**Abstract.** A trace formula related to $p$-almost commuting subalgebras $X$ and $Y$ is established. By means of this formula, homomorphisms from $K_0(X)$ to $H^{\text{odd}}_\lambda(Y)$ and from $K_1(X)$ to $H^{\text{even}}_\lambda(Y)$ are established. An index map from $K_0(X) \times K_1(Y)$ to $\mathbb{Z}$ is also given.

1. Introduction

Let $\mathcal{A}$ be an algebra over $\mathbb{C}$ with trace ideal $J$ and trace $\tau$. Assume $X$ and $Y$ are two subalgebras of $\mathcal{A}$ satisfying the condition that there is a natural number $p$ such that

$$[x^1, y^1] \cdots [x^p, y^p] \in J$$

for $x^j \in X$ and $y^j \in Y$, where $[x, y]$ is the commutator $xy - yx$. Then we say that $X$ and $Y$ are $p$-almost commuting. The present note is a continuation of [6], [7], [8], [9] to study the Chern characters associated with this pair of $X$ and $Y$ in the context of A. Connes’ theory of noncommutative geometry. First in §2, we establish a trace formula (8) of

$$\xi_n(x^0, \ldots, x^n; y^0, \ldots, y^{n-1}) \ \text{def} = \tau x^0 [x^1, y^0] \cdots [x^n, y^n], \quad n \geq p,$$

which gives the relation of the cyclic cochains $A_xA_y \xi_k$ and $A_xA_y \xi_{k+2m}$ in terms of cyclic cohomology (see Theorem 1). As a corollary, it reduces to the trace formula of

$$\varphi_n(x^0, \ldots, x^n; y^0, \ldots, y^n) = \tau [x^0, y^0] \cdots [x^n, y^n], \quad n \geq p - 1,$$

in [7] and [8].

Based on Theorem 1 and A. Connes’ theory, in Theorem 2 and Theorem 3 we establish homomorphisms from $K_0(X)$ to $H^{\text{odd}}_\lambda(Y)$ and $K_1(X)$ to $H^{\text{even}}_\lambda(Y)$ by means of Chern characters. But for convenience, in the statement of Theorem 3, we switch the roles of $X$ and $Y$. In Theorem 4, we give the index map from $K_0(X) \times K_1(Y)$ to $\mathbb{Z}$ in the case when $\mathcal{A}$ is an algebra of operators on a separable complex Hilbert space. We also give a simple example to show that the index map is not trivial.

Received by the editors August 16, 1995.

1991 Mathematics Subject Classification. Primary 47A55; Secondary 47G05.

Key words and phrases. Cyclic cohomology, Chern character, K-theory, almost commuting, deformed commutator, twisted commutator.

This work is supported in part by NSF grant DMS-9400766.

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In §4, we generalize Theorem 1 to the $q$-deformed (or $q$-twisted) commutator case (see Theorem 1'). Besides, we give a theorem (Theorem 5) of Chern character of odd dimension which is just a remark of the previous work in [9].

2. Basic formulas

Let $X$ be an algebra over $\mathbb{C}$. Let $C^n = C^n(X)$ be the space of multilinear functions $f_n(x^0, \ldots, x^n)$, $x^i \in X$, and $t : C^n \to C^n$ be the operation

$$(tf_n)(x^0, \ldots, x^n) = (-1)^n f_n(x^n, x^0, \ldots, x^{n-1}),$$

let $C^n_{\lambda} = \{ f \in C^n : tf = f \}$, and let $Af = (1 + t + \cdots + t^n)f$, $f \in C^n$. Let $b$ be the Hochschild operation $C^n \to C^{n+1}$,

$$(bf)(x^0, \ldots, x^{n+1}) = (b'f)(x^0, \ldots, x^{n+1}) + (-1)^{n+1} f(x^{n+1}, x^0, \ldots, x^n),$$

where $(b'f)(x^0, \ldots, x^{n+1}) = \sum_{j=0}^{n} (-1)^j f_n(x^0, \ldots, x^j x^{j+1}, \ldots, x^n)$. Let $p$ be the operation $pf = \sum_{j=0}^{n} (-1)^j f_n(x^0, \ldots, x^j x^{j+1}, \ldots, x^n)$. Let $p$ be the operation $pf = \sum_{j=0}^{n} (n - j)b^j f$, $f \in C^n$, where $f^0 = 1$. Define $S = 2\pi ibp'$. Then this operator $S$ coincides with A. Connes' operator [1] $S$ at $Z^n = \{ f \in C^n_{\lambda} : bf = 0 \}$. Let $q$ be the operation [8] $qf = nAf/2 - ptf$, $f \in C^n$. Let $r$ be the operation [8] $rf = r_n(t)f$, $f \in C^n$, for $n > 0$, where $r_n(\cdot)$ is a polynomial of degree $\leq n - 1$ satisfying $qf = (1 - t)rf$, $f \in C^n$. For convenience, define $\hat{S} = bpb'$ (which is the operator $S$ in [8] and [9]).

The following well-known identities will be needed. Notice that $(1 - t)b = b'(1-t)$ (cf. [4], [1]) and that $p(1 - t)f = (n + 1 - A)f$, for $f \in C^n$ (cf. [8]). Hence

(1) $$\hat{S}(1 - t) = -bAb$$

and

(2) $$(1 - t)\hat{S} = -b'bA.$$

If $f \in C^n_{\lambda}$, then $Sbf \in C^{n+3}_{\lambda}$ and (cf. [8])

(3) $$\hat{S}bf = \frac{1}{n+1} bA\hat{S}f, \quad f \in C^n_{\lambda}.$$
Lemma 1. For \( n \geq p \),

\[
\begin{align*}
\xi_{n+1} &= b_x \psi_n, \quad \eta_{n+1} = -t_x b_y \psi_n, \\
(1 - t_x) \xi_n &= b'_x \phi_{n-1}, \quad (1 - t_y) \eta_n = b'_y \phi_{n-1}, \\
(1 - t_y^{-1}) \xi_n &= b_x \phi_{n-1}, \quad (1 - t_x^{-1}) \eta_n = b_y \phi_{n-1}, \\
(1 - t_y) \psi_n &= b'_x \xi_n - t_y \phi_n, \quad (1 - t_x) \psi_n = b'_x \eta_n + \phi_n.
\end{align*}
\]

Theorem 1. Let \( k \geq p \) be an even number, \( m \) be a natural number, and \( n = k+2m \). Then there are \( \theta_j \in C^1(X) \otimes C^1(Y) \), \( j = n-1, n-2 \) (\( \theta_j \) also depending on \( k \)), such that

\[
A_x A_y \xi_n = b_x \theta_{n-1} + A_x S_x b_y \theta_{n-2} + \hat{\xi}_n,
\]

where

\[
\hat{\xi}_n = \frac{1}{(2\pi)^{2m}} \frac{k! (k-1)!}{n!(n-1)!} (A_x S_x)^m (A_y S_y)^m A_x A_y \xi_k.
\]

Proof. Based on Lemma 1, in [8] (cf. (20) of [8]) it is proved that

\[
A_x A_y \psi_j = (j+1)^2 \psi_j - (j+1)p_y b'_y \xi_j - p_x b'_x A_y \eta_j - (j+1)q_y \phi_j,
\]

for \( j \geq p \). From (4), (10) and the fact that \( b_x \xi_j = 0 \), we have

\[
\xi_{j+1} = \frac{1}{(j+1)^2} (b_x A_x A_y \psi_j + \hat{S}_x A_y \eta_j) + \frac{1}{j+1} b_x q_y \phi_j,
\]

for \( j \geq p \). Thus

\[
A_x A_y \xi_{j+1} = \frac{j+2}{j+1} b_x A_x A_y \psi_j + \frac{1}{j+1} A_x \hat{S}_x A_y \eta_j,
\]

for \( j \geq p \), since \( A_y q_y = 0 \).

Similarly, we have

\[
\eta_{j+1} = -\frac{1}{(j+1)^2} (A_x b_y A_y \psi_j + A_x \hat{S}_y \xi_j) - \frac{1}{j+1} t_x b_y q_y \phi_j,
\]

for \( j \geq p \). It is easy to see that \( r_x \phi_j = r_y \phi_j \), since \( t_x t_y \phi_j = \phi_j \). Denote \( r_x \phi_j \) by \( r \phi_j \) (cf. [8]). From (1), we have

\[
\hat{S}_x t_x q_y \phi_j = \hat{S}_x t_x (1 - t_y) r_y \phi_j = -\hat{S}_x (1 - t_x) r_y \phi_j
\]

\[
= b_x A_x b_x \phi_j,
\]

for \( j \geq p - 1 \). From (6), it is easy to see that \( t_y \xi_j = \xi_j + \sum_{v=1}^{j} t^v_y b_x \phi_{j-1} \). Thus

\[
A_y \xi_j = j \xi_j + b_x p_y t_y \phi_{j-1}, \quad j \geq p.
\]

Similar to (14), we have \( \hat{S}_y q_y \phi_j = -b_y A_y b_y r \phi_j \). From (7), \( A_y \phi_j = b_y A_y \xi_j \). Hence \( b_x A_y \phi_j = 0 \). Thus

\[
b_x \hat{S}_y t_y p_y \phi_j = b_x \hat{S}_y (j A_y \phi_j/2 - q_y \phi_j) = b_x b_y A_y b_y r \phi_j.
\]
From (12), (13), (14), (15) and (16), we have, for \( j \geq p \),
\[
A_x A_y \xi_{j+2} = \frac{j+3}{j+2} b_x A_x A_y (\psi_{j+1} - \frac{1}{j+1} b_x b_y r \phi_{j+1})
\]
(17)
\[
- \frac{1}{(j+1)^2} A_x \hat{S}_x A_x b_y A_y (\psi_j - \frac{1}{j} b_x b_y r \phi_{j-1})
\]
\[
- \frac{1}{(j+2)(j+1)^2} A_x \hat{S}_x A_x A_y \hat{S}_y A_y \xi_j.
\]
As in [8], define \( \Theta_{k-1} = \Theta_{k-2} = 0 \),
\[
\xi_j = \psi_j - \frac{1}{j} b_x b_y r \phi_{j-1} - \frac{1}{j(j-1)^2} \hat{S}_x \hat{S}_y \Theta_{j-2}
\]
and \( \Theta_j = \frac{1}{j+1} A_x A_y \xi_j \) for \( j \geq k \) by mathematical induction. Define
\[
\hat{\xi}_j = A_x A_y \xi_j - (j+1) b_x \Theta_{j-1} + \frac{1}{j-1} A_x \hat{S}_x b_y \Theta_{j-2}.
\]
From (3) and (17), we have
\[
\hat{\xi}_{i+2} = -A_x \hat{S}_x A_y \hat{S}_y \hat{\xi}_i / (j+2)(j+1)^2 j, \quad j \geq k,
\]
which proves (8) and (9) where \( \theta_{n-1} = (n+1) \Theta_{n-1} \) and \( \theta_{n-2} = -\Theta_{n-2}/(n-1)2\pi i \).

Since \( A_x \phi_n = A_y \phi_n \), denote it by \( A\phi_n \). Applying \( b_y \) to (8), we get the following.

**Corollary 1** [8]. Let \( k \geq p \) be an even number, \( m \) be a natural number and \( n = k + 2m \). Then there is \( \Theta_{n-1} \in C_\lambda^{n-1}(X) \otimes C_\lambda^{n-1}(Y) \) such that
\[
A\phi_n = b_x b_y \Theta_{n-1} + \tilde{\phi}_n,
\]
where \( \tilde{\phi}_n = k! S_x^m S_y^m A\phi_n/(2\pi i)^{2m} n! \).

### 3. Chern characters

Let \( \ell \geq 1 \). For \( (x_{ij}) \in M_\ell(X) \), let \( [(x_{ij}), y] = ([x_{ij}], y) \), \( y \in Y \), where \( M_\ell(X) \) is the algebra of \( \ell \times \ell \) matrices over \( X \). Define
\[
(\text{tr} \xi_{n}) (x^0, \ldots, x^n; y^0, \ldots, y^{n-1}) = (\text{tr} \xi_{r}) (x^0 [x^1, y^0] \ldots [x^n, y^{n-1}]),
\]
(18)
for \( x^j \in M_\ell(X) \), \( n \geq p \). For odd \( n \geq p - 1 \) and \( e \in \text{Proj}(M_\ell(X)) \), let
\[
\text{ch}_{e,n} (y^0, \ldots, y^n) = \frac{n!}{(n+1)!} (-2\pi i)^{n+1} A_y (\text{tr} \xi_{n+1}) (e, \ldots, e; y^0, \ldots, y^n).
\]
For \( f \in Z^{n}_\lambda \), let \( [f] = f + bC^{n-1}_\lambda \in H^{n}_\lambda = Z^{n}_\lambda / bC^{n-1}_\lambda \). Let \( H^{\text{odd}}_\lambda(Y) \) be the group defined in [1].

**Theorem 2.** The mapping \([e] \mapsto \text{ch}_{e,2m-1}\) is a homomorphism from \( K_0(X) \) to \( H^{2m-1}_\lambda(Y) \), \( 2m \geq p \). It satisfies
\[
[S\text{ch}_{e,2m-1}] = [\text{ch}_{e,2m+1}]
\]
(19)
and it defines a homomorphism from \( K_0(X) \) to \( H^{\text{odd}}_\lambda(Y) \).
Proof. We only have to consider the case $e \in \text{Proj}(X)$. It is easy to see (cf. [1]) that for even $n$,
\begin{equation}
(hf)(e, \ldots, e) = f(e, \ldots, e) = 0, \quad f \in C^n_{\lambda},
\end{equation}
and
\begin{equation}
(Af)(e, \ldots, e) = (n+3)(n+2)f(e, \ldots, e)/2, \quad f \in C^n.
\end{equation}
By (7), $b_yA_y\xi_n(e, \ldots, e; y) = A_y\phi_n(e, \ldots, e; y) = A_y\tau[e, y^0] \ldots [e, y^n] = 0$ for odd $n \geq p - 1$. Thus $\text{ch}_{e, n} \in \mathcal{Z}_{\lambda}^n(Y)$. On the other hand, $A_x\xi_n(e, \ldots, e; y) = (n+1)\xi_n(e, \ldots, e; y)$ for even $n$. From (8), (9), (20) and (21), we have
\[
A_y\xi_n(e, \ldots, e; y^0, \ldots, y^{n-1}) = b_y(A_x\hat{S}_x \theta_{n-2})(e, \ldots, e; y^0, \ldots, y^{n-1})
+ (-1)^n - (k-1)!/(n-1)!^{2m} (A_y\hat{S}_y)^m A_y\xi_k(e, \ldots, e; y^0, \ldots, y^{n-1}),
\]
for $n = 2m + k$ and $k \geq p$, which proves (18). The rest of the proof is similar to the proof of Proposition 14 of Chapter II of [1].

Remark. By (12) and (21), for odd $n \geq p - 1$, we also have
\[
\text{ch}_{e, n}(y^0, \ldots, y^n) = n!(n+2)/(2\pi i)^{n+1} (A_y \text{tr} \#\eta_n)(e, \ldots, e; y^0, \ldots, y^n).
\]
For $(x, y) \in M_t(Y)$, let $[x, (y)] = ([x, y], e) \in X$. Define $\text{tr} \#\xi_n$ by (18) for $y^j \in M_t(Y)$ and $n \geq p$. For $u \in GL_t(Y)$ and even $n \geq p$, define
\begin{equation}
\text{ch}_{u, n}(x^0, \ldots, x^n) = k_n(A_x A_y \text{tr} \#\xi_n)(x^0, \ldots, x^n; u^{-1}, u, \ldots, u^{-1}, u),
\end{equation}
where $k_n = (-2\pi i)^n/2!/(n+1)(\pi i)^2$. Theorem 3. The mapping $[u] \mapsto [\text{ch}_{u, 2m}]$ is a homomorphism from $K_1(Y)$ to $H_\lambda^{2m}(X)$, for $2m \geq p$. It satisfies
\begin{equation}
[S \text{ch}_{u, 2m}] = [\text{ch}_{u, 2(m+1)}]
\end{equation}
and it defines a homomorphism from $K_1(Y)$ to $H_\lambda^{\text{even}}(X)$.

Proof. We only have to consider the case that $u \in Y$ and $u^{-1} \in Y$. It is easy to see that
\begin{equation}
(A\phi_{2m-1})(x^0, \ldots, x^{2m-1}; u^{-1}, u, \ldots, u^{-1}, u) = 0.
\end{equation}
For $f \in C^n$, $n = 2m$,
\begin{equation}
bAgf - b(f - t^j f)(a^j a^{j+1}, \ldots, a^{n+1}, a^0, \ldots).
\end{equation}
By (4) and (7) we have
\[
(1 - t_x)\xi_n = (1 - t_x)b_x\psi_{n-1} = b'_x(1 - t_x)\psi_{n-1} = b'_x\phi_{n-1}, \quad n \geq p + 1.
\]
From (24), it follows that $(1 - t_x)A_y\xi_{2m}(x^0, \ldots, x^n; u^{-1}, u, \ldots, u^{-1}, u) = 0$. By $b_x\xi_n = 0$ and (25), we have
\[
(b_x A_x A_y\xi_{2m})(x^0, \ldots, x^{2m+1}; u^{-1}, u, \ldots, u^{-1}, u) = 0.
\]
Thus $\text{ch}_{u,2m} \in Z^2_\lambda$. Formula (23) comes from Theorem 1. Let us follow the lines of the proof of Proposition 15 of Chapter II of [1]. It is easy to see that

$$\xi_n(x^0, \ldots, x^n; y^0, \ldots, y^{n-1}) = 0$$

if one of the $y$'s is 1. Thus in (22), $u^{-1}$ and $u$ may be replaced by $u^{-1} - 1$ and $u - 1$ respectively.

For $f \in C^2_{\lambda-1}$, it is easy to calculate that

$$(A\hat{S}f)(u^{-1}, u, \ldots, u^{-1}, u) = 4(m + 1)mf(u^{-1}, u, \ldots, u^{-1}, u).$$

Similar to the proof of Proposition 15 of Chapter II of [1], we may prove that $\text{ch}_{u,2m} - \text{ch}_{u,2m} - \text{ch}_{v,2m}$ is the boundary of a cyclic cochain in $C^2_{\lambda-1}(X)$ since

$$b_yA_y\xi_n = A\phi_n = -b_xA_x\eta_n$$

by (7), which proves the theorem.

**Remark.** Similar to (15), by (6), we have $A_x\eta_m = m\eta_m + t_xp_zb_y\phi_{m-1}$. Suppose $n$ is even and $\geq p + 1$. By (24), we have

$$(A_x\eta_{n-1})(x^0, \ldots, x^{n-2}; u^{-1} - 1, u - 1, \ldots, u^{-1} - 1, u - 1)$$

$$= (n - 1)\eta_{n-1}(x^0, \ldots, x^{n-2}; u^{-1} - 1, u - 1, \ldots, u^{-1} - 1, u - 1).$$

By (12), we may prove that

$$(A_xA_y\xi_n - \frac{(n + 1)}{n(n - 1)}A_xA_y\eta_{n-1})(\cdot; u^{-1} - 1, \ldots, u - 1) \in bC_{\lambda-1}(X).$$

Thus for even $n \geq p + 1$, we may define

$$\text{ch}_{u,n}(x^0, \ldots, x^n)$$

$$= \hat{k}_n(A_xA_y\text{tr}g\eta_{n+1})(x^0, \ldots, x^n; u^{-1} - 1, u - 1, \ldots, u^{-1} - 1, u - 1),$$

where $\hat{k}_n = (-2\pi i)^{n/2}n!/\left(\frac{n+2}{2}\right)^2$.

Now let us consider the case that $X$ and $Y$ are $p$-almost commuting subalgebras of the operator algebra on a complex separable Hilbert space $\mathcal{H}$ and $\tau$ is the usual trace. In this case, the elements of $M_t(X)$ or $M_t(Y)$ may be regarded as operators on $\mathcal{H} \otimes \mathbb{C}^t$.

**Theorem 4.** The index map

$$\langle [e], [u] \rangle = \frac{1}{m}(-1)^m\left(A_y\text{tr}g\xi_{2m}\right)(e, \ldots, e; u^{-1}, u, \ldots, u^{-1}, u)$$

$$= \text{index } eu\big|_{\text{range}(e)},$$

where $e \in \text{Proj } M_t(X)$ and $u \in \text{GL}_t(Y)$, is a homomorphism from $K_0(X) \times K_1(Y)$ to $\mathbb{Z}$.

**Proof.** It is easy to calculate that

$$\langle [e], [u] \rangle = \text{trace}\left((I - ca)^m - (I - ae)^m\right),$$

where $a = eu|_{\text{range}(e)}$ and $c = eu^{-1}|_{\text{range}(e)}$. The rest of the proof is similar to the corresponding part of Chapter I of [1].

**Example.** Suppose $e$ is the orthogonal projection from $L^2(\mathbb{T})$ to $H^2(\mathbb{T})$, and $u$ is the backward bilateral shift $(uf)(z) = \overline{zf}(z)$. Then $\langle [e], [u] \rangle = 1$. 


4. Deformed commutator case

Let $\mathcal{A}$ be an algebra over $\mathbb{C}$ with trace ideal $J$ and trace $\tau$. Let $X$ and $Y$ be subgroups of $\mathcal{A}$. Assume that there is a function $q : X \times Y \to \mathbb{C}$ satisfying
\[
q(x^i x^j ; y^k y^l) = \prod_{i,j=1}^2 q(x^i, y^j) \quad \text{and} \quad q(x, 1) = q(1, y) = 1
\]
for $x^i, x \in X$ and $y^j, y \in Y$. Let $\{x, y\} = xy - q(x, y)yx$ be the $q$-deformed commutator. Assume that there is a natural number $p$ such that
\[
\{x^1, y^1\} \ldots \{x^p, y^p\} \in J.
\]
(We say that $X$ and $Y$ are $p$-almost $q$-deformed commuting.) Let
\[
M^{m,n} = \{(x^0, \ldots, x^m, y^0, \ldots, y^n) \in X^{m+1} \times Y^{n+1} : q(x^0 \ldots x^m, y^0 \ldots y^n) = 1\}.
\]
By means of the operation defined in [9], we may generalize Theorem 1 to the following.

**Theorem 1'.** Let $k$ be an even number, $k \geq p$, $m \in \mathbb{N}$, and $n = k+2m$. Then there exists $\theta_j = \theta_j(x^0, \ldots, x^j; y^0, \ldots, y^j)$ satisfying $\tau \theta_j = \tau \theta_j$ on $M^{j,j}$ for $j = n-1$ and $n-2$ such that
\[
\alpha_x \alpha_y \xi_n = \delta_x \theta_{n-1} - \alpha_x S_x \delta_y \theta_{n-2} + \tilde{\xi}_n \quad \text{on } M^{n,n-1},
\]
where
\[
\tilde{\xi}_n = \frac{1}{(2\pi)^{2m} n!(n-1)!} (\alpha_x S_x)^m (\alpha_y S_y)^m \alpha_x \alpha_y \xi_k.
\]
Let $Q = \{(x, y) : x \in X, y \in Y\}$. Let $\mathcal{W} = \{(x, y, c) : x \in X, y \in Y, c \in Q\}$ be the group with multiplication $(x^0, y^0, c^0)(x^1, y^1, c^1) = (x^0 x^1, y^0 y^1, c^0 c^1 q(x^0, y^1))$. Let $p : \mathcal{W} \to \mathcal{A}$ be the mapping $p(x, y, c) = cyx$. Then the “curvature” of this mapping is defined as $\omega(w^0, w^1) = p(w^0 w^1) - p(w^0)p(w^1)$, $w^0, w^1 \in \mathcal{W}$. For $n \geq p$, define the Chern character of dimension $2n - 1$ (see [3] and [5]) as
\[
\mathrm{ch}_{2n-1}(w^0, \ldots, w^{2n-1}) = \tau (\omega(w^0, w^1) \ldots \omega(w^{2n-2}, w^{2n-1}) - \omega(w^{2n-1}, w^0) \ldots \omega(w^{2n-3}, w^{2n-2})).
\]

**Theorem 5.** If $p = 1$ or $2$, then there are $(2n - 2)$-cyclic cochains $f_{2n-2}$ such that
\[
\text{ch}_{2n-1} = bf_{2n-2} \quad \text{off } M^{2n-1,2n-1}
\]
for $n \geq p$.

**Proof.** By the proof of the corollary of Theorem 2 in [9], it is easy to see that (26) holds for $n = p$. Then the formula (26) for $n > p$ follows from the fact that there is a constant $k_n$ such that $\mathrm{ch}_{2n+1}$ and $k_n S \mathrm{ch}_{2n-1}$ are in the same cohomology class by Proposition 15 of Chapter II of [1].

**References**


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