherwise. It then follows from (6) that

\[ \frac{d\pi_1(s)}{ds} = \int_{\theta(s)}^s w_1(s, t)dF(t) \]

for \( s \geq b \) and 0 otherwise. Integrating the above equation yields

\[ \pi_1(s) = \pi_{10} + \int_b^s \int_{\theta(y)}^y w_1(y, t)dF(t)dy \]
for \( s \geq b \), where \( \pi_{10} \) is a constant. Changing integration order in the above expression yields

\[
\pi_1(s) = \pi_{10} + K(s) - \int_b^s [w(t,t) - r]dF(t),
\]

where

\[
K(s) = \int_s^\theta [w(s,t) - r]dF(t).
\]

The profit expression for buyer 2 can be derived symmetrically.

> From (5),

\[
\pi_1(s) = E_t[Q_1(s,t) \max\{w(s,t) - r, 0\}] + E_t P_1(s,t).
\]

It follows that

\[
E_t P_1(s,t) = \pi_1(s) - K(s)
\]

for \( s \geq b \) and is equal to \( \pi_1(s) \) if \( s \leq b \). Thus,

\[
E_{(s,t)} P_1(s,t) = E_s \pi_1(s) - \int_b^1 K(s) dF(s)
\]

\[
= \pi_{10} - \int_b^1 \int_b^s [w(t,t) - r] dF(t) dF(s).
\]

Using integration by parts, we obtain

\[
E_{(s,t)} P_1(s,t) = \pi_{10} - \int_b^1 [w(t,t) - r][1 - F(t)] dF(t).
\]

Ex ante budget balance implies \( E_{(s,t)} \{P_1(s,t) + P_2(t,s)\} = 0 \). It follows that

\[
\pi_{10} + \pi_{20} = 2 \int_b^1 [w(t,t) - r][1 - F(t)] dF(t).
\]
Proof of Proposition 9:

First notice that the profit for a buyer with signal $s$ from the seller’s auction, $\pi^{NC}(s)$, is equal to zero for $s \leq a$, and strictly increasing in $s$ for $s \geq a$. Moreover, for $s > a$,

$$\frac{d\pi^{NC}(s)}{ds} = \int_0^s w_1(s, x)dF(x).$$

On the other hand, by Theorem 1, the profit for a bidder with signal $s$ from an efficient, incentive ring mechanism, $\pi^C(s)$, is a positive constant when $s \leq b$, and strictly increasing in $s$ for $s \geq b$. Furthermore, for $s > b$,

$$\frac{d\pi^C(s)}{ds} = \int_0^s w_1(s, x)dF(x).$$

Since $a > b$, it follows that, for any $s > b$,

$$\frac{d\pi^{NC}(s)}{ds} \geq \frac{d\pi^C(s)}{ds}.$$

Therefore, $\pi^C(s) \geq \pi^{NC}(s), \forall s \in [0, 1]$ if and only if $\pi^C(1) \geq \pi^{NC}(1)$. The claim follows.

Q.E.D.

Proof of Proposition 10:

Let

$$u_1(s, \tilde{s}) = E_t[\max[w(s, t) - r, 0]Q_1(\tilde{s}, t) - P_1(\tilde{s}, t)]$$

and

$$u_1(s) = u_1(s, s).$$

$u_2(t)$ and $u_2(t, \tilde{t})$ are similarly defined. An allocation mechanism $[Q, P]$ is incentive compatible if and only if

$$u_1(s, \tilde{s}) \leq u_1(s)$$

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and
\[ u_2(t, \tilde{t}) \leq u_2(t) \]
for all \( s, t, \tilde{s}, \tilde{t} \in [0, 1] \). Let \( M \) be the set of incentive compatible and ex ante budget balance allocation mechanisms. We first show that if

\[ w_1(s, t) \frac{F(s)}{f(s)} \geq w_1(t, s) \frac{F(t)}{f(t)} \]

if and only if \( s \geq t \), then \( u_1(1) + u_2(1) \) is maximized in \( M \) when

\[
Q_1(s, t) = \begin{cases} 
1, & \text{if } s > t \\
1/2, & \text{if } s = t \\
0, & \text{otherwise}
\end{cases}
\]

\[ Q_2(t, s) = 1 - Q_1(s, t). \]

Let \( \Omega(s) = \{ t \mid w(s, t) \geq r \} \) and \( \overline{\Omega} = \{ (s, t) \mid w(s, t) \geq r \} \). Then necessary and sufficient conditions for IC are

(i) \( u_1'(s) = \int_{\Omega(s)} w_1(s, t)Q_1(s, t)dF(t) \)

and

(ii) \( \int_{\overline{\Omega}(s)} w_1(s, t)Q_1(\tilde{s}, t)dF(t) \) weakly increases with \( \tilde{s} \)

for buyer 1 and similar conditions for buyer 2.

Since

\[ Q_1(s, t) + Q_2(t, s) = 1, \]
we let $Q(s,t) = Q_1(s,t)$ and $Q_2(t,s) = 1 - Q(s,t)$. Let $u_1(0) = u_{01}$. It follows that

$$E_s u_1(s) = \int_0^1 u_1(s) dF$$

$$= u_{01} + \int_0^1 u_1' [1 - F] ds$$

$$= u_{01} + \int_0^1 \int_{\Omega(s)} \left( w_1(s,t) \frac{1 - F(s)}{f(s)} Q(s,t) \right) dFdF.$$

Since, by definition,

$$u_1(s) = E_t [\max[w(s,t) - r, 0]Q(s,t) - P_1(s,t)],$$

it follows that

$$E_{s,t} P_1(s,t) = E_{s,t} [\max[w(s,t) - r, 0]Q(s,t)] - E_s u(s)$$

$$= -u_{01} + \int_0^1 \int_{\Omega(s)} I(s,t) Q(s,t) dFdF$$

$$= -u_{01} + \int_{\Omega} I(s,t) Q(s,t) dFdF,$$

where

$$I(s,t) = w(s,t) - r - w_1(s,t) \frac{1 - F(s)}{f(s)}.$$

Symmetrically, we have

$$E_{t,s} P_2(t,s) = -u_{02} + \int_{\Omega} I(t,s)[1 - Q(s,t)] dFdF.$$

Ex ante budget balance requires that

$$0 = E_{s,t} P_1(s,t) + E_{s,t} P_1(s,t)$$

$$= -u_{01} - u_{02} + \int_{\Omega} (I(t,s) + [I(s,t) - I(t,s)] Q(s,t)) dFdF.$$
It follows that
\[ u_{01} + u_{02} = \int_{\Omega} (I(t, s) + \{I(s, t) - I(t, s)\}Q(s, t)) \, dF dF. \]

Thus,
\[
\begin{align*}
  u_1(1) + u_2(1) &= \int_{\Omega} (I(t, s) + \{I(s, t) - I(t, s)\}Q(s, t)) \, dF dF \\
  &+ \int_{\Omega} \frac{w_1(s, t)}{f(s)}Q(s, t) \, dF dF + \int_{\Omega} \frac{w_1(t, s)}{f(t)}[1 - Q(s, t)] \, dF dF \\
  &= \int_{\Omega} I(t, s) \, dF dF + \int_{\Omega} [J(s, t) - J(t, s)]Q(s, t) \, dF dF,
\end{align*}
\]

where
\[ J(s, t) = w(s, t) - r + w_1(s, t) \frac{F(s)}{f(s)}. \]

Since \( w(s, t) - r = w(t, s) - r \), it follows that
\[ J(s, t) - J(t, s) = w_1(s, t) \frac{F(s)}{f(s)} - w_1(t, s) \frac{F(t)}{f(t)}. \]

It follows from the assumption that \( J(s, t) - J(t, s) \geq 0 \) if and only if \( s \geq t \). Therefore, \( u_1(1) + u_2(1) \) is maximized in \( M \) when the efficient allocation rule is used.

Since the first-price knockout auction implements the efficient allocation rule, to check whether the interim participation constraints are violated it is without loss of generality to compare the payoff from the knockout auction and that from the competitive bidding at the highest signal. The claim follows. Q.E.D.
Appendix B

In this appendix, we present the expressions for payoffs and cutoff points used in Section 5.

B1. Status Quo Mechanism with Two Buyers

The cutoff point in the status quo mechanism can be calculated as follows: $a_2 = 0$ if $r \leq \mu$, $a_2 = 1$ if $r \geq (1 + 3q)/(1 + q)$, and

$$a_2 = \frac{1 + q}{1 + 2q}(r - \mu)$$

otherwise, where $\mu = q/(1 + q)$. Using equation (1), the equilibrium payoff is given by

$$\pi^{NC}(s) = \frac{1}{1 + q}(s^{1+q} - a_2^{1+q})$$

for $s \geq a_2$ and zero otherwise.

B2. Knockout Auction with Two Buyers

A first-price knockout auction generates the following profits to a buyer

$$\pi^K(s) = \pi_0 + \int_{\theta(s)}^{s} (s + x + \mu - r)dx^q - \int_{b_2}^{s} (2x + \mu - r)dx^q$$

for $s > b_2 = \max[0, (r - \mu)/2]$ and is equal to $\pi_0$ otherwise, where

$$\pi_0 = \int_{b_2}^{1} (2x + \mu - r)(1 - x^q)dx^q$$

and $\theta(s) = \max\{r - \mu - s, 0\}$. Note that

$$\pi^K(1) = \pi_0 + \int_{\theta(1)}^{1} (1 + x + \mu - r)dx^q - \int_{b_2}^{1} (2x + \mu - r)dx^q$$

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where

\[ \pi_0 = \int_{b_2}^{1} (2x + \mu - r)(1 - x^q)dx^q \]

\[ = \frac{2q^2}{(1 + q)(1 + 2q)} - b_2 + \frac{2}{1 + q}b_2^{1+q} - \frac{1}{1 + 2q}b_2^{1+2q}. \]

The second term in the equation for \( \pi^K(1) \) can be expressed as

\[ \int_{\theta(1)}^{1} (1 + x + \mu - r)dx^q = 2 + \mu - r - \frac{1}{1 + q} + \frac{1}{1 + q}\theta(1)^{1+q} \]

and the third term can be expressed as

\[ \int_{b_2}^{1} (2x + \mu - r)dx^q = 2 + \mu - r - 2\int_{b_2}^{1} x^qdx \]

\[ = 2 + \mu - r - \frac{2}{1 + q} + \frac{2}{1 + q}b_2^{1+q}. \]

Putting the terms together yields

\[ \pi^K(1) = \frac{2q^2 + 2q + 1}{(1 + q)(1 + 2q)} - \frac{r - \mu}{2} - \frac{1}{1 + 2q}b_2^{1+2q} \]

\[ + \frac{1}{1 + q}\theta(1)^{1+q}. \]

**B3. Equal Sharing Mechanism with Two Buyers**

The payoff for a buyer with signal \( s \) is given by

\[ \pi^E(s) = \frac{1}{2} \int_{\theta(s)}^{1} (s + x + \mu - r)dx^q \]
which can be simplified as follows,

\[
\pi^E(s) = \frac{q}{1+q} - \frac{r}{2} + \frac{s}{2} + \frac{\theta(s)^{1+q}}{2(1+q)}
\]

for all \( s \in [\theta(1), 1] \) and 0 otherwise.

**B4. Status Quo Mechanism with Three Buyers**

Define

\[
w(s, t) = E[s_1 + s_2 + s_3 | s_1 = s, \max\{s_2, s_3\} = t].
\]

In the status quo mechanism, it can be verified that there is a symmetric equilibrium bidding strategy \( B(s) + r \), where

\[
B(s) = \frac{\int_{a_3}^{s} [w(x, x) - r] dF^2}{F(s)^2}
\]

and

\[
\int_0^{a_3} [w(a_3, x) - r] dF^2 = 0.
\]

The payoff is

\[
\pi^{NC}(s) = \int_0^{s} [w(s, x) - r] dF^2 - \int_{a_3}^{s} [w(x, x) - r] dF^2.
\]

Since in the example

\[
w(s, t) = s + \frac{1+2q}{1+q} t
\]

it can be computed that

\[
a_3 = \frac{1+q}{1+3q} r
\]

and

\[
\pi^{NC}(s) = \frac{1}{1+2q} (s^{1+2q} - a_3^{1+2q}).
\]

Note that \( a_3 > a_2 \) if and only if \( r < (1+3q)/(1+q) \). If \( r \geq (1+3q)/(1+q) \) then
\(a_2 = a_3 = 1.\)

**B5. Knockout Auction with Three Buyers**

To determine the cartel payoffs, define

\[
\bar{w}(s, t) = E[\max[0, s_1 + s_2 + s_3 - r]|s_1 = s, \max\{s_2, s_3\} = t].
\]

It can be shown that the interim payoff for a cartel member with signal \(s\) is given by

\[
\pi^K(s) = \int_0^1 \int_0^s \left( \bar{w}(s, t) - \frac{1 - F(s)}{f(s)} \bar{w}_1(s, t) \right) dF(s) dF^2(t) + \int_0^s \bar{w}_1(x, t) dF^2 dx.
\]

It follows that

\[
\pi^K(1) = \int_0^1 \int_0^s J(s, t) dF^2(t) dF(s)
\]

where

\[
J(s, t) = \bar{w}(s, t) + \frac{F(s)}{f(s)} \bar{w}_1(s, t).
\]

In the example,

\[
\bar{w}(s, t) = \int_0^t \max\{s + t + x - r, 0\} dx^q/t^q.
\]

If \(s + t > r\), then

\[
\bar{w}(s, t) = s + \frac{1 + 2q}{1 + q} t - r.
\]

If \(s + t \leq r\) and \(s + 2t \geq r\), then

\[
\bar{w}(s, t) = s + \frac{1 + 2q}{1 + q} t - r + \frac{(r - s - t)^{1+q}}{(1+q)t^q}.
\]

If \(s + 2t < r\), then \(\bar{w}(s, t) = 0\).
Since the slope of the cartel payoff is zero when $\bar{w}(s, t) = 0$ for all $t \leq s$, it follows that $b_3 = r/3$. Note that $b_3 < b_2$ if and only if $r > 3\mu$. Moreover, since

$$a_3 - b_3 = \frac{2r}{3(1 + 3q)},$$

$$a_2 - b_2 = \frac{r - \mu}{2(1 + 2q)},$$

it can be verified that $a_3 - b_3 > a_2 - b_2$ since $r \leq 3$. $\pi^K(1)$ can be computed numerically.
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Figure 1

\[ \pi^E(1) > \pi^{NC}(1) \]
\[ \pi^K(1) < \pi^{NC}(1) \]
\[ \pi^K(1) > \pi^{NC}(1) \]
Figure 2

Two buyers

Three buyers

$\pi^K(1) > \pi^{NC}(1)$

$\pi^K(1) < \pi^{NC}(1)$