Finite-Life Private Information Theory of Unsecured Debt

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1 Motivation

In this paper we construct a theory of unsecured borrowing in the presence of enforcement problems (i.e. a bankruptcy law). Our theory is constructed for life cycle agents which excludes the possibility of having trigger strategies as an enforcement mechanism. Moreover, the environment is such that agents cannot be excluded from market activities based on payoff irrelevant actions, in other words agents who do not pay back their debt cannot be excluded from contracting based on whether they have defaulted because free entry of firms guarantees that if there are gains from trade a firm would offer these agents a contract. Finally, our theory is not based on getting disutility directly from not honoring contracts: there is no stigma attached to any behavior.

So what is our theory based upon? It is based on the existence of private information about various economic characteristics including type and on agents using payback behavior to SIGNAL the type. Agents that look to be of the good type are rewarded in the market, and good agents are more capable of taking actions that make them look good. In particular, in our model agents differ in patience (good guys are the more patient guys) and in the likelihood of a loss. The latter feature being an outcome of more patient agents undertaking costly activities that make them be better insurance risks than impatient agents. Patience makes good insurance risk agents value more the future than the present which gives them an easier time given signals of being good types. This signal take the form of credit repayment. In our economy there is no credit if all types are equal or if type is public information. Our theory matches a plethora of qualitative features of the U.S. economy. Bankruptcy is both legal and pervasive. Moreover in states with a high homestead threshold (Texas Florida) access to credit do not seem to be much more difficult than in the rest of the country despite providing essentially the unilateral right to default at no cost while saving sizeable amounts in the form of housing wealth. Credit histories are used both by the credit industry and by the insurance industry (car and home insurance) as indicators of good performance. People with good credit histories do have better credit terms, do face cheaper insurance premiums,
and in turn default less often and file substantially less claims.

Needless to say, our model economy also replicates the feature that all agents die in finite time.

2 Environment

There is a single good. There are 3 periods, denoted \( t = 1, 2, \) and 3 and a unit measure of people. We will describe people who live in this economy, the legal environment they face, the market arrangement and the timing of events.

2.1 People

There are two types of people, denoted \( i = g, b. \) The measure of type \( g \) is \( 0 < \gamma < 1. \) The preferences of each type is given by

\[
E u^i(c_1) + \beta^i u^i(c_2) + (\beta^i)^2 u^i(c_3)
\]

where \( c_t \) is consumption in period \( t \) of the single good. Thus, type is assumed to affect a person’s discount factor and his momentary utility function. For each type, \( u^i \) is taken to be strictly concave and twice continuously differentiable.

The endowment of a person is random and is drawn in an \( iid \) fashion from a time-invariant common distribution with cdf \( F(e) \). The support of \( F \) is taken to be \([\bar{e}, \bar{v}]\). We denote by \( e_t \) a person’s earnings in period \( t \).

In period 3, a person of type \( i \) faces a probability \( \pi^i > 0 \) of experiencing a loss in wealth \( L > 0 \). Thus, the probability of loss is also type-dependent. We will denote the loss incurred by a person in period 3 by \( z \in \{0, L\} \).

Finally, for each person the person’s type, earnings and consumptions are all assumed to be
private information.

2.2 Legal Environment

A key feature of the environment is the existence of a bankruptcy law. This law gives people the right to disavow their financial obligations. As is generally true for actual bankruptcy law, this “right to bankruptcy” is assumed to be inalienable – meaning that a debtor cannot waive his or her right to bankruptcy.

We assume a person’s asset position is observable and that a person who invokes the right to bankruptcy is not permitted to leave that period with any assets. The motivation for this assumption is that in a real bankruptcy proceeding a person is required to relinquish available assets (with some exceptions) to creditors when obtaining discharge of debt. Accumulation of assets in the period of bankruptcy is, therefore, inconsistent with a creditors’ right to seize assets in that period.

For simplicity, we assume that invoking the right to bankruptcy does not cost the debtor any fees or expenses.

2.3 Market Arrangements

There are two sets of markets. In one market people borrow from or lend to banks and in the other market people purchase insurance against the loss $L$. Since household type is private information, and type will matter for the propensity of a person to declare bankruptcy or suffer a loss, banks and insurance companies must make an assessment of a person’s type when selling a loan or an insurance policy. We will study a market structure that permits the terms of financial contracts to depend on such an assessment. We think of the assessment as being supplied by an information-processing agency that keeps track of a person’s financial history.
The asset market operates in periods 1 and 2 - there is obviously no role for an asset market in period 3 since no one lives beyond period 3. In each period, the loan market offers a finite set of one-period loan contracts \( L = \{\ell_1, \ell_2, \ldots, \ell_N\} \). Price of each loan depends on an individual’s financial history. Prior to initiating a new transaction with a person in period \( t \), banks obtain an assessment of the person’s type from the information processing agency. This assessment, denoted \( s^B_t \), is the probability that the person is of type \( g \). Banks also obtain from the information-processing agency a person’s beginning of period asset position \( \ell_t \). For each assessment and asset position, the market offers a set of loan prices \( \{q(\ell_1, \ell_t, s^B_t), q(\ell_2, \ell_t, s^B_t), \ldots, q(\ell_N, \ell_t, s^B_t)\} \), where a person who purchases the contract \( q(\ell_n, \ell_t, s^B_t) \) pays \( q(\ell_n, \ell_t, s^B_t) \cdot \ell_n \) in period \( t \) and receives \( \ell_n \) in period \( t + 1 \). The “loan” \( \ell \) could be positive or negative, where a positive \( \ell \) signifies a deposit and a negative \( \ell \) signifies a loan.

The insurance market works in a similar fashion. Prior to initiating a new transaction with a person in period 3, insurance companies obtain an assessment of the person’s type. Denote this assessment by \( s^I_3 \) and it is, once again, the probability that the person is of type \( g \). In addition to this assessment insurers also obtain information on a person’s wealth \( \ell_3 \). For each pair of assessment and wealth, the market offers a finite set of insurance contracts \( I(\ell_3, s^I_3) = \{p(x_1, \ell_3, s^I_3), p(x_2, \ell_3, s^I_3), \ldots, p(x_J(\ell_3, s^I_3), \ell_3, s^I_3)\} \). A person who purchases the contract \( p(x_j, \ell_3, s^I_3) \) pays \( p(x_j, \ell_3, s^I_3) \cdot x_j \) as premium and, in the event of loss, collects \( x_j \). One difference between the loan and insurance market is that in the latter the number of insurance contracts offered is permitted to depend on a person’s assessment and wealth – which is why \( J \) depends on \( \ell_3, s^I_3 \).

A financial firm takes the set of contracts \( \{q(\ell_n, \ell, s)\} \) and \( \{(p(x_j, \ell, s)\} \) as given and any given contract is viewed as a distinct financial product. There is free-entry in the provision of each of these financial products. In equilibrium each of these financial products will fetch zero profits in expectations.
3 Decision Problems

In this section we describe the decision problem of people, insurers and banks.

3.1 People

It’s convenient to start with the final period and work backwards.

3.1.1 Period 3

The timing of events in period 3 is as follows. At the start of the period the insurance companies obtain an updated assessment of a person’s type, denoted $s^I_3$ and the person’s wealth $\ell_3$. Next, the person chooses how much insurance to purchase against the loss $\Lambda$. The set of contracts available to choose from is given by $I(\ell_3, s^I_3)$. At the time of this choice it is assumed that each person has access to his or her minimum earnings $e$ and that this minimum earnings is large enough purchase insurance (sufficient conditions for this to be true will be supplied when the equilibrium is discussed). After insurance is purchased the person observes his earnings $e_3$ and his loss $z$. Finally, the person decides whether to pay back any loans and then consumes and dies.

Post insurance market, a person’s decision problem is to choose whether to pay back any loans. Therefore, post-insurance decision problem is

$$\max_{d_3 \in \{0, 1\}} u^i(e_3 + \ell_3(1 - d_3) + z - p(x_3, \ell_3, s^I_3) \cdot x_3 + x_3 \cdot 1_{\{z=\Lambda\}}).$$

We will denote the decision rule for this problem by $d_3(e_3, x_3, z, \ell_3, s_3)$. This decision rule of course takes a very simple form: if $\ell_3 \geq 0$ then $d_3(e_3, x_3, z, \ell_3, s_3) = 0$ and if $\ell_3 < 0$ then $d_3(e_3, x_3, z, \ell_3, s_3) = 1$. Because the default decision is taken after the insurance purchase has been made, there is no cost to defaulting on a loan.
The person's insurance decision problem is as follows.

\[
V^i_3(\ell_3, s^I_3) = \\
\max : \pi^i \int u^i(e + \ell_3(1 - d_3(e, x, \Lambda, \ell_3, s_3) - p(x, \ell_3, s^I_3) \cdot x - \Lambda + x)dF(e) \\
+(1 - \pi^i) \int u^i(e + \ell_3(1 - d_3(e, x, 0, \ell_3, s_3) - p(x, \ell_3, s^I_3) \cdot x)dF(e) \\
\text{s.t.} \\
\ x \in I(\ell_3, s^I_3)
\]

We will denote a type \(i\)'s period 3 decision rule as \(x^i_3(\ell_3, s^I_3)\).

A person’s choice in the insurance market is potentially revealing about a person’s type. This point will be important in the functioning of the insurance market. We denote a person’s post insurance assessment as \(s'_3\) and assume that this assessment is given by

\[
s'_3 = \phi_3(x_3, \ell_3, s^I_3).
\]

3.1.2 Period 2

The timing of events in period 2 is as follows. At the start of the period the banks obtain an updated assessment of a person’s type, denoted \(s^B_2\). Next a person learns of his earnings \(e_2\). A debtor must then choose whether to declare bankruptcy. If the person chooses not to declare bankruptcy, or if bankruptcy is not an option because the the person is not a debtor, the person must choose \(\ell_3\) from the set \(L\). In making this choice the person recognizes that his or her assessment of type at the start of period 3 is influenced by his or her asset market choices in period 2.

If a person of type \(i\) is a debtor and chooses default then the person’s utility is given by

\[
V^{i,1}_2(e_2, \ell_2, s^B_2) = u^i(e_2) + \beta^i : V^i_3(0, s^I_3) \\
s^I_3 = \phi_2(1, 0, \ell_2, s^B_2)
\]
If the person of type $i$ chooses not to default, then utility is given by

$$V^{i,0}_2(e_2, \ell_2, s^B_2) = \max : u^i(c^i_2) + \beta^i : V^i_3(\ell_3, s^I_3)$$

s.t.

$$c^i_2 = e_2 + \ell_2 - q(\ell_3, \ell_2, s^B_2) \cdot \ell_3$$

$$s^I_3 = \phi_2(0, \ell_3, \ell_2, s^B_2)$$

$$c^i_2 \geq 0, : \ell_3 \in L$$

Therefore,

$$V^i_2(e_2, \ell_2, s^B_2) = \max : \{V^{i,1}_2(e_2, \ell_2, s^B_2), : V^{i,0}_2(e_2, \ell_2, s^B_2)\}$$

At the end of the period the person consumes and his choice in the asset market is reported to the information processing agency. We will denote a type $i$’s period 2 decision rules as $d^i(e_2, \ell_2, s^B_2)$ and $\ell^i(e_2, \ell_2, s^B_2)$.

### 3.1.3 Period 1

At the start of the period financial firms obtain the assessment of each person’s type from the information-processing agency. Since no one in period 1 has a prior financial history, we assume that the assessment of each person is simply $\gamma$, the fraction of type $g$ in the population. Therefore

$$s^B_1 = \gamma.$$ 

Thus in period 1 there is only a single “risk category” and for this one risk category there is a set of loan contracts $L(\gamma)$. The person observes his earnings $e_1$ and must choose how much to borrow or save. In making this decision, the person recognizes that his choice will affect his assessment at the start of next period. The decision problem of a person of type $i$
is then

\[ V(e_1, \gamma) = \max : u^i(c_1^i) + \beta^i : V^i_2(\ell_2, s_2^B) \]

s.t.

\[ c_1^i = e_1 - q(\ell_2, \gamma) \cdot \ell_2 \]

\[ s_2^B = \phi_1(\ell_2, \gamma) \]

\[ c_1^i \geq 0, : \ell_2 \in L \]

After a person picks \( \ell_2 \) the choice is reported to the information processing agency. We will denote a person’s \( i \)'s period 1 decision rule by \( \ell_2^i(e_1, \gamma) \).

### 3.2 Insurers

For each risk category \( s_3^l \) and wealth \( \ell_3 \), insurers face a set of insurance contracts \( \{ p(x, \ell_3, s_3^l) \} \).

The decision problem of insurers is to choose how many of these different types of contracts to sell. Clearly, insurers will have an incentive to sell infinite amounts of any contract that makes strictly positive profits in expectation, sell none of the contracts that make strictly negative profits in expectation, and sell any amount of the contracts that make exactly zero profits in expectation. In other words, insurers will participate in selling any contract that makes non-negative profits in expectation. That is if

\[ p(x, \ell_3, s_3^l) \cdot x \geq \phi_3(x, \ell_3, s_3^l) \pi^g \cdot x + (1 - \phi_3(x, \ell_3, s_3^l)) \pi^b \cdot x, \]

where, recall, \( \phi_3(x, \ell_3, s_3^l) \) is the probability that a person of risk category \( s_3^l \) and wealth \( \ell_3 \), who chooses insurance \( p(x, \ell_3, s_3^l) \) is of type \( g \). Eliminating \( x \), this condition reduces to

\[ p(x, \ell_3, s_3^l) \geq \pi^b - \phi_3(x, \ell_3, s_3^l) [\pi^b - \pi^g]. \]
3.3 Banks

In period 1, banks face a set of loan contracts \( q_1(\ell, \gamma) \). As in the case of insurers, the decision problem of banks is to choose how many of these different types of contracts to sell. And, as in the case of insurers, banks will participate in selling only those contracts that make non-negative profits in expectation. For \( \ell < 0 \), non-negative profits requires

\[
q_1(\ell, \gamma) \geq 
\phi_1(\ell, \gamma) [1 - \mu^g_1(\ell, \phi_1(\ell, \gamma))] \frac{\ell}{(1 + r)} + (1 - \phi_1(\ell, \gamma)) [1 - \mu^h_1(\ell, \phi_1(\ell, \gamma))] \frac{\ell}{(1 + r)},
\]

where \( r \) is the risk-free rate available to banks and \( \mu^i_1(\ell, \phi_1(\ell, \gamma)) \) is the probability that a person of type \( i \) whose updated assessment is \( \phi_1(\ell, \gamma) \) will default on a loan of size \( \ell \).

Eliminating \( \ell \), yields the condition

\[
q_1(\ell, \gamma) \geq 
\phi_1(\ell, \gamma) [1 - \mu^g_1(\ell, \phi_1(\ell, \gamma))] + (1 - \phi_1(\ell, \gamma)) [1 - \mu^h_1(\ell, \phi_1(\ell, \gamma))] (1 + r)^{-1}.
\]

For \( \ell > 0 \), non-negative profits require that

\[
q_1(\ell, \gamma) \leq (1 + r)^{-1}.
\]

A similar set of conditions hold for period 2. In this period, for each pair \( \ell_2, s_2 \) banks face a set of contracts \( q_2(\ell, \ell_2, s_2) \). For \( \ell < 0 \), non-negative profits requires

\[
q_2(\ell, \ell_2, s_2) \cdot \ell \geq 
[\phi_2(0, \ell, \ell_2, s_2)(1 - \mu^g_2(\ell, \phi_2(0, \ell, \ell_2, s_2))) + (1 - \phi_2(0, \ell, \ell_2, s_2))(1 - \mu^h_2(\ell, \phi_2(0, \ell, \ell_2, s_2)))] \frac{\ell}{(1 + r)},
\]

where \( \mu^i_2(\ell, \phi_2(0, \ell, \ell_2, s_2)) \) is the probability that a person of type \( i \) who takes out a loan \( \ell \) and has updated assessment of \( \phi_2(0, \ell, \ell_2, s_2) \) defaults. For \( \ell > 0 \), non-negative profits require

\[
q_2(\ell, \phi_2(0, \ell, \ell_2, s_2)) \leq (1 + r)^{-1}.
\]
4 The Structure of Loan and Insurance Contracts

As noted above, participants in the loan and insurance markets take the set of contracts available for trade as given – the only decision that people and financial firms make is whether to participate in trading these contracts. In this section we discuss which contracts are available to be traded.

4.1 Loan Markets

For any risk assessment $s$, the loan market offers the opportunity to trade virtually all contracts with face-values between a minimum of $\ell_{\text{min}} < 0$ and a maximum of $\ell_{\text{max}} > 0$. That is

$$L = \{\ell_{\text{min}}, \ell_{\text{min}} + \frac{\delta}{N}, \ell_{\text{min}} + \frac{2\delta}{N}, \ldots, \ell_{\text{max}}\}, \quad \delta = \frac{\ell_{\text{max}} - \ell_{\text{min}}}{N}$$

where $N$ is a large number. The upper and lower bounds on the set of loan contracts are chosen so as to be non-binding in any equilibrium. $\ell_{\text{max}}$ is chosen to be greater than $\bar{e}(1 + r) + \bar{e}$, which is the highest level of savings possible in this economy, and $\ell_{\text{min}}$ is to be chosen to be less than $\bar{e} + \frac{\bar{e}}{(1+r)}$ which is the most a person can borrow in this economy and hope to pay back. The finiteness of $L$ is assumed for technical simplicity. In what follows we will denote the set of loan contracts described above as $L^*$. 

4.2 Insurance Market

The insurance market offers the opportunity to trade contracts that would be offered if insurers competed to offer the best contracts to their customers. The microeconomic literature on the provision of insurance indicates that in a population with two hidden types, competition among insurers will result in one of two kinds of equilibrium - pooling or separating. In a pooling equilibrium all insurers offer the same full-insurance contract at a price that reflects
the composition of low- and high-risk types in the population. In contrast, in a separating equilibrium insurers offer two contracts, one with limited insurance at a low price designed to attract only the low-risk types and another with full insurance at a higher price for the remaining high-risk types.

A fundamental insight of this literature is that if faced with a pooling contract, insurers have an incentive to entice away the low-risk types by offering them a cheaper but limited-insurance contract. However, as argued in detail in Wilson (1977), this enticement strategy can work only if two conditions are satisfied. First, the low-risk types must (weakly) prefer the best limited insurance contract that can be offered to them to the pooling contract. Second, the high-risk types must (weakly) prefer the best full-insurance contract that can be offered to them to the best limited-insurance contract offered to the low-risk types. This second condition must hold because once the low-risk types are enticed away, the pooling contract becomes unavailable to the high-risk type. Therefore, for the enticement strategy to work the high-risk types must not have an incentive to pool with the low-risk types if offered the best full-insurance contract.

In the context of this paper, this insight translates into the following observation. The kind of insurance contracts a person can participate in will depend on the person’s risk assessment $s^I_3$ and wealth $\ell_3$. Specifically, people with wealth $\ell_3$ and risk assessment higher than a cut-off value $s^* (\text{which will depend on } \ell_3)$ will be offered pooling contracts and people with wealth $\ell_3$ and risk assessment equal to or less than $s^*$ will be offered separating contracts. The reason is that for a low-risk person who belongs to a risk category that is mostly composed of low-risk types (i.e., someone with a high $s^I_3$) the pooling contract is only slightly more expensive than the cheaper, limited-insurance, contract. Consequently, such a person will prefer the slightly cheaper separating contract if the insurance offered is only slightly less than full insurance. But for an “almost-full-insurance” contract, the a high-risk type will prefer to pool with the low-risk types and the second condition noted above for separation will fail. The rest of this subsection develops the observation in detail.
Recall that the probability that a type-$i$ person suffers the loss $L$ is $\pi^i$, where $0 < \pi^g < \pi^b < 1$. For a given $s$ and $\ell$ define $B(\ell, s)$ to be the set of all $\tilde{\Lambda} < \Lambda$ such that (i) the limited insurance contract $p(\tilde{\Lambda}, s)$, with $p = \pi^g$, is weakly preferred by type $g$ with asset $\ell$ to the pooling contract $p(\Lambda, s)$, with $p = \pi^a - s [\pi^b - \pi^g]$ and (ii) the full insurance contract $p(\Lambda, s)$ with $p = \pi^b$ is weakly preferred by type $b$ with asset $\ell$ to the limited insurance contract $p(\tilde{\Lambda}, s)$ with $p = \pi^g$.

Then, for each $\ell, s$, we define the set $I^*(\ell, s)$ as follows. $I^*(\ell, s) = \{(p(\Lambda, \ell, s), with p = \pi^g - s [\pi^b - \pi^g]\}$ unless the set $B(\ell, s)$ is non-empty, in which case $I^*(\ell, s) = \{p(\tilde{\Lambda}, \ell, s)$ with $p = \pi^g and p(\Lambda, \ell, s)$, with $p = \pi^b\}$, where $\tilde{\Lambda} < \Lambda$ is the largest element of $B(\ell, s)$.

This determination of the structure of insurance markets incorporates the logic of Wilson’s argument, namely, that the market offers the pooling contract unless there exists a separating contracts that give the low-risk types at least as much utility as the pooling contract and does not give the high-risk types incentive to pool with the low-risk types.

In what follows, we will assume that $u^g = u^b$ so the only taste difference between the two types is in their discount factors. Now consider the high-risk types (type $b$) with wealth $\ell_3$ and assessment $s_3^I$. Their utility from purchasing the expensive full-insurance contract is given by

$$V^b(L; \ell_3, s_3) = \int u(e + \ell_3 - \pi^b L)dF(e),$$

and their utility from purchasing a cheap but limited insurance contract is given by

$$V^b(X, \ell_3, s_3) = \int [\pi^b u(e + \ell_3 - \pi^g X - \Lambda + X) + (1 - \pi^b)u(e + \ell_3 - \pi^g X)] dF(e).$$

Then, since $\pi^g < \pi^b$, it’s clear that

$$V^b(X = \Lambda, \ell_3, s_3) > V^b(\Lambda; \ell_3, s_3).$$

It’s also clear that

$$V^b(X = 0, \ell_3, s_3) < V^b(\Lambda; \ell_3, s_3),$$
which says that no insurance is worse than expensive but actuarially fair full insurance. This is true because $u$ is strictly concave. Furthermore, it’s clear that $V^b(X, \ell_3, s_3)$ is continuous and increasing in $X \in [0, L]$. Therefore, there exists an $\Lambda^b(\ell_3) \in (0, X)$ such that

$$V^b(X = \Lambda^b(\ell_3), \ell_3, s_3) = V^b(L; \ell_3, s_3).$$

$\Lambda^b(\ell_3)$ is the most generous limited insurance contract that can be offered to low-risk types without necessarily attracting the high-risk type.

Next, consider the low-risk types (type $g$) with state variables $\ell_3$ and $s_3$. Their utility from purchasing the pooling contract is given by

$$V^g(L; \ell_3, s_3) = \int [\pi^g u(e + \ell_3 - [\pi^b - s_3 (\pi^b - \pi^g)] \Lambda)] dF(e)$$

and their utility from purchasing the cheap but limited insurance contract is given by

$$V^g(X, \ell_3, s_3) = \int [\pi^g u(e + \ell_3 - \pi^g X - \Lambda + X) + (1 - \pi^g) u(e + \ell_3 - \pi^g X)] dF(e).$$

Again, it’s clear that $V^a(X, \ell_3, s_3)$ is continuous and increasing in $X$. Furthermore, for any $s_3 \in [0, 1]$, we have

$$V^g(X = \Lambda, \ell_3, s_3) \geq V^g(\Lambda; \ell_3, s_3)$$

where the equality holds only for $s_3 = 1$. Now suppose that $\ell_3$ is such that

$$V^g(X = 0, \ell_3, s_3) < V^g(\Lambda; \ell_3, s_3).$$

That is, no insurance is worse than the pooling contract. Then it follows that there exists $\Lambda^g(\ell_3, s_3) \in (0, L]$ such that

$$V^g(X = \Lambda^g, \ell_3, s_3) = V^g(\Lambda; \ell_3, s_3),$$

where $\Lambda^g(\ell_3, s_3) \leq \Lambda$ is the least generous limited insurance contract that a low-risk type with wealth $\ell_3$ and risk assessment $s_3$ would be willing to accept over the pooling contract. Observe that the equality holds only if $s_3 = 1$. If no insurance is not worse than the pooling
contract, the least generous limited-insurance contract that can be offered to the low-risk type and still get their participation is obviously is a limited “insurance” contract is with $\tilde{\Lambda} = 0$. In this case we can take $\Lambda^g(\ell_3, s_3) = 0$. Finally, observe that $\Lambda^g(\ell_3, s_3)$ is increasing in $s_3$ because $V^g(\Lambda; \ell_3, s_3)$ is increasing in $s_3$ but $V^g(X = \Lambda^a, \ell_3, s_3)$ is independent of $s_3$.

Now for a claim: $B(\ell_3, s_3)$ is non-empty if and only if $\Lambda^g(\ell_3, s_3) \leq \Lambda^b(\ell_3)$ To see necessity, observe that $B(\ell_3, s_3)$ non-empty implies there is a cheap but limited insurance contract $\tilde{\Lambda}$ that the low-risk types weakly prefer over the pooling contract. Therefore, $\tilde{\Lambda} \geq \Lambda^g(\ell_3, s_3)$. And, the high risk types weakly prefer the expensive full insurance contract to the cheap but limited insurance contract. Therefore, $\tilde{\Lambda} \leq \Lambda^b(\ell_3)$. Hence $\Lambda^g(\ell_3, s_3) \leq \Lambda^b(\ell_3)$ To see sufficiency, observe that if $\Lambda^g(\ell_3, s_3) \leq \Lambda^b(\ell_3)$ then any $\tilde{\Lambda} \in [\Lambda^g(\ell_3, s_3), \Lambda^b(\ell_3)]$ is an element of $B(\ell_3, s_3)$.

Now we ready to state and prove the two main results of this subsection.

**Proposition 1.** If $u^g = u^b$, for each $\ell$ there is an $0 \leq s^* \leq 1$ such that for all $s > s^* B(\ell, s) = \emptyset$ and for all $s \leq s^* B(\ell, s) \neq \emptyset$.

**Proof.** Begin with $s_3 = 1$. We know that $\Lambda^g(\ell_3, s_3) = \Lambda > \Lambda^b(\ell_3)$. Therefore $B(\ell_3, s_3)$ is empty and only the pooling contract is offered (actually in this case there is no pooling since everyone is type $g$). As we drop $s_3$, $\Lambda^g(\ell_3, s_3)$ falls. Hence there will be an $s^*(\ell_3)$ such that $\Lambda^g(\ell_3, s_3) \leq \Lambda^b(\ell_3)$ for all $s \leq s^*(\ell_3)$ but not otherwise.

Coupled with the definition of the set $I^*(\ell, s)$, Proposition 1 implies that people with wealth $\ell$ and risk assessment $s > s^*(\ell)$ will face an insurance market in which only the pooling contract is offered while people with $s < s^*(\ell)$ will face an insurance market in which two contracts are offered – one for limited insurance $\Lambda(\ell) < \Lambda$ at price $\pi^g$ and another for full insurance at price $\pi^b$.

For future reference we also note an important property of period 3 value function for both types which follows from the structure of insurance markets people face.
Proposition 2. If \( u^g = u^b \), \( V^i_3(\ell, s) \) is independent of \( s \) for \( s \leq s^* \) and increasing in \( s \) for \( s > s^* \).

**Proof.** When \( s \leq s^*(\ell) \), the separating contracts are offered. Each of these contracts is independent of \( s \). Therefore, \( V^i_3(\ell, s) \) is independent of \( s \). For \( s > s^*(\ell) \), the pooling contract is offered and the price of this contract falls with \( s \). Therefore, \( V^i_3(\ell, s) \) is increasing in \( s \) in this range.

5 Equilibrium

We are now in a position to define the equilibrium of this model and explore some of its properties. The definition of equilibrium takes as given the structure of loan and insurance markets discussed in the previous section. That is, it assumes that the loan markets in periods 1 and 2 offer contracts in the set \( \mathcal{L}^* \) and the insurance market in period 3 offers contracts \( I^*(\ell, s) \), \( \ell \in \mathcal{L}^* \) and \( s \in [0, 1] \).

**Definition** An equilibrium is (i) a set of loan prices \( q^*_i(\ell, \gamma) \) and \( q^*_j(\ell, \ell_i, s) \), (ii) a set of default probabilities \( \mu^*_i(\ell, s) \) and \( \mu^*_j(\ell, s) \), (iii) a set of decision rules \( l^*_i(e, \gamma), l^*_j(e, \ell, s), d^*_i(e, \ell, s), x^*_i(\ell, s) \) and \( d^*_j(e, x, z, \ell, s) \), and (iii) a set of updating function \( \phi^*_i(\ell, \gamma), \phi^*_j(d, \ell, s) \) and \( \phi^*_j(x, \ell, s) \) such that

1. Each loan in \( \mathcal{L}^* \) earns zero profits.

   (a) For \( \ell_i \geq 0 \) and \( \ell_j \geq 0 \), this requires

   \[
   q^*_i(\ell, \gamma) = (1 + r)^{-1} \quad \text{and} \quad q^*_j(\ell, \gamma) = (1 + r)^{-1}.
   \]  

   (2)

   (b) For \( \ell_i < 0 \), this requires

   \[
   q^*_i(\ell, \gamma) = \phi^*_i(\ell, \gamma) [1 - \mu^*_i(\ell, \phi^*_i(\ell, \gamma))] + (1 - \phi^*_i(\ell, \gamma)) [1 - \mu^*_i(\ell, \phi^*_i(\ell, \gamma))] \cdot (1 + r)^{-1}.
   \]  

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(c) For \( \ell_3 < 0 \) this requires

\[
q_2^*(\ell_3, \ell_2, s_2) = \left\{ \begin{array}{l}
\phi_2^*(0, \ell_3, \ell_2, s_2) [1 - \mu_2^*(\ell_3, \phi_2^*(0, \ell_3, \ell_2, s_2))] \\
+ (1 - \phi_2^*(0, \ell_3, \ell_2, s_2)) [1 - \mu_2^*(\ell_3, \phi_2^*(0, \ell_3, \ell_2, s_2))] \end{array} \right\} \cdot (1 + r)^{-1}
\]

where we have recognized that only people who do not default in period 2 can potentially borrow (i.e., we need only consider the case where \( d = 0 \)).

2. Default probabilities are consistent with decision rules. For \( i \in \{g, b\} \) and \( \ell_2 < 0 \), this requires

\[
\mu_i^*(\ell_2, s_2) = \int d_i^*(e, \ell_2, s_2) dF(e),
\]

and for \( \ell_3 < 0 \), this requires

\[
\mu_2^*(\ell_3, s_3) = (1 - \pi^i) \int d_3^*(e, x_3^*(\ell_3, s_3), 0, \ell_3, s_3) dF(e) + \pi^i \int d_3^*(e, x_3^*(\ell_3, s_3), L, \ell_3, s_3) dF(e).
\]

3. Updating functions are consistent with decision rules and satisfy Bayes’ Rule whenever possible. To state these conditions, define

\[
E_i^*(\ell; \gamma) = \{ e_1 : \ell_2^*(e_1, \gamma) = \ell \}
\]

\[
E_2^*(d; \ell; \ell_2, s_2) = \{ e_2 : \ell_3^*(e_2, \ell_2, s_2) = \ell \text{ and } d_3^*(e_2, \ell_2, s_2) = d \}
\]

(a) For \( \phi_1^*(\ell, \gamma) \) this requires

\[
\phi_1^*(\ell, \gamma) = \frac{\gamma \int 1_{\{e \in E_1^*(\ell; \gamma)\}} dF(e)}{\gamma \int 1_{\{e \in E_1^*(\ell; \gamma)\}} dF(e) + (1 - \gamma) \int 1_{\{e \in E_1^*(\ell; \gamma)\}} dF(e)}
\]

provided the denominator is positive – that is, provided a positive measure of people choose \( \ell \) in period 1.
(b) For \( \phi^*_2(d, \ell, \ell_2, s_2) \) this requires

\[
\phi^*_2(d, \ell, \ell_2, s_2) = \frac{s_2 \int 1_{\{e \in E^*_2(d,\ell,\ell_2,s_2)\}} dF(e)}{s_2 \int 1_{\{e \in E^*_2(d,\ell,\ell_2,s_2)\}} dF(e) + (1 - s_2) \int 1_{\{e \in E^*_2(d,\ell,\ell_2,s_2)\}} dF(e)}
\]

provided, again, the denominator is positive.

(c) For \( \phi^*_3(x, \ell_3, s_3) \), this requires that for \( x \in I^*(\ell_3, s_3) \)

\[
\phi^*_3(x, \ell_3, s_3) = \frac{s_3 \cdot 1\{x = x^*_g(\ell_3, s_3)\}}{s_3 \cdot 1\{x = x^*_g(\ell_3, s_3)\} + (1 - s_3) \cdot 1\{x = x^*_b(\ell_3, s_3)\}}.
\]

6 Properties of Equilibrium

We can now proceed to a discussion of the properties of equilibrium defined in the previous section. As noted in the introduction, the main interest of this study is in understanding the role of (market’s) perception of one’s type in controlling opportunistic behavior in the credit market. We now have a structure that can shed light on one channel through which perception of one’s type can enforce good behavior, namely, via the adverse effect of opportunistic behavior in credit market on the terms of trade in the insurance market.

**Proposition 3.** For \( \ell < 0 \), \( q_2(\ell, \ell_2, s_2) = 0 \) for all \( \ell_2 \) and \( s_2 \).

**Proof.** Observe that, as noted earlier, \( d^*_i(e, x, 0, \ell_3, s_3) \) is 1 whenever \( \ell_3 < 0 \).

Hence, it follows from (6) that

\[
\mu^*_i(\ell, s) = 1 \text{ for } i = g, b.
\]

and therefore, from (4), that \( q^*_2(\ell, \ell_2, s_2) = 0 \).

In the last period, there is no reason to pay back a loan and therefore in the next-to-last period loans have a zero price. This result depends on the assumption that the default
decision is made after insurance purchase is made. If, instead, insurance purchase followed
the default decision, the debtor would have to take account of any adverse change in the
market’s assessment of his or her type resulting from opportunistic behavior. Indeed, this is
the channel through which it may be possible to constrain opportunistic behavior behavior
with regard to loans taken out in period 1 because for these loans the insurance purchase
will follow the default decision. It is to this possibility we now turn.

As a preliminary step, we will examine whether any lending can be supported in period 1
when there is no difference between the two types in the probability of loss. In this case we
would expect that induced changes in the market’s assessment of a person’s type will play
no role in constraining opportunistic behavior in the credit market. This, in fact, is true.

**Proposition 4.** Suppose $\pi^g = \pi^b = \pi$. Then $\phi^*_2(d, \ell, \ell_2, s_2)$ (the updating rule in
period 2) does not affect the probability of default on loans made in period 1.

**Proof.** When there is no difference in the probability of loss between types,
separating contracts do not exist and insurers only offer a “pooling” contract for
full insurance at price $\pi$. Therefore $V^i_3(\ell_3, s_3)$, the value function of a person of
type $i$ in period 3, is independent of $s_3$. Now consider a person of type $i$ with debt
$\ell_2 < 0$ and assessment $s_2$. This person’s utility from default is clearly independent
of $\phi^*_2(d, \ell, \ell_2, s_2)$ because $V^i_3(0, s_3)$ is independent of $s_3$. This person’s utility from
not defaulting is also independent of $\phi^*_2(d, \ell, \ell_2, s_2)$ because by Proposition 3 all
loan prices are zero and by (2) in the definition of equilibrium the deposit rate
just depends on the risk-free rate $r$. Consequently, the decision to default is
independent of $\phi^*_2(d, \ell, \ell_2, s_2)$ and the result follows.

But, surprisingly (perhaps), it is still possible for some lending to be supported in period
1 because a person who defaults is not permitted to save: in some instances, a person
may prefer to pay back a loan in order to save. In what follows it will be convenient to
eliminate this outcome so that we can focus solely on what the need for insurance can do
to sustain borrowing and lending. This outcome can be eliminated if we assume that people who default can save in the period of default. This is not an unreasonable assumption. Real world bankruptcy proceedings permit bankrupt households to keep certain assets – most commonly the equity in the person’s home. In some U.S. states these exemptions are unbounded. With this change we have

**Proposition 5.** Suppose \( \pi^g = \pi^b = \pi \) and people who default can save. Then lending cannot be sustained in equilibrium in period 1. That is, for any \( \ell < 0 \), \( q_1^*(\ell, \gamma) = 0 \).

**Proof.** Consider a person of type \( i \) with debt \( \ell_2 < 0 \) and assessment \( s_2 \). By the same logic as in Proposition 4, the net benefit default is independent of \( \phi_2^*(d, \ell, \ell_2, s_2) \). Furthermore, since default does not preclude saving, any consumption-saving pair the person can choose when not defaulting can also be chosen by the person when he or she defaults. In addition, the person gets to consume the value of the debt under default. Therefore, default is always superior to no-default for any \( \ell_2 \).

Therefore we now have a situation where lack of differences in the probability of loss completely shuts down the loan market in the economy. Now let’s turn to the case where the types do differ in the probability of loss. We will assume that the type that is more patient (has a higher \( \beta \)) is also the type that has a lower probability of loss. We will show that under this assumption it will be possible to borrowing and lending in period 1, provided the loan size is small enough. We have the final proposition of the paper:

**Proposition 6.** Suppose (i) \( 0 < \pi^g < \pi^b < 1 \) (ii) \( \beta^g > \beta^b \) and (iii) people who default can save. Then it is possible for \( \mu_1^*(\ell, s) < 1 \) for some \( \ell \) and \( s \).

**Proof.** Consider two people of different types with the same debt \( \ell_2 < 0 \) and assessment \( s_2 \). We will prove the statement by simply constructing an equilibrium in which there is repayment some of the time. We will guess that in this
equilibrium only type $g$ people pay back a loan. If this guess is correct, $\phi_2^*$ will have the property that $\phi_2^*(0, \ell, \ell_2, s_2) = 1$ for any $\ell$. Type $g$ people who choose not to default will therefore have $s_3 = 1$ and hence obtain full insurance at the price $\pi^g$. For this to be an equilibrium, we need to verify two facts: (i) that type $g$ people who pay back cannot do better by defaulting and (ii) any type $b$ person cannot do better by paying back.

(i) Turning to the first condition, suppose the a type $g$ person who pays back in equilibrium chooses to default. Since all type $b$ people default but only some (may be none) type $g$ people do, it follows that default will lower the person’s assessment from 1 to some lower number. It follows from Proposition 2 that by defaulting a type $g$ person will raise his cost of insurance in period 3. For the first condition to be satisfied this increase in cost must be higher than the benefit from declaring default. The benefit is of course directly related to the size of the loan, so for a small enough loan this condition will be satisfied.

(ii) Now consider a type $b$ person who chooses to pay back. Since only type $g$ people ever pay back a loan, doing so would raise the person assessment from some number less than 1 to 1. This will reduce the cost of insurance for this person in period 3. The cost of paying back the loan of course is reduced consumption in period 2. Recall that type $b$ people are also less patient than type $g$ people and are therefore less willing to incur the cost of lower consumption today for higher consumption (via lower insurance costs) tomorrow. Therefore, for a low enough $\beta^b$ condition (ii) will be satisfied as well.

7 Extensions

As this argument in Proposition 6 makes clear, the logic of repayment in this model relies on two things – the good types (type $g$) have a lower probability of loss and therefore have
an incentive to separate themselves from the type \( b \) in the insurance market (this is why “looking good” is valuable to the good types) and the bad types (type \( b \)) do not have an incentive to mimic the good types because the rewards to “looking good” come in the future and the bad types do not care sufficiently about the future.

The assumption that the good types are both better insurance risks and more patient may seem artificial. However, there is an extension to this model that is in progress where we endogenize the risk of loss. In this extension, the two types differ only in their discount factor but people can make an unobservable investment in period 2 that lowers the probability of loss in period 3. Now, the patient type has more of an incentive to incur the costs of investment because they care more about increased consumption in the future. In this model, patient types also tend to be the less risky types and a result analogous to Proposition 6 holds.

So far we have analyzed environments in which opportunistic behavior in the credit market has consequences for how the person is treated in the insurance market. But we can also imagine situations where actions in the insurance market can have consequences for behavior in the credit market. This issue is being looked at in a variant of the model where people purchase insurance in periods 2 and 3.

8 References

(To be added)