Moral Hazard and Customer Loyalty Programs *

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Abstract

Frequent flier plans (FFPs) -- operated by almost all of the world’s major airlines -- have become the most famous, and probably the most valuable, customer loyalty programs in the world. In addition, plans created on the FFP model are now offered by sellers in a number of other industries such as hotels, car rental agencies and credit cards. In this paper we present a theory of FFPs that models them as efforts to take advantage of the agency relationship between employers -- who pay for airline tickets -- and employees -- who book those tickets. In this view, FFP benefits constitute “bribes”, inducing employees to book flights on airlines with higher prices. The model considers two airlines competing in a differentiated product environment and shows that a single airline offering an FFP has a large advantage. However, when both airlines operate plans, it is very possible that, while raising prices, competition (now via FFP benefits) will be intensified so much that the airlines end up worse off than had they not created the plans.

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1. Introduction

There can be little doubt that airline frequent flier plans (FFPs) are one of the most significant developments in the history of relationship marketing and customer loyalty plans. While varying somewhat one to another, these plans in general reward loyal (and frequent) customers with free flights, upgrades, products and other services. Given their current size in terms of number of participating airlines, plan members and miles collected and redeemed, it may be hard to believe that the oldest plans are barely 25 years old. For example, the Economist magazine recently estimated that the total stock of unredeemed frequent-flier miles is now worth more (at $700 billion US) than all the dollar bills in circulation around the world.¹

While these plans have attracted a great deal of attention in the transportation and marketing literatures, there has been relatively little formal modeling of FFPs by economists. The purpose of this paper is to analyze an often recognized, but never (to our knowledge) modeled aspect of FFPs -- namely that they exploit an agency relationship between employers who pay for business travel and employees who book the travel and collect the benefits of the FFP.² We also comment upon the use of loyalty programs more generally: we observe programs built on the FFP model for hotels, rental car agencies, and some other products and services, and it is not obvious why these programs are present in some industries but not others.

An employee selecting an airline (or hotel, or car rental agency) will not necessarily have the right incentives to find the lowest possible price when the employer

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¹ “In Terminal decline?”, Economist, January 6, 2005.
² Economists are certainly among those who have recognized this property of FFPs. See, e.g. Tretheway [1989] and the Los Angeles Times "Viewpoints" piece by Klemperer and Png [1986]. It was also discussed by Levine [1987, p. 452].
is paying the bill. With respect to business travel on airlines there are two reasons to be concerned. First, there is the normal agency relationship between the employer who pays most (possibly all) the costs of the ticket and the employee who selects the airline to be flown. In addition there are the added temptations for employees stemming from frequent flier plans. Even a relatively small enticement in the form of a frequent flier benefit may be enough to induce morally hazardous behavior.

In this paper, we show how these programs can give an advantage to a lone airline adopter, but that imitation by competitors can dissipate the benefits such that the airlines are no better with them than they were without them; and that they might even be worse off. Employee/passengers who collect the FFP points clearly benefit from the programs while employers will pay higher prices for employee travel.

The next section of the paper provides some basic background on frequent flier plans and presents an overview of the model. Section III presents the formal analysis while Section IV provides our concluding thoughts. While we will present the analysis with respect to FFPs, the reader should keep in mind that our results will also apply to similar loyalty programs in other industries.

II. Background and overview of the model

Since American Airlines launched the first major FFP, its AAdvantage program, in 1981, most major airlines have adopted their own FFPs. In the typical FFP, rewards are based upon the number of “miles” travelers have accumulated by flying with the sponsoring airline. Rewards are most often free flights, but other benefits (e.g. flight

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3 See Shaw [2004, p. 235].
class upgrades) are available in many programs as well. As suggested by the Economist article cited above, these programs are now huge, both in terms of the number of members and their accumulated miles.\(^4\) There are a number of purposes FFPs can and do serve that helps to explain their current popularity with airlines.

(i) \textit{Price Discrimination}: FFPs function as second degree price discrimination by serving as a sort of quantity discount scheme. To the extent that some buyers will not earn enough miles to collect rewards, through the FFP, the airline effectively charges them a higher price than it charges its larger volume purchasers.

(ii) \textit{Switching Costs}: A common concern with FFPs is that they create switching costs as travelers are inclined to stick with one airline to accumulate the largest balances possible so as to reach the necessary threshold levels. This aspect of FFPs, recognized by Klemperer [1987, p. 376 and 1995, pp. 517-518], has been formally modeled by Banerjee and Summers [1987] and Kim et al. [2001]. In Banerjee and Summers’ model, the switching costs created by FFPs can soften competition so drastically that the monopoly price is supportable as a non-cooperative equilibrium. Kim et al. contrast the competitive effects of more and less costly award programs. In some cases, firms may choose to adopt more costly award programs in order to reduce the intensity of price competition.\(^5\)

(iii) \textit{Other Barriers to Entry}: In addition to creating switching costs, FFPs can add to barriers to entry in other ways. Tretheway [1989] and Levine [1987] recognize that airlines with more extensive route networks may be at a competitive advantage when it

\(^4\) The same Economist article estimated that by the end of 2004, 14 trillion frequent flier miles had been accumulated worldwide. There are at least eight airlines—All Nippon Airways, American, Continental, Delta, JAL, Northwest, United, and US Air—that each have a worldwide membership in the tens of millions.

\(^5\) Carlsson and Lofgren [2004] estimate the switching costs between domestic airlines in Sweden and found that SAS's frequent flier plan contributed to those switching costs.
comes to offering FFPs since they have many more destinations to offer plan members, raising the value of their points without necessarily raising the costs of providing those benefits. Borenstein [1996] shows how an airline could use its dominance in a particular hub together with its FFP to deter entry by smaller, though possibly more efficient firms.⁶

(iv) **Customer Tracking and Database Marketing**: FFPs provide airlines with valuable information about their customers which may be useful for them and for others (to whom they may sell access to the data) in future marketing activities.⁷

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**Frequent flier plans and the agency relationship.**

Another perspective on FFPs suggests that they may be, at least in part, tools by which airlines take advantage of the agency relationship that exists when the party who books air travel and collects the FFP benefits is not the one who pays for the ticket. In this view, FFPs function as bribes to business travelers, tempting them to select higher priced airfares and possibly even take unnecessary (or less convenient) flights, paid for by their employer, in exchange for a small personal benefit in the form of program miles or points.⁸

While there has been very little in the way of formal modeling of this effect, the potential for abuse has been well-recognized. Dean [1988] and Arnesen et al. [1997]

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⁶ In an interesting recent paper, Lederman [2004] studies the impact of frequent flier programs on airline demand with a technique that allows her to disentangle the impact of FFPs from other advantages enjoyed by dominant airlines.

⁷ Shaw [2004, p. 238] argues that “effective use of the database for database marketing is now the key to airlines obtaining value-for-money from FFP investment.”

⁸ While it is clear that business travel represents a substantial fraction of all air travel, it is difficult to get precise measures of this share. As customers do not have to reveal the purpose of their trip when they book their ticket, this information is typically only collected through survey research. The (U.S.) Travel Industry Association’s *Domestic Market Report* [2005] reports that 28% of domestic trips by air in 2004 were for business/convention purposes and another 9% were for combined business and pleasure. A number of surveys have been done for specific airports as well. For example, a survey done for the Vancouver International Airport revealed that 30% of trips by air through that airport were for business purposes. *(YVR Skytalk*, April 2006, p. 3). Pels et al. [2001] report a share of 39% for the San Francisco airport.
consider the ethical implications of FFPs and find them to be suspect. Using survey data, Stephenson and Fox [1992] find that a significant fraction of corporate travel managers reported that FFPs had resulted in companies having to pay higher fares than necessary (consistent with the expected effect from the agency relationship we consider here) and also paying for unnecessary air travel. Based on this survey, Stephenson and Fox estimated that FFPs cost the companies an additional 7-9% of their travel costs annually and projecting this led them to estimate a total effect across the U.S. of $6.3 billion of excess costs annually.9 Nako [1992] and Proussaloglou and Koppleman [1999] find econometric evidence of the effects of FFPs on buyers' willingness to pay and on prices actually paid.10

The model

Our purpose here is to model explicitly the principal-agent problem characteristic of FFPs, so that we may understand better their effect on competition and prices in oligopoly markets. Our model will allow us to determine how different features of the marketplace and FFPs themselves -- such as the degree to which employees’ travel is paid by their employers and the costs of providing FFP benefits -- will influence the relationship between FFPs and market prices.

The existing work closest to what we do here is the paper by Cairns and Galbraith [1990], who incorporate elements of the agency problem into their switching cost model

9 Tretheway [1989, p. 197] cites sources suggesting that 13-20% of business travel under FFPs is unnecessary. In a widely-cited interview in U.S. News and World Report (April 22, 1985), Judith Dettinger of American Express reported that “A recent survey showed that 25 percent of the frequent travelers polled admitted to taking trips that were totally unnecessary in order to rack up miles.”

10 The data in Proussaloglou and Koppelman [1999] came from a stated preference survey of travelers, with the "mock" choices made treated as raw data in the econometric work. They report, for example, that high frequency business travelers were willing to pay a premium of $72 to travel with the carrier in whose program they most actively accumulate mileage points.
(their employee pays an exogenous fraction of the cost of the ticket). However, their purposes are quite different from ours. While we wish to explore the full equilibrium implications of FFPs in the agency environment, their focus is more on the implications of different airline network sizes for the conditions under which an incumbent can use an FFP to deter entry.\footnote{Their model can effectively be interpreted as a model of switching costs. Their Proposition 3 is the one point of significant overlap between our sets of results. It corresponds closely with our Proposition 2 on the equilibrium with two identical airlines both with FFPs.}

In our model we treat the FFP as a simple cash transfer to travelers. In effect, then, each airline sets two prices--one (P) is the price to be paid by the ultimate purchaser of the ticket, and the second (F) is the FFP side-payment from the airline to the traveler.\footnote{In fact, FFP benefits are typically not in the form of cash, thus F should be thought of as the present discounted value of monetary equivalent of those benefits.} We consider the possibility that transferring a dollar to travelers in the form of F may cost the airline more or less than one dollar. We model the cost to the airline of delivering a benefit of F to the traveler as $\gamma F$, where $\gamma$ may be greater than, equal to, or less than 1. If the airline has a great deal of excess capacity it can apply to deliver benefits (e.g. via free flights), it may perceive that $\gamma < 1$, while if the plans are costly to set up and maintain it may be the case that $\gamma > 1$.

We assume that all travelers for whom these prices apply are employees (we will also use the term “workers”) traveling on company business and that they are all members of the FFPs. Workers’ airfares are largely paid for them by their employers, but we do assume that workers bear some (possibly very small or even zero) share, $\theta$, of the price of the ticket, where $0 \leq \theta \leq 1$.\footnote{What we want to consider is that there is some cost to employees, however small, of paying higher prices for tickets. It could be that they bear some of this cost --- as it leaves less money in their expense account budgets for other items perhaps. Or it could be that this cost represents the expected costs to them of being audited and punished for careless use of their employer’s resources. We will assume that this} Certainly airlines do serve leisure travelers;
however, we assume that the airlines’ much vaunted ability to segregate their markets via various means is sufficient to allow us to focus on employee travel as independent from the leisure travel market. In our discussion section below we will consider some of the implications of introducing leisure travelers into the market we study here.

There are two airlines in the model, and the flights they offer are differentiated such that workers have preferences for one airline over another. We model this as a duopoly with the two firms at either end of a Hotelling line with unit length and a density of $M$. Given equal prices and FFP benefits, workers would prefer to travel with the “closer” airline, perhaps because of the kinds of services it offers. The employer does not perceive this differentiation and therefore would like the employee to book travel at the lowest price possible. But the employer does not observe the possible prices and can only approve (or not) travel on the employee’s selected airline. Hence, workers are free to pick either airline they wish as long as the price charged is below their employer’s reservation price, $V<\infty$.

The full game studied has two stages. In the first, airlines decide whether or not to have FFPs. In the second they simultaneously pick their prices ($P_i$, $i=1,2$) and, if relevant, the levels of their FFP benefits ($F_i$, $i=1,2$). We look for subgame perfect equilibria, employing backward induction techniques to solve the game. Hence, we begin by analyzing the second-stage subgame and use those results to determine whether one airline, both or neither will choose to have a FFP, and (when there are FFPs) what they do to the level of prices paid in the market and to airline profits.

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fraction is constant and the same for all workers. A natural extension of this model would involve adding an earlier stage in which firms choose $\theta$. 


An alternative structure for the timing in this game might have us allow airlines to choose their levels of FFP benefits before they choose their levels of prices, however this implies a level of commitment to the $F_i$ that we do not believe exists. Airlines can easily alter the value of their awards by making award redemption easier or harder, for example by allocating more or fewer seats for award travel. In contrast we do believe there is significant commitment value to a firm’s choice at stage one not to have an FFP. Without taking the steps to create the program at this first stage, the airline will not have any ability to offer a positive $F_i$. Thus, we can think of the decision to create an FFP at stage one as a decision to remain flexible with respect to $F_i$ in the second stage, rather than committing it to be zero.

If a worker located at $z$ buys from airline 1, her utility can be represented by:

$$ W - \theta P_1 - zt + F_1 $$

while if she buys from airline 2 it will be:

$$ W - \theta P_2 - (1 - z)t + F_2 $$

where $(P_i, F_i)$ are the price and level of FFP benefit offered by airline i, $z$ is the “distance” (in real or product space) of the worker from airline 1, and $t$ is the cost per unit of product-space distance for workers flying with an airline located some distance from the worker’s own location. We assume that the worker must choose an airline as a condition of remaining employed, i.e. we effectively assume that $W \rightarrow \infty$.\(^\text{14}\)

One more piece of notation is useful, related to a critical level of $\theta$ and an assumption about its magnitude.

**Assumption:** Let $\theta = t / V$. We assume that $\theta < 1$, or equivalently $t < V$.

\(^{14}\) To be clear, if the employer rejects travel because it is too expensive ($P > V$), the worker will not lose her job. She would lose it only by refusing to travel.
This assumption guarantees that the two firms would actually compete absent FFPs, i.e. that it would not be an equilibrium for them to charge the reservation value and take exactly half the market each.

Finally, airlines are assumed to have constant marginal costs of providing seats, which without any further loss of generality we set to zero. Therefore, the only airline costs in our model are those related to the frequent flier benefits they pay to workers. If marginal costs are positive, the prices and frequent flier benefits we solve for below can be viewed in terms of the markup of price over marginal cost (unless price is equal to the reservation price, in which case the marginal cost is not a consideration in pricing).

III. Analysis of the Model

We first note that, whenever $P_1 \leq V$ or $P_2 \leq V$, the market will be fully covered. If $P_1, P_2 > V$, then the market is shut down: the market can never be partially covered. However, the case $P_i > V$ and $P_j \leq V$ cannot arise in equilibrium: firm $i$ would have no demand, so it would have an incentive to lower its price. Therefore, we are sure that we are in the case $P_1, P_2 \leq V$ and the market is always fully covered.

III.1 0-FFP Case

We begin by examining the subgame in which neither airline has elected to offer an FFP. Since the market is fully covered, the demands are given by:

$$D_1(P_1, P_2) = Mz(P_1, P_2)$$
$$D_2(P_1, P_2) = M(1 - z(P_1, P_2))$$
where the location of the worker indifferent between the two airlines is given by:

\[ z(P_1, P_2) = \frac{1}{2} + \theta \frac{P_2 - P_1}{2t} \]

First-order conditions on profit maximization lead in a direct manner to

\[ P_1 = P_2 = \bar{P} = \frac{t}{\theta} \text{ and } \pi_1 = \pi_2 = \bar{\pi} = \frac{tM}{2\theta} \]

Note that, if \( \theta = 1 \), we are back in a usual Hotelling model where the third party payer problem does not arise and both prices will equal \( t \), the transport cost parameter. When \( \theta < 1 \), however, prices rise above normal Hotelling levels. This is as a result of the moral hazard present in the model even without FFPs that was created by the fact that a third party (the employer) is paying at least part of the cost of the worker’s ticket. This makes the worker’s demand less elastic and pushes up equilibrium prices.

However, there will be a limit to how high prices can rise. For \( \theta \) sufficiently low, \( \bar{P} = V \), the employers’ reservation price. Recalling that \( \theta = t/V \) this tells us that we get two sets of equilibrium prices and profits as summarized in the following proposition.\(^{15}\)

**Proposition 1**: In the 0-FFP subgame:

If \( \theta > \vartheta \), then \( P_1 = P_2 = \bar{P} = \frac{t}{\theta} \text{ and } \pi_1 = \pi_2 = \bar{\pi} = \frac{tM}{2\theta} \)

If \( \theta \leq \vartheta \), then \( P_1 = P_2 = \bar{P} = V \text{ and } \pi_1 = \pi_2 = \bar{\pi} = \frac{VM}{2} \)

\(^{15}\) The proof of Proposition 1 is straightforward and is omitted here, but is available upon request from the authors.
Prices and profits both rise as $\theta$ falls, until price hits the reservation value. Thus, the moral hazard is good for the airlines but costly to the employers.

III.2 2-FFP Case

The subgame in which both airlines offer FFPs is also quite straightforward, given the symmetry of the situation. As indicated above, the FFPs are modeled here as simple cash transfers (“bribes”) between the airlines and the workers. Consistent with the way FFPs are managed, the employers are not permitted to capture these benefits. Airline $i$ now offers workers the pair $(P_i, F_i)$. While $F_i$ is the amount of benefit the worker will receive, the cost to the airline of providing this benefit is $\gamma F_i$, where $\gamma$ may be less than, equal to, or greater than 1.

The demands are now given by:

$$D_1(P_1, P_2, F_1, F_2) = M z(P_1, P_2, F_1, F_2)$$

$$D_2(P_1, P_2, F_1, F_2) = M (1 - z(P_1, P_2, F_1, F_2))$$

and hence the profits are

$$\pi^1 = M z(P_1, P_2, F_1, F_2)(P_1 - \gamma F_1)$$

$$\pi^2 = M (1 - z(P_1, P_2, F_1, F_2))(P_2 - \gamma F_2)$$

but now the indifferent consumer is located at:

$$z(P_1, P_2, F_1, F_2) = \frac{1}{2} + \frac{\theta(P_2 - P_1) + F_1 - F_2}{2t}.$$

A check on the concavity properties of the profit functions reveals some complications. We have that
\[
\frac{\partial^2 \pi^1}{\partial P_i^2} = -\frac{M\theta}{t}, \quad \frac{\partial^2 \pi^1}{\partial F_i^2} = -\frac{My}{t}, \quad \frac{\partial^2 \pi^1}{\partial P_i^2} \frac{\partial^2 \pi^1}{\partial F_i^2} = \left( \frac{\partial^2 \pi^1}{\partial P_i \partial F_i} \right)^2 = -\frac{M^2 (1 - \gamma \theta)^2}{4t^2}
\]

with analogous expressions for firm 2. The Hessian of the profit function has a determinant that is always negative; hence, second order conditions would never be satisfied. The open-loop game, in which firms choose prices and bribes simultaneously, is not well-behaved in that the first order condition approach does not work. However, the following two lemmas will simplify things importantly, both here and in the 1-FFP case. Proofs can be found in Appendix A.

**Lemma 1:** When \(\gamma > 1/\theta\) (or equivalently \(\gamma > 1\)), if firm \(i\) has an FFP, the Nash equilibrium must involve \(F_i = 0\) and hence, equilibrium prices and profits will be as in the 0-FFP case (Proposition 1). While the airline(s) may formally have an FFP, they will choose to set the level of benefits to zero because they are too costly. The intuition behind this lemma is straightforward. Airlines attract new business by making their offering more attractive. To put a dollar benefit into a buyer’s pocket using the FFP will require an increase in \(F_i\) of $1 which will cost the airline \(\gamma\). To put a dollar into the buyer’s pocket by lowering price, the price will have to fall by an amount \(1/\theta\) (since the buyer only benefits by \(\theta\) when price falls by $1). Therefore, if \(\gamma > 1/\theta\), it is less costly to the airline to use lower prices to attract business then to use the FFP and it will choose to set \(F_i = 0\).
The following lemma describes the choices firms will make when it is less costly to use the FFP to attract business than lower prices. Firms will use FFPs as much as possible, raising prices up to the employer’s reservation value.

**Lemma 2**: When $\gamma < 1/\theta$ (or equivalently, $\gamma \theta < 1$), if firm $i$ has an FFP, the Nash equilibrium must involve $P_i = V$.

Therefore, Lemma 1 tells us that we do not need to worry about the case in which $\gamma \theta > 1$. The outcome is known from Proposition 1. When $\gamma \theta < 1$, Lemma 2 tells us that both firms will set their prices to $V$, and hence we only need to worry about their choice of $F$. This is convenient as the second order conditions for $F$ do hold, $\partial^2 \pi^i / \partial F_i^2 < 0$, and hence a first-order approach is feasible. Taking the first order conditions, $\partial \pi^i / \partial F_i = 0$, and solving for the equilibrium we obtain:

$$F_i^* = \frac{V - t \gamma}{\gamma}$$

Comparative statics reveal that:

$$\frac{dF_i^*}{dV} = \frac{1}{\gamma}, \quad \frac{dF_i^*}{dt} = -1, \quad \frac{dF_i^*}{d\gamma} = -\frac{V}{\gamma^2}, \quad \frac{dF_i^*}{d\theta} = 0$$

These results tell us that the level of frequent flier plan benefits ($F$) is:

(i) increasing in the reservation price ($V$);

(ii) decreasing in the degree of differentiation ($t$);

(iii) decreasing in the cost of the FFP program ($\gamma$); and

(iv) independent of the degree to which workers pay for their flights ($\theta$).

The intuition behind observation (iv) is that, since $F$ is a direct transfer to workers, the proportion of price paid by the workers does not affect airlines’ ability to use
F to capture workers. As we shall see below, price does not depend directly on $\theta$ either, although the range of $\theta$ influences whether price is equal to the reservation price. Note that, if $\gamma \geq 1/\theta$, then $F_1 = F_2 = F = 0$ and $\pi_1 = \pi_2 = \pi' = \frac{MV}{2}$; the FFPs are, again, (endogenously) inactive and we are back into the 0-FFP case, just as when $\gamma \theta > 1$. If $\gamma < 1/\theta$, then $F_i^* = \frac{V - t\gamma}{\gamma}$, which leads to $\pi_1 = \pi_2 = \pi' = \frac{tM\gamma}{2}$.

Finally, in the case when $\gamma \theta = 1$, the profits would be exactly as in $\gamma \theta < 1$ but we would have a multiplicity of equilibria; many $(P, F)$ pairs would sustain those profits (this can be easily checked from the proofs of Lemmas 1 and 2). Restricting our attention to the Nash equilibrium with higher prices in the case $\gamma \theta = 1$, we can summarize the 2-FFP case in the following proposition:

**Proposition 2:** In the 2-FFP subgame:

When $\gamma \leq \min\left\{\frac{1}{\theta}, \frac{1}{\theta}\right\}$:

\[ P_1 = P_2 = P^f = V, \quad F_1 = F_2 = F = \frac{V}{\gamma} - t, \quad \text{and} \quad \pi_1 = \pi_2 = \pi' = \frac{tM\gamma}{2} \]

When $\gamma > \min\left\{\frac{1}{\theta}, \frac{1}{\theta}\right\}$:

If $\theta < \frac{1}{\theta}$, $P_1 = P_2 = P^f = V, \quad F_1 = F_2 = F = 0$, and $\pi_1 = \pi_2 = \pi' = \frac{MV}{2}$

If $\theta > \frac{1}{\theta}$, $P_1 = P_2 = P^f = t/\theta, \quad F_1 = F_2 = F = 0$, and $\pi_1 = \pi_2 = \pi' = \frac{tM}{2\theta}$
Graphically, Proposition 2 leads to the following distribution of profits:

Notice that when $\gamma=1$, the profits of the airlines are equal to the profits that would be observed in a typical Hotelling equilibrium in which there was no moral hazard (i.e. $\theta=1$ and no FFP). That is, all the extra profits that could have been made from exploitation of the third-party payer feature (as in the 0-FFP case) are dissipated. Moreover, when $\gamma<1$ profits are even lower than in the typical Hotelling model. Not only are the extra profits stemming from the third-party payer feature dissipated, but some of the profits due to differentiation are lost as well. In the extreme case in which $\gamma=0$, in fact, profits are zero, just as if there were no differentiation at all (i.e. $t=0$). Lower profits are realized despite the fact that prices are now higher than with no FFPs. When employees do not bear the entire cost of a flight ($\theta<1$), competition is softer than in the typical Hotelling model.
The agency problem leads to higher prices, even in the absence of FFPs. Since FFP benefits are a direct transfer to workers, the addition of FFPs as an instrument intensifies the competition between airlines. The result is, from the airlines’ perspective, more like typical price competition, since the extra revenues from higher prices are given away through the FFPs. When both airlines offer FFPs, both the airlines and the employers are worse off (prices are higher but profits are lower); only the employees benefit. This result that FFPs can intensify competition contrasts sharply with the view of FFPs from the switching cost literature which suggests such plans will reduce competition and raise airline profits.

The effectiveness of this added competitive instrument depends on its cost – and airlines are better off when it is less efficient. We see here that airlines’ profits increase with $\gamma$: the more expensive the FFP program, the higher are airline profits. Hence, when airlines have competitive FFP programs, they would prefer them to be expensive;\(^{16}\) any small increment in the cost of the FFPs would imply that the firm would choose smaller bribes in equilibrium, softening the competition between them. If $\gamma$ is high enough, bribes can be driven to zero. But in the best case, the airlines would achieve the same profits they would get if the FFPs were not feasible: comparing $\pi^f$ to $\pi$, it is easy to see that the latter weakly dominates the former. They are equal only when $\gamma \geq \min\left\{\frac{1}{\theta}, \frac{1}{\bar{\theta}}\right\}$. We formalize this in the following proposition.

**Proposition 3:** Equilibrium profits in the 0-FFP case weakly dominate equilibrium profits in the 2-FFP case, that is $\pi \geq \pi^f$. $\pi^f$ is equal to $\pi$ if and only if

\(^{16}\) More precisely, they would prefer that their rival’s FFP is expensive. Here, however, both FFPs have the same cost parameter and in this case airlines are better off when both have more costly programs.
\[ \gamma \geq \min \left\{ \frac{1}{\theta}, \frac{1}{\bar{\theta}} \right\} \] (and, in this case, the level of FFP benefits will be set equal to zero).

In particular, if workers do not pay the full amount of their ticket \((\theta < 1)\), the absence of FFPs will be always more profitable if \(\gamma \leq 1\).

The last aspect of the proposition, that \(\bar{\pi} > \pi'\) if \(\gamma \leq 1\), implies that, even if FFPs are inexpensive for airlines \((\gamma < 1)\), they cannot be used effectively as ‘money pumps’ because competition precludes it.

**III.3 1-FFP Case**

When only one firm offers an FFP, we assume, without loss of generality, that firm 1 is the one that offers an FFP. Since the market is still fully covered, but only firm 1 has the additional pricing instrument, the demands are given by:

\[
D_1(P_1, P_2, F_1) = Mz(P_1, P_2, F_1)
\]

\[
D_2(P_1, P_2, F_1) = M(1 - z(P_1, P_2, F_1))
\]

and profits are

\[
\pi^1 = Mz(P_1, P_2, F_1)(P_1 - \gamma F_1)
\]

\[
\pi^2 = M(1 - z(P_1, P_2, F_1))P_2
\]

where the indifferent consumer is \(z(P_1, P_2, F_1) = \frac{1}{2} + \frac{\theta(P_2 - P_1) + F_1}{2t}\).

Checking again the concavity of the profit functions we have that
\[
\frac{\partial^2 \pi^1}{\partial P_1^2} = -\frac{M\theta}{t}, \quad \frac{\partial^2 \pi^1}{\partial F_1^2} = -\frac{M\gamma}{t}, \quad \frac{\partial^2 \pi^1}{\partial P_1^2} \frac{\partial^2 \pi^1}{\partial F_1^2} = \left(\frac{\partial^2 \pi^1}{\partial P_1^2 \partial F_1^2}\right)^2 = -\frac{M^2 (1-\gamma\theta)^2}{4t^2}
\]

Again, second order conditions will never be satisfied for firm 1. As before, the open-loop game, where firms simultaneously choose prices and firm 1 chooses F_1 is not well-behaved in that the first order condition approach can be used only for firm 2. But, Lemmas 1 and 2 above apply, meaning that we do not need to worry about the case in which \(\gamma\theta > 1\). The outcome is known from Proposition 1. When \(\gamma\theta < 1\) we know that \(P_i = V\) and we have now a game with two decision variables, in which the second-order conditions do hold. First order conditions lead to:

\[
P_2^* = \frac{3t\gamma - V(1-\gamma\theta)}{3\gamma\theta}, \quad F_1^* = \frac{V(2 + \gamma\theta) - 3t\gamma}{3\gamma} \quad \text{and} \quad z^* = \frac{V(1-\gamma\theta) + 3t\gamma}{6t\gamma}
\]

Now that we have an asymmetric solution, the possibility of corner solutions is considerably greater. Here, there are three possible corners that may be hit:

1. \(P_2^* < 0\) and \(z^* > 1\) if \(\gamma < g_1(\theta) = \frac{V}{3t + V\theta}\)

2. \(P_2^* > V\) if \(\gamma > g_2(\theta) = \frac{V}{3t - 2V\theta}\), for \(\theta \in [0, 3\theta/2]\).

3. \(F_1^* < 0\) if \(\gamma > g_3(\theta) = \frac{2V}{3t - V\theta}\), for \(\theta \in [0, 3\theta]\).

It is straightforward to establish the properties of \(g_1, g_2\) and \(g_3\) in order to define the four regions in Figure 2. For the purpose of illustration here, we make the additional
assumption that $V/t < 3/2$, which simply establishes that the intercepts in the figure are all less than 1.\footnote{It is easy to verify that: (i) the intercept of $g_3$ is the largest and the three intercepts are below $1/\theta$. If $V/t > 3$, then the three intercepts would be above 1, if $3 > V/t > 3/2$ then only $g_3$’s intercept would be above 1, an if $V/t < 3/2$ the three intercept would be below one (note that when $V$ is big, $\theta$ is small). (ii) $g_1$ lies below $g_3$ and $\gamma = 1/\theta$ in the interval $[\theta, \theta]$ and (iii) $g_2$ and $g_3$ intersect exactly when $\theta = \theta$ and, at that point, they achieve a value of $\gamma$ that is exactly equal to $1/\theta$. Assuming arbitrarily that $V/t < 3/2$ and, hence, that the intercepts are below one, the curves look like those in Figure 2.}

![Figure 2: Regions in the 1-FFP Case](image)

Interior solutions are found in region B and correspond to $P_1 = V$, $0 < P_2 < V$, and $F_1 > 0$.

In region A, $P_2 = 0$ and $z = 1$. Firm 2 is then effectively out of the market. In the limit, when $\gamma \to g_1(\theta)$ from above and $P_2 \to 0$ and $z \to 1$, $F_1 > 0$. This would be the equilibrium in region A. However, if firm 2 can decide whether to enter or not, it
would not, since it would not expect to make positive profits. Firm 1 would then set $F=0$ because it would get all the demand and earn $\pi_1 = MV$. For the full game, the only thing that matters is that in this region, firm 1 does not have actual competition, so it makes the largest profits and firm 2 makes nothing.

The boundary between regions C and D is actually not relevant, because when moving from interior solutions (region B) to corner solutions, the frontier between B and C, $g_2(\theta)$, is hit first. Above $g_2$, firm 2 finds it optimal to choose $P_2 = V$. But firm 1, despite charging $V$, may still find it optimal to choose $F_1 > 0$ because that allows it to attract more demand. That value of $F_1$ is obtained by optimizing with respect to $F_1$ given $P_1 = P_2 = V$ for both firms. We then obtain $F_1 = \frac{V - \gamma \theta}{2 \gamma}$. Note that $F_1 > 0$ as long as $\gamma < 1/\theta$. Otherwise, the FFP is too expensive, and firm 1 does not want to use its FFP. It sets $F = 0$ and both firms charge prices of $V$.

If $\gamma \theta = 1$, the profits would be the same as for $\gamma \theta < 1$ but we would have multiplicity of equilibria. Restricting our attention to the Nash equilibrium with higher prices when $\gamma \theta = 1$, and recalling that when $\gamma \theta > 1$ the outcome is as in the 0-FFP case (Lemma 1), we can summarize the 1-FFP case in the following proposition:

**Proposition 4**: In the 1-FFP subgame:

When $\gamma < g_1(\theta)$:

$$P_1 = V, P_2 = 0 \text{ and } F_1 = 0, \pi_1 = MV \text{ and } \pi_2 = 0$$

When $g_1(\theta) \leq \gamma < \min\{g_2(\theta); 1/\theta\}$:
\[ P_1 = V, \quad P_2 = \frac{3t\gamma - V(1 - \gamma\theta)}{2\gamma\theta} \quad \text{and} \quad F_1 = \frac{V(2 - \gamma\theta) - 3t\gamma}{3\gamma} \]

\[ \pi_1 = \frac{M(V(1 - \gamma\theta) + 3t\gamma)^2}{18t\gamma} \quad \text{and} \quad \pi_2 = \frac{M(3t\gamma - V(1 - \gamma\theta))^2}{18t\gamma^2\theta} \]

When \( g_2(\theta) \leq \gamma < 1/\underline{\theta} \), for \( \theta \in [0, \underline{\theta}] \):

\[ P_1 = V, \quad P_2 = V \quad \text{and} \quad F_1 = \frac{V - t\gamma}{2\gamma} \]

\[ \pi_1 = \frac{M(V + t\gamma)^2}{8t\gamma} \quad \text{and} \quad \pi_2 = \frac{MV(3t\gamma - V)}{4t\gamma} \]

When \( \gamma > \min\{1/\Theta, 1/\underline{\theta}\} \):

If \( \theta < \Theta \), \( P_1 = P_2 = P^{\dagger} = V \), \( F_1 = F_2 = F = 0 \), and \( \pi_1 = \pi_2 = \pi^{\dagger} = \frac{MV}{2} \)

If \( \theta > \Theta \), \( P_1 = P_2 = P^{\dagger} = t / \theta \), \( F_1 = F_2 = F = 0 \), and \( \pi_1 = \pi_2 = \pi^{\dagger} = \frac{tM}{2\theta} \)

Graphically, Proposition 4 leads to the following distribution of profits:
III.4 Equilibria of the Full Game

We can now determine the equilibrium of the full game. To do this we simply compare the profits earned by each airline in the 0-FFP, 1-FFP and 2-FFP scenarios outlined above and ask whether firms, anticipating these levels of profits, would elect to create FFPs. This analysis is straightforward but tedious and is found in Appendix B. Here we simply summarize the results.\(^{18}\)

\[\begin{align*}
\pi_1 &= MV \\
\pi_2 &= 0 \\
\pi_{1,2} &= \frac{M(1-\gamma(1-\gamma))}{8t\gamma} \\
\pi_1 &= \frac{M(V+\gamma)}{8t\gamma} \\
\pi_2 &= \frac{M(V-3t\gamma-V)}{4t\gamma} \\
\end{align*}\]

---

\(^{18}\) For each \((\theta, \gamma)\) point at stage 1 we have a simple two-by-two matrix game, in which the strategies available to each airline are to create a FFP or not to do so. The Nash equilibrium choice at this stage will determine the course of the full game.
Proposition 5: In the full game, in which airlines choose whether to offer FFPs in the first period and compete in prices and FFP benefits in the second period:

(i) If $\gamma$ is sufficiently low (region H), both airlines offer FFPs, but profits are lower than they would be if neither airline offered an FFP (prisoner’s dilemma).

(ii) If $\gamma$ is intermediate (region I), one airline offers an FFP, as this is more profitable given that its competitor does not offer an FFP. It is not profitable for the second airline to offer an FFP also (battle of the sexes).
(iii) If $\gamma$ is sufficiently high (region J), there are effectively no FFPs. Zero, one, or two FFPs are all equilibria, but in all cases FFP benefits are set to zero.\footnote{If there were any small set-up cost for a FFP, only the 0-FFP equilibrium would survive.}

Thus we see that in the region (H) which includes the less costly FFPs and the more heavily subsidized travel for workers, we are most likely to find both airlines creating FFPs but, as in the prisoner’s dilemma, suffering lower profits than if neither of them had done so. As the FFP becomes more costly, it is less effective as a competitive tool and we enter region I, where only one firm elects to have a FFP and then into J where neither airline adopts an FFP.

The size of these regions clearly depends on the parameter values in the model, the ratio of $V/t$ being most critical. As can be seen in Figure 4, as $V/t$ increases, both the border between H and I, and the border between I and J rise, while $\theta$ approaches 0. This expands the region (in $\gamma$-$\theta$ space) in which the 2-FFP prisoner’s dilemma holds. If $V/t$ is sufficiently high, the full length of the line $\gamma = 1$ will be in region H, implying that even if the FFP is costless redistribution it may lead to a prisoners’ dilemma and lower profits for both airlines (and in that case, area I would be tall but thin).

It is interesting to contrast these results with those that we would obtain were we to remove the first stage of the game and simply let the two firms chose their prices and frequent-flier benefits simultaneously, permitting them to set $F_i = 0$ if they wished. In our model, the first stage does not allow a firm to commit to offering an FFP since it can set $F_i = 0$ in the second stage, but it does permit firms to commit \textit{not} to offer an FFP, and there is value to that commitment. Without the ability to commit, the game would be...
exactly as in our subgame with 2 FFPs. Hence, the entire region I in Figure 4 in which the
two-stage game leads to an asymmetric battle-of-the-sexes outcome with one FFP will be
added to the range of two-FFP prisoner dilemmas, as in Figure 1. In this region, the
action of one airline to commit not to employ an FFP softens competition and benefits
both airlines.

IV. Discussion and Conclusions

Frequent flier plans, now almost universal among the world’s major airlines, may
serve many purposes, including price discrimination and establishing switching costs.
Similar kinds of loyalty plans exist in other industries, most notably the hotel business.
In this paper we have explored one aspect of FFPs that had not been formally modeled by
economists before as far as we know. Our focus has been on the role that FFPs can play
in helping airlines take advantage of the agency relationship between employers who pay
for tickets and employees who select the airline. Since the employee is the one to receive
the FFP benefit, the plan provides a mechanism through which airlines can “bribe”
employees to select them over possibly lower-priced rivals. Our results indicate that,
consistent with survey reports in the transportation literature, FFPs can lead to higher
prices to employers and lower profits for airlines.20 Significantly, this contrasts with the
“switching cost” approach in which the FFPs raise price and airline profits. We also
showed that more costly FFPs may actually lead to higher profits than less costly plans,

20 On the view that FFPs may be more trouble than they are worth (perhaps suggesting they may be a
prisoner's dilemma) see, for example Hu et al [1988] and especially Kearney [1990].]
as they provide less efficient ways for firms to compete. The inefficiency of the FFPs mitigates the degree to which FFPs can intensify competition.

It is interesting to consider why the moral hazard associated with purchases for business purposes has led to these programs for air travel, hotels and car rentals, but not to any great extent for all the other inputs that employees order and their employers pay for. We offer two thoughts on this point. For most of the significant purchases a firm makes there will be specific employees tasked with doing the purchasing – a purchasing department, for example. When purchasing is concentrated, while bribery is still possible, the monitoring of employees is less costly. In the case of employee travel, however, purchases are made on a more distributed basis: that is, individual employees often arrange their own travel. This may well be efficient because the right choice of flight or hotel will often depend on personal characteristics of the traveler such as what time of day he or she is available to fly. With so many different people making purchasing decisions, monitoring their behavior is more costly. It may also be the case that loyalty programs of this type may be less costly for airlines, which can provide benefits out of excess capacity at a lower cost (i.e. airlines may have a lower $\gamma$) than firms in other industries.

There is much additional work that can be done in this area. We suggest a few directions here. First, on the assumption that airlines can discriminate in price between those flying on business (at the expense of their employer) and leisure travelers (people paying for their own seats), we have included only workers in our model. It would clearly be valuable to consider a model in which this market segmentation is not so perfect so that the market studied included some leisure travelers as well. Preliminary
work in this direction suggests that the qualitative nature of the results is largely the
same, such as the existence of battle-of-the-sexes and prisoner’s-dilemma equilibria. One
complication is that, when both airlines offer FFPs, prices and FFP benefits can be
decreasing in $\theta$ (rather than being independent of $\theta$ as in the all-worker case). The
presence of leisure travelers (i.e. consumers for whom $\theta = 1$) gives airlines some
incentive to lower price below $V$, and $F$ is adjusted accordingly. This incentive is
increasing in $\theta$ (and decreasing in the relative proportion of workers). These results
clearly hold in some region of the parameter space; however, the concavity issues that
arise in the all-worker case become much more severe in the model that includes leisure
travelers. Since the model with leisure travelers is far less tractable and does not appear
to offer any crucial insights that the all-worker model lacks, we have excluded leisure
travelers from our analysis.

Second, we could explore the implications were airlines to have different costs
associated with FFPs. It has been argued that airlines with more extensive networks are
at a competitive advantage with respect to delivering FFP benefits to travelers because
the travelers have more choices of places to fly. In our model this would translate into
allowing one firm to have a $\gamma$ parameter smaller than that of the other firm. Our study of
the 1-FFP subgame hints at what we might find: we suspect that when one airline has an
advantage of this kind, it will translate into higher profits.

Third, we have not really explored the welfare consequences of FFPs in this
model, in part because the unit nature of demand, and the fact that the market is always
fully covered, makes them somewhat less interesting. In this model, when $\gamma = 1$ (so that
benefits are costless transfers), as long as everyone is flying with the closest airline, we
have full efficiency. Another direction in which to take this work would then be to allow travelers (or their employers) to have downward-sloping demand curves for flights. Workers could then select the airline, but the employer would determine how many flights they could take at that airline’s price. The inflation in airline prices we have observed here would then have some efficiency consequences as prices above marginal costs create deadweight loss.

Fourth, we have not integrated tax considerations into our analysis and this could be useful. The fact that travelers receive a benefit that is typically not taxed has a number of implications. To the extent that such tax-free benefits are incorporated into compensation packages with correspondingly lower taxable wages, employers have a reason to favor FFPs. In this case, the plans induce another type of wealth transfer, from taxpayers to firms and employees.

Beyond extensions to the theoretical framework employed here, we believe it would be valuable to get more systematic empirical evidence of the extent to which the moral hazard at the core of our model is important. It would also be useful to explore the extent to which the effects we have described here exist in other markets such as hotels and car rentals. We think it is no coincidence that these very large loyalty programs exist in areas in which a large fraction of purchases are work-related and therefore where these moral hazard problems present themselves. We thus have perhaps a more compelling explanation for the presence of FFPs than those previously offered, since we can also explain why similar programs do not exist in many industries.

21 The efficiency story is a little more complicated if $\gamma > 1$ or if $\gamma < 1$. In the former case, any use of the FFP at all is inefficient in that the costly transfers lower total surplus. In the latter case, FFPs generate welfare themselves (since the benefits are valued more than the associated costs) and in this respect the use of the FFPs creates wealth. Of course, if they lead fliers to travel with the “wrong” airline they would create some offsetting costs.
Appendix A

Proofs of Lemmas 1 and 2

Lemma 1: Let us consider firm 1. We will show that, if $F_1 > 0$, firm 1 has a profitable unilateral deviation. Firm 1’s profit is given by $\pi_1 = Mz(P_1 - \mathcal{F}_1)$. A deviation $(dP_1, dF_1)$ yields the following change in profits:

$$d\pi_1 = \frac{\partial \pi_1}{\partial P_1} dP_1 + \frac{\partial \pi_1}{\partial F_1} dF_1 = M \left[ \left( \frac{\partial z}{\partial P_1} (P_1 - \mathcal{F}_1) + z \right) dP_1 + \left( \frac{\partial z}{\partial F_1} (P_1 - \mathcal{F}_1) - \gamma \cdot z \right) dF_1 \right]$$

$$d\pi_1 = M \left[ \left( -\frac{\theta}{2t} (P_1 - \mathcal{F}_1) + z \right) dP_1 + \left( \frac{1}{2t} (P_1 - \mathcal{F}_1) - \gamma \cdot z \right) dF_1 \right]$$

Now, consider a deviation such that $dP_1 = dF_1 / \theta$. In that case, we get:

$$d\pi_1 = \frac{z \cdot dF_1 (1 - \gamma \theta)}{\theta}.$$ Hence, if $\gamma \theta > 1$, $d\pi_1$ can be made positive by a deviation that involves $dP_1 = dF_1 / \theta$ and $dP_1$ and $dF_1$ negative, which is possible if $F_1 > 0$. The proof for firm 2 is analogous.

Lemma 2: Let us consider firm 1. We will show that, if $P_1 < V$, firm 1 has a profitable unilateral deviation. From the proof of lemma 1, we know that a deviation $dP_1 = dF_1 / \theta$ yields the following change in profits for firm 1: $d\pi_1 = \frac{z \cdot dF_1 (1 - \gamma \theta)}{\theta}$. Hence, if $\gamma \theta < 1$, $d\pi_1$ can be made positive by a deviation that involves $dP_1 = dF_1 / \theta$ and $dP_1$ and $dF_1$ positive, which is possible if $P_1 < V$. The proof for firm 2 is analogous.
Appendix B

Equilibrium of the Full Game

The full game has two stages: (1) airlines decide whether or not to offer FFPs; (2) they choose prices and FFP benefits. We want to find the equilibria in each of these regions:

\[ \gamma_1 \theta_1 = \theta \]

\[ \gamma_1 \theta_1 V_t = \theta_1 / V_t \]

Figure 5: Regions for equilibria of the complete game

As discussed in Section 3.3, given our assumption that \( V/t < \frac{3}{2} \), the common intercept of \( g_1 \) and \( g_2 \), \( V/3t \), will be below one half.

Using Figures 1 and 3 we can easily see that in region A, the equilibrium is 2-FFP and is a prisoner’s dilemma by virtue of Proposition 4; and that in regions E-G, the
equilibria are 0-FFP, or 1-FFP or 2-FFP, but the F’s are zero. Note that, if there is any small set-up cost for an FFP, the equilibrium would be 0-FFP for regions E-G.

In regions B, C, and D it is evident that \( \pi_1^{1-FFP} > \pi_1^{0-FFP} > \pi_1^{2-FFP} \). The first inequality follows from the fact that \( F_1 > 0 \); if the profits for the firm with the FFP are smaller than when it does not have an FFP, the firm can always set \( F \) equal to zero and make the same profits as in the 0-FFP case (as in regions E and G). The second inequality follows from Proposition 4.

Second, from Proposition 4 we know that \( \pi_2^{0-FFP} > \pi_2^{2-FFP} \) in the three regions.

Third, along the curve \( \gamma = g_1(\theta) \), we have that \( \pi_2^{1-FFP} = 0 \)

Now, consider region B. It is easy to see that \( \pi_2^{1-FFP}(\gamma = 1/\theta) = \frac{tM}{2}\theta \). Therefore, in region B we know that \( \pi_2^{1-FFP} < \pi_2^{0-FFP} \). But since \( \pi_2^{1-FFP} \) varies from 0 to \( \frac{tM}{2}\theta \) as \( \gamma \) varies from \( g_1 \) to \( 1/\theta \), there is a locus inside region B in which \( \pi_2^{1-FFP} = \pi_2^{2-FFP} \). This locus is given by:

\[
\gamma_B(\theta) = \frac{6tV + V^2\theta + \sqrt{V^3\theta(12t + V\theta)}}{18t^2}
\]

which is upward sloping in \( \theta \). It intersects \( \gamma = \frac{1}{\theta} \) exactly at the vertical asymptote of \( g_2 \), which is located at \( \theta = \frac{3t}{2V} \). This is to the right of \( \theta = 1 \) if \( \frac{V}{t} < \frac{3}{2} \) and hence, \( \gamma_B \) intersect \( \theta = 1 \) below \( \gamma = 1 \). Therefore, region B is divided into two sub-regions: an upper region in which \( \pi_2^{1-FFP} > \pi_2^{2-FFP} \), and a lower region in which \( \pi_2^{1-FFP} < \pi_2^{2-FFP} \).
Region C is similarly divided into an upper and a lower region, by the same locus. The locus intersects $g_2$ at $\frac{V}{2t}$, which is evidently below $\gamma = 1$ (see figure 2 or 3).

Given the inequalities of profits in the lower and upper regions inside B and C, we know that the equilibrium in the lower regions is that both airlines offer FFPs, and it is a prisoner’s dilemma; in the upper regions, the equilibrium is that one firm offers an FFP and it is a battle of the sexes.

Finally, consider region D. Since $\pi_1^{1-\text{FFP}} > MV / 2$ and the maximum joint profit is $MV$, it follows that $\pi_2^{(1-\text{FFP})} < MV / 2 = \pi_2^{0-\text{FFP}}$. Noting that $\pi_2^{1-\text{FFP}}$ does not depend on $\theta$ in region D, and that $\pi_2^{1-\text{FFP}} (\gamma = V / 3t) = 0$ and $\pi_2^{1-\text{FFP}} (\gamma = \theta) = MV / 2$, we see that there is a locus inside region D in which $\pi_2^{1-\text{FFP}} = \pi_2^{2-\text{FFP}}$. This locus is given by:

$$\gamma_D = \frac{V}{2t}$$

which obviously connect with $\gamma_B$ at $g_2$. The equilibrium of the game is thus as given by Proposition 5 and Figure 4.
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