A Sharing Model of the Household:
Explaining the Deaton-Paxson Paradox and Contesting the Collective Model

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Abstract
This paper presents a new model of the household where each member fully controls his/her individual income and voluntarily shares purchased goods with other members. The nature of goods and norms in the household play a central role in the allocation of resources. The model is able to explain a variety of consumption patterns that standard models cannot describe. Most notably, it provides an intuitive explanation for the fact that per capita food consumption tends to decline as household size increases, which contradicts the Barten model (the so called Deaton-Paxson paradox). The sharing model is an alternative to the popular collective model without imposing efficiency in the intra-household allocation of resources. Although efficiency is implied by one of the sharing model equilibria, the paper argues that this is not the equilibrium played. The paper shows why a collective model test has limited power against the sharing model for a broad set of goods, reconciling contradictory evidence in the literature. Empirically, the paper revisits the Deaton-Paxson paradox exploiting household splits in longitudinal data and computes the elusive economies of scale coefficients.

JEL: D13, J12, O15

Keywords: sharing model, collective model, Deaton-Paxson paradox, household economies of scale

1 Introduction

Although the economic theory of individual decision making is almost undisputed, theoretical models explaining household choice remain controversial. Since most people live in multi-person households, modeling group behavior correctly is crucial in many theoretical and empirical studies. Previous research indicates

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that ignoring the interaction of household members affects the measurement of demand elasticities (Brown-ning and Chiappori (1998)), family labor supply (Chiappori (1988), Fortin and Lacroix (1997), Browning and Meghir (1991)) and the impact of social policies (Lundberg et al. (1997), Duflo (2003)).

This paper presents a new model for the intra-household allocation of resources where each member fully controls his/her income and voluntarily shares purchased goods with other members. Two main ingredients differentiate this model, denominated here the “sharing model”, from others commonly used in the literature. First, there is a clear distinction between quantities purchased and quantities consumed by each member resulting in an explicit system of transfers within the household. Second, the allocation of goods after they are purchased is determined by their nature and norms in the household. In this respect, the model contemplates three types of commodities: public goods, private goods and common-pool goods.

Public goods have the characteristic of being non-rival in consumption. Then, the purchases made by a household member are fully enjoyed (i.e. consumed) by her. But, these goods are also non-excludable and susceptible to the well-known free rider problem where each member contributes less than optimally. Private goods are on the other side of the spectrum. Because they are rival and excludable, the model assumes that a member who purchases a given quantity of such goods can freely decide how much to consume and how much to transfer to each of the other members in the household. The excludability condition guarantees that the allocation of private goods is exactly as desired. - e.g. a mother buying a new dress for one her daughters - and the benefits are fully appropriable by the buyer.

The third type of commodities contemplated in the model is common-pool goods. The difference between these and private goods is important and not considered in standard theories of the household. Common-pool goods are rival in consumption but non-excludable. Thus, when a household member purchases such goods, she knows that they have to be shared with others. An example of a common-pool good is food at home that can be consumed by any household member irrespectively of who purchased it. The non-excludability condition of common-pool goods does not mean the absence of a rule to allocate them. It means that the buyer cannot fully appropriate the benefits because other members are allowed to consume them. The existence of common-pool goods is in the essence of the household. For example, the United Nations define a multi-person household “as a group of two or more persons living together who make common provision for food or other essentials for living” (emphasis added). Similarily, the majority of surveys define the household as a function of common goods, usually food.

Household members in the model maximize their individual utility considering not just their budget constraints but the actions taken by other members. The fact that each type of commodity has its own allocation/sharing rule differentially affects the purchasing incentives. This household interaction ‘game’

\[\text{http://unstats.un.org/unsd/demographic/sconcerns/fam/fammethods.htm}\]

\[\text{2For example, the Mexican Family Life Survey used in this study defines a household as “a person or group of people, related or unrelated by biological bonds, who usually live together in a part of, or in an entire building/ dwelling and usually consume meals provided by a common budget on the same stove/oven and may even use the same utensils for preparing meals”}\]
generates multiple equilibria when played repeatedly. One equilibrium implies an efficient allocation of outcomes (folk theorem), which would make the sharing model consistent with the collective model described below. However, this paper argues that efficiency is incompatible with a series of consumptions patterns observed in the data. A standard Nash equilibrium in the static game - also supported in the dynamic case - best describes the empirical regularities.

The sharing model contrasts with the two theories of the household most commonly evoked in the literature, the unitary and the collective models. None of these theories are explicit about who makes the decisions in the households. Then, they cannot account for the fact that purchasing goods and allocating them among household members are related but different processes. The two models treat aggregate household demands only implicitly as a result of multiple individuals simultaneously maximizing their utility and interacting with each other. Although this lack of specificity is occasionally considered an advantage because it precludes the models from making strong assumptions about the underlying decision process, it comes at a price.

The unitary model assumes that households can be treated ‘as if’ they were individuals. Specifically, the mechanism governing household members’ interactions, whatever this is, results in well-defined group preferences capable of being represented by an aggregate utility function. Although some situations lead naturally to the unitary model, such as households where only one person makes all the decisions or households where all members have identical preferences, they cannot account for the generality. Testing the unitary model relies either on its income pooling property (i.e. the identity of the income earner should not affect household demands for goods) or on the symmetry of the Slutsky matrix. The empirical evidence is now ample against the unitary model (Thomas (1990), Schultz (1990), Lundberg et al. (1997), Deaton and Paxson (2003), Browning and Chiappori (1998), Attanasio and Lechene (2002)).

The collective model, originally developed by Chiappori (1988) and later extended by Blundell et al. (1993), Browning and Chiappori (1998), Bourguignon et al. (2009) and Chiappori and Ekeland (2009) among others, is a commonly used and accepted alternative to the unitary model. It recognizes that households are formed by individuals with different preferences. However, this theory does not explicitly model the group decision process. It simply assumes that such process leads to an efficient allocation of resources in the household. The collective model is appealing because it makes only one extra assumption on top of individual rationality and is compatible with a great number of mechanisms.

Theoretical papers related to the collective model deal with the question of whether the model is too general to obtain meaningful conclusions and testable conditions. These papers take Pareto-optimality as given and derive the conclusions that emerge from it. On the other hand, empirical studies take the implications of the collective model to test whether this theory is a valid representation of households’ behavior. Despite the attractiveness of the collective setting, the evidence is not conclusive. While some studies cannot reject the collective model hypothesis (Browning and Chiappori (1998), Bobonis (2009),
Attanasio and Lechene (2014)), others find strong evidence against Pareto-optimality within the household (Udry (1996), Duflo and Udry (2004), Dercon and Krishnan (2003)).

This apparent lack of consensus in the literature is at least partially resolved in section 4. Tests for the collective model in the absence of price variation rely on the existence of ‘distribution factors’. These are variables that impact the negotiation power of household members but do not affect individual preferences or the household budget constraint. In the collective model, distribution factors enter the demand system in a specific and testable way. Using Mexican data, Attanasio and Lechene (2014) follow this strategy but implement the test only on the components of total food expenditure. All these goods are plausibly common-pool goods as previously described. In such case, the Attanasio-Lechene test is likely to have no power against the sharing model.

The most important evidence in favor of the sharing model is the fact that, holding total household per capita expenditures fixed, per capita food consumption tends to decline with household size. This empirical regularity, which was first described by Deaton and Paxson (1998), contradicts the predictions of the Barten (1964) model in relation to the presence of household economies of scale. This consumption pattern cannot be explained with existing models of the household.

Household economies of scale are thought to come from the presence of public goods (e.g. housing). Because these goods are non-rival in consumption, per capita monetary contributions to maintain a given level of consumption declines as household size increases. The freed resources obtained from adding a new member are used to increase the purchases of normal goods. Then, the share of food on total expenditures is expected to increase with household size, but the opposite is observed in most of the countries (Deaton and Paxson (1998)). The sharing model in this paper gives a simple and intuitive explanation of this paradox. Since food is likely to be a common-pool good, as households become larger, the individual consumption for each additional unit of food purchased declines. Consequently, household members have incentives to reallocate resources away from food to private goods (e.g. shoes) for which the buyer can fully appropriate the benefits.

The sharing model is able to explain several consumption patterns that neither the unitary nor the collective models can. In addition to the fact that i) food as a share of total expenditure tends to decline with household size, the model explains ii) why the expenditures on food as a share of food plus housing increases with household size (Gan and Vernon (2003) explanation of the paradox) despite the observed decline in per capita food expenditures (Deaton and Paxson (2003) response to Gan and Vernon), and iii) why food consumed away from home tends to increase with household size. Moreover, if the sharing model is enriched by allowing the altruistic parameter to vary with income as the experimental evidence indicates (Chowdhury and Jeon (2014)), it can explain iv) why the Deaton-Paxson paradox is more prevalent in low-income countries and v) why the Engel’s curve tends to be hump-shaped in poor regions.

The empirical section of the paper is divided in two parts. First, it revisits the Deaton-Paxson paradox
augmenting the standard method to estimate economies of scale with Engel’s curves. In addition to analyzing how food consumption covaries with household size, the method exploits households’ splits in longitudinal data. The Mexican Family Life Survey (MxFLS) used in this study follows individuals over time irrespectively of whether they remain in the same household or they form a new one. Thus, consumption is observed for all members in the original sample. The estimation of economies of scale is done comparing households that split to others that do not split over time. Holding family size constant, the division of a household necessarily implies a duplication of public good expenditures per unit consumed. The predictions of the Barten model indicate that food as a share of total expenditures should decline among split households, however the opposite is observed reinforcing the Deaton-Paxson paradox. To my knowledge, using split households to measures economies of scale is new in the literature. In addition to solving some potential endogeneity problems, the method resembles the ideal experiment to achieve such goal.

The second part of the empirical section develops and implements a method to estimate household economies of scale coefficients. That is, a measure of the amount of money a household should receive to increase the welfare of its members in the same magnitude as an additional member. The measurement of economies of scale is fundamental for the analysis of individual living standards using household data. It affects all poverty and inequality indexes. Therefore, the policy implications are enormous since many social programs determine eligibility on the basis of poverty status. Nonetheless, the estimation of economies of scale has been elusive since the Deaton-Paxson paradox remained unresolved.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. It first describes the different types of goods consumed in the household and why the rules to allocate them differ. (section 2.1). Then, it discusses the implications of the popular unitary and collective models (section 2.2) to finally present the sharing model (section 2.3). Section 3 discusses the Deaton-Paxson paradox and other empirical regularities unexplained by existing models. Section 3.1 specify a functional for the utility functions to obtain close form solutions and show how the sharing model is able to explain the consumption patterns described in the previous section. Section 4 explains why a collective model test has limited power against the sharing model. Section 5 presents a method to measure household economies of scale. Section 6 shows the results of the estimated Engel’s curve and the economies of scale coefficients. Finally, section 7 presents the conclusions of the paper.

2 Theoretical Framework

This section presents a general version of the sharing model. It first characterizes the goods consumed in the household. Then, it discusses the implications of the popular unitary and collective models. Finally, it

\footnote{Most conditional cash transfer programs determine eligibility on the basis of living standards, for example Progresa-Oportunidades in Mexico and Juntos in Peru.}
describes the components of the sharing model.

2.1 Public goods, appropriable goods and non-appropriable goods in the household

The literature on intra-household allocation of resources has traditionally dealt with only two types of goods consumed in the household, public goods and private goods. The distinction between them is made on the principles of rivalry and excludability. Public goods are modeled as non-rival and non-excludable in the sense that they are consumed simultaneously by all members in the household, while private goods are modeled as fully rival and excludable since they are assigned to and consumed by only one person at a time.\(^4\)

Completely ignored in the literature are goods that can be consumed by only one person, but cannot be ex-ante appropriated by any member of the household. That is, goods that are rival but non-excludable in consumption. These ‘common-pool’ goods are expected to constitute an important share of the household budget. According to the United Nations:

A multi-person household [is] defined as a group of two or more persons living together who make common provision for food or other essentials for living.\(^5\)(emphasis added)

Excludability within the household arises due to intrinsic characteristics of the good - e.g. items that are size specific such as shoes - or as a consequence of norms - e.g. no underage alcohol consumption. Non-excludable goods are likely to be those that are sharable by nature such as food. In this paper the terms ‘common-pool goods’, ‘sharable goods’ and ‘non-appropriable goods’ are used interchangeably. Non-excludability in this context does not mean a complete absence of rules to allocate common-pool goods in the household. It means that once a member purchases a quantity of such goods, he or she cannot fully appropriate it. For example, food presumably has to be shared somehow.

**Definition 1.** There are three types of goods consumed by member \(i = 1, \ldots, n\) in the household. Private goods, which consumption is denoted by \(x_i\), are rival and excludable (e.g. clothing specific to each member). Public goods, denoted by \(Q\), are non-rival and non-excludable (e.g. housing). Sharable goods, non-appropriable goods or common-pool goods, which consumption is \(s_i\), are rival but non-excludable (e.g. food).

The three types of commodities in definition 1 are certainly extreme cases. Goods vary continuously in terms of rivalry - i.e. different degrees of congestions - and excludability - i.e. goods that are sharable for a subset of the household such as tobacco. However, these intermediate cases add little or no insight to

\(^4\)The literature has also covered the intermediate case of public goods with congestion where the principle of rivalry is only partially satisfied. In this case, the addition of a household member affects the quantity or quality of the public good consumed by others.

\(^5\)http://unstats.un.org/unsd/demographic/sconcerns/fam/fammethods.htm
the model at the cost of reducing tractability. Nonetheless, the empirical section 6.4 analyzes the role of congestion in public goods.

Previous studies in the literature have not differentiated the purchasing process from the allocating process. Then, they could not model how the non-excludability condition of some rival goods affects their allocation in the household. The next section clarifies this point.

### 2.2 The unitary and the collective models: implications

This section briefly describes two popular models of the household with the objective of contrasting them with the sharing model introduced in the next section. Both, the unitary and the collective models can be written within a common framework (Attanasio and Lechene (2014)). The household budget constraint is identical in both cases.

$$p_x \sum_{i=1}^{n} x_i + p_s \sum_{i=1}^{n} s_i + p_q Q = Y$$  \hspace{1cm} (1)

The total household income $Y = \sum_{i=1}^{n} y_i$ is used to purchase private goods $x_i$, public goods $Q$ and sharable goods $s_i$ for all members $i = 1,..,n$ in the household. $p_x$, $p_s$ and $p_q$ are the prices of the good composites. Given the budget constraint, households maximize a weighted sum of individual preferences.

$$\max \sum_{i=1}^{n} \mu_i U_i(x_i, s_i, Q)$$  \hspace{1cm} (2)

The difference between the unitary and the collective models is in the treatment of individual weights. With fixed weights $\mu_i$, summation (2) is a function $U(x_1, ..., x_n, s_1, ..., s_n, Q)$ that inherits all the properties of a utility function. This is the unitary model where preferences of household members can be jointly represented by a unique utility function. In contrast, the collective model allows individual weight $\mu_i(Y, p, z)$ to depend on total household income, prices $p = (p_x, p_s, p_q)$ and a vector of variables $z$, called ‘distribution factors’, which affects the negotiation process within the household but not individual preferences $U_i(\cdot)$.

The unitary model is nested in the collective model. As such, it is more restrictive. In particular, the unitary model has the *income pooling* property. That is, holding total household income constant, the identity of the person who earns the money is irrelevant to the decision process. The collective model is more general. It only imposes efficiency in the allocation of resources within the household, letting the negotiation process to be affected by multiple factors.

Neither the unitary model nor the collective model is explicit about the mechanism used by household members to allocate goods. Although these theories recognize that group decisions are the result of the interaction of heterogeneous individuals, the choice set that each household member faces is not specified. One of the consequences of treating household decisions as a black box is that the act of allocating goods cannot be disentangled from the act of purchasing goods. Notice that in model (1)-(2), there is no difference in the way $s_i$ and $x_i$ are treated. The role of the excludability condition cannot be contemplated in these
models. Once quantities of a rival good are allocated to household members, whether the good is excludable or not becomes irrelevant.

2.3 A sharing model of the household

Each person \( i \) in a household with \( n \) members maximizes his/her own utility function.

\[
\max \ U_i(x_i, s_i, Q) + \tau_i \left( \frac{1}{n-1} \sum_{j \neq i}^{n} \alpha_{ij} U_j(x_j, s_j, Q) \right)
\]

The function \( U_i(.) \) maps own consumption of private goods \( x_i \), public goods \( Q \) and common-pool or sharable goods \( s_i \) on utility. The parameter \( \tau_i \geq 0 \) is the level of altruism. It indicates how much individual \( i \) values the (weighted average) utility that other members of the household derive from consumption. The weights \( \alpha_{ij} \) measure altruism heterogeneity from \( i \) to other members (e.g. a mother may care more about her children than her in-laws.).

The characteristics of goods and norms in the household imply that purchased and consumed quantities may differ. For private goods (e.g. shoes), the quantity consumed \( x_i \) equals the purchases \( \tilde{x}_{ji} \) made by all the \( j = 1, \ldots, n \) members in individual \( i \)'s behalf.\(^6\)

\[
x_i = \sum_{j=1}^{n} \tilde{x}_{ji}
\]

The sharable good \( s_i \) (e.g. food) is rival but non-excludable in consumption. Thus, the buyer cannot allocate it to a specific person. The purchases \( \tilde{s}_{j} \) made by each \( j \) member go to a common pool. Then, the quantity consumed \( s_i \) by member \( i \) is a share \( \delta_i \) of total purchases in the household (equation (5)). The determination of \( \delta_i \) is discussed below.

\[
s_i = \delta_i \sum_{j=1}^{n} \tilde{s}_{j}
\]

Being non-rival and non-excludable, the total consumption of the public good \( Q \) equals the sum of all members’ purchases \( \tilde{Q}_j \), for \( j = 1, \ldots, n \).

\[
Q = \sum_{j=1}^{n} \tilde{Q}_j
\]

Individuals are in full control of his/her resources and voluntarily decides to purchase goods. Thus, an individual with income \( y_i \) facing prices \( p_x, p_s \) and \( p_q \) have the following budget constraint.

\[
p_x \sum_{j=1}^{n} \tilde{x}_{ij} + p_s \tilde{s}_i + p_q \tilde{Q}_i = y_i
\]

Notice that the subscripts for the purchases of private goods are interchanged from (4) to (7). The summation in the first case indicates the goods purchased by all household members which are given to

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\(^6\)For example, private goods consumed by a teenager in the household could be clothing that his mother bought for him, a motorcycle his father gave him and video games he bought for himself.
member i, while the summation in the budget constraint indicates all the private goods purchased by member i in her’s and other’s behalf.

Given the quantities purchased by other household members, individual i maximizes utility (3) subject to equalities (4) to (7) and non-negative constraints. The result is a system of conditional demands that represents the best response functions in a static non-cooperative game.

\[
\begin{align*}
\tilde{x}_{ij} &= g_{ij}^c \left( y_i, p_1, p_2, p_q, n, \left\{ \sum_{k \neq i}^{n} \tilde{x}_{kr} \right\}_{r=1}^{n}, \sum_{k \neq i}^{n} \tilde{s}_k, \sum_{k \neq i}^{n} \tilde{Q}_k \right) \\
\tilde{s}_i &= h_i^c \left( y_i, p_1, p_2, p_q, n, \left\{ \sum_{k \neq i}^{n} \tilde{x}_{kr} \right\}_{r=1}^{n}, \sum_{k \neq i}^{n} \tilde{s}_k, \sum_{k \neq i}^{n} \tilde{Q}_k \right) \\
\tilde{Q}^c_i &= l_i^c \left( y_i, p_1, p_2, p_q, n, \left\{ \sum_{k \neq i}^{n} \tilde{x}_{kr} \right\}_{r=1}^{n}, \sum_{k \neq i}^{n} \tilde{s}_k, \sum_{k \neq i}^{n} \tilde{Q}_k \right)
\end{align*}
\]

The Marshallian demands are obtained when the household reaches a Nash equilibrium.

\[
\begin{align*}
\tilde{x}_{ij} &= g_{ij} \left( y_i, p_1, p_2, p_q, n \right) \\
\tilde{s}_i &= h_i \left( y_i, p_1, p_2, p_q, n \right) \\
\tilde{Q}^c_i &= l_i \left( y_i, p_1, p_2, p_q, n \right)
\end{align*}
\]

Intuitively, when altruism is imperfect, i.e. \( \tau < (n - 1) \), the equilibrium described by (8) is not efficient since individuals have the incentive to purchase sub-optimal quantities of sharable goods and public goods (the free rider problem) and purchase above-optimal quantities of private goods.

New in this model is that the miss-allocation of resources occurs even when there is a single breadwinner in the household. In this case, the household head can perfectly control the consumption of the private good for each member in the household (e.g. buying clothes for each child and the spouse) but not the consumption of the sharable good (e.g. food) since everyone eats from one common pot. As a result, members with zero income will consume more food and less clothing than what they would in an efficient equilibrium.

A natural question in such situation is to what extent the breadwinner is able to perfectly control \( \delta_i \) \( \forall i = 1, .., n \), which would make sharable goods indistinguishable from private goods. There are two reasons to believe that this is not the case. First, the breadwinner should be able to perfectly monitor at no cost what others eat. This is unlikely. Second and most importantly, social norms related to households and families seem to agree with the existence of common-property goods (see U.N. definition of the household in section 2.1). The allocation of such goods may not be completed determined by the breadwinner. For example, a stay-at-home mother may be the one who makes the decisions regarding food consumption.

Parameters \( \delta_i \) are determined by a variety of factors: i) what a breadwinner is supposed to provide for her/his family according to the society where he/she lives, ii) the negotiation power of household members, and iii) individual incomes, prices, etc. As discussed elsewhere in the paper (appendix II using the model specificities from section 3.1), \( \delta_i \) parameters are closely related to the weights attributed to household members in the collective model (i.e. \( \mu_i(Y, p, z) \) in equation (2)) but operating only over a subset of commodities.
For the sake of simplicity, $\delta_i$ for $i = 1, ..., n$ are fixed in the rest of the paper. No conclusion relies on this assumption.

**Recurrence and the folk theorem** The household is a relatively stable unit where the process of allocating resources is repeated over time. Without a certain date of household dissolution, the problem given by equations (3)-(7) does not have a unique equilibrium. The game theory literature indicates that two equilibria in this household “game” of allocating resources over time are i) the **Nash equilibrium in the static game** and ii) **the subgame perfect equilibrium supporting an efficient outcome** (folk theorem).\(^7\)

The collective model previously described, which imposes efficiency within the household, is conceived with the idea that the folk theorem holds, see Browning and Chiappori (1998). That is, the recurrence interaction of household members generates mechanisms to avoid any miss-allocation of resources. Although, this equilibrium is supported by problem (3)-(7) in a repeated setting, this paper states that it is not the one prevailing. Instead, the observed consumption patterns described below are highly consistent with a Nash equilibrium in the static game as that described in (8). A well-known result in the literature is that a Nash equilibrium in a static game is always a subgame perfect equilibrium in the repeated game. Then, the allocation described for the static model (3)-(7) is also an equilibrium in a dynamic setting.

3 The sharing model, the Deaton-Paxson paradox and other unexplained regularities of the Engel’s curve

This section presents empirical regularities in consumption that are incompatible with the unitary and the collective models, but can be explained by the sharing model from previous section.

Deaton and Paxson (1998) seminal paper showed that some household behaviors in relation to food consumption cannot be explained by standard microeconomic theory. Specifically, the existence of economies of scale implies that, as household size increases, per capita expenditure on public goods (e.g. housing) should decrease, freeing resources to increase the consumption of private goods (e.g. food). However, evidence from developed and developing countries consistently shows the opposite.

Formally, a model that accounts for economies of scale can be written using equations (1) and (2). Imposing that all household members have identical preferences and weights ($\mu_i = 1 \ \forall i$) gives the Deaton-Paxson model (and the Barten (1964) model with pure private and pure public goods.). The $n$-member

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\(^7\)Infinitely repeated games generate a continuum of equilibria. This paper focuses on the static Nash equilibrium, which is capable of explaining consumption patterns.
household objective function simplifies as follows.\(^8\)

\[
\text{max } nU(x,s,Q)
\]

(9)

and the per-capita budget constraint is given by

\[
px + ps + \frac{pq}{n} Q = \frac{Y}{n}
\]

(10)

In model (9)-(10), all household members are identical, thus symmetry is pre-imposed and the variables \(x, s\) and \(Q\) are per-capita consumptions. The effect of household size is clear from the budget constraint (10). Holding total per capita expenditures constant (i.e. \(Y/n\) fixed), an extra member in the household decreases the average price of the public good \(Q\). The condition for per capita consumption of food \(s\) to increase with \(n\) is as follows:\(^9\)

\[-\eta_{spq} + \eta_{sy}\omega_q > 0\]

(11)

The compensated cross-price elasticity of food with respect to the public good \(\eta_{spq}\) is presumably low and the income elasticity of food \(\eta_{sy}\) at the equilibrium share of the public good \(\omega_q = p_q Q/Y\) is expected to be relatively large, particularly in low-income countries. Nonetheless, the empirical evidence is inconsistent with inequality (11). As household size increases, the share of food expenditures declines. This paradox together with other related regularities unexplained by the unitary and the collective models are summarized in remark 1.\(^{10}\)

**Remark 1. Consumption regularities:** i) As household size increases, the share of food in total expenditures declines (Deaton-Paxson paradox), ii) food as a share of food plus housing increases with household size (consistent with the Barten model) despite a simultaneous decline in per capita food expenditure, iii) food consumed away from home increases with household size, iv) The Engel’s curve tends to be hump-shaped in poor countries, v) The Deaton-Paxson paradox is more evident in low-income countries.

Regularities i) and iii) are described in Deaton and Paxson (1998). The first part of regularity ii) is shown by Gan and Vernon (2003) in response to the paradox, claiming that the evidence is consistent with the Barten model when one includes food and a good that is clearly more public than food. Nonetheless, Deaton and Paxson (2003) reply to Gan and Vernon arguing that the puzzle is not solved since the decline in per capital food consumption remains unexplained (second part of regularity ii). Regularity iv) has received

\(^8\)Deaton and Paxson only consider private goods and public goods (and the intermediate case of partially rival goods). The inclusion of sharable goods is new in this model.

\(^9\)The per capita demand for food is a function \(s = s(p_x, p_s, p_q, \frac{Y}{n})\). Holding per capita income constant (i.e. increasing \(Y\) in the same proportion as \(n\)), the elasticity of \(s\) with respect to \(n\) equals the negative elasticity of \(s\) with respect to \(p_q\). Using the Slutsky equation, inequality (11) is obtained.

\(^{10}\)The expression in Deaton and Paxson (1998) looks different than (11). However, considering only the existence of a public good \(Q\) and a food composite \(s\) as in their paper, and using the results \(-\eta_{spq} = \eta_{sp_s}\) and \(\omega_q = 1 - \omega_s\), then expression (11) becomes \(\eta_{sp_s} + \eta_{sy}(1 - \omega_s) > 0\) as in page 901 in their paper.
less attention, but it is equally important. For several countries, the share of food in total expenditures
increases with living standards among the poor, which appears to be inconsistent with standard micro theory.
Regularity v), also described in Deaton and Paxson (1998), strengthens the paradox. In poor countries, price
elasticities of food are expected to be relatively low and the income elasticity of food relatively high, making
inequality (11) more likely to hold. The next section shows how the sharing model can explain all the
regularities in remark 1.

3.1 Sharing Model: Symmetric case with Stone-Geary utilities

This section specifies the functional form for the utility function (3) to make the model tractable and obtain
close form solutions that are capable of explaining the consumption patterns described in remark 1.

Consider a household with \( n \) identical members in relation to preferences and endowments. There is no
heterogeneity in the level of altruism (\( \alpha_{ij} = 1 \ \forall i,j \)). Then, the utility function (3) for member 1 reduces to:

\[
\max \left( \frac{x_1^{1-\sigma}}{1-\sigma} + \frac{(s_1 - \gamma)^{1-\sigma}}{1-\sigma} + \frac{Q_1^{1-\sigma}}{1-\sigma} \right) + \tau \left( \frac{x_2^{1-\sigma}}{1-\sigma} + \frac{(s_2 - \gamma)^{1-\sigma}}{1-\sigma} + \frac{Q_1^{1-\sigma}}{1-\sigma} \right)
\]

(12)

The utility derived from own consumption \( U_1(\cdot) \) has a Stone-Geary shape. The constant \( \gamma \) is usually
interpreted as the minimum food consumption for subsistence. While \( \gamma = 0 \) simplifies the expression to a
CES utility, \( \gamma > 0 \) breaks the homotheticity and guarantees the 1st Engel’s law to hold (i.e. the share of
food declines with income). All the other \( (n-1) \) members of the household are identical. Thus, setting the
consumption of all of them to be the same imposes no constraint. The altruistic term in (3) simplifies to the
utility that any of the other members derives from consumption, such as member 2.\(^{11}\)

The rules (4)-(6) simplify. The total consumption of member 1’s private good \( x_1 \) equals the quantity she
purchases to be consumed by her \( \hat{x}_{11} \) plus the \( (n-1) \) identical quantities purchased by each of the other
members of the household in her’s behalf \( \hat{x}_{21} \), equation (13). On the other hand, the private goods consumed
by each of the other household members \( x_2 \) equals the quantity member 1 gives to each of them \( \hat{x}_{12} \), the
quantities they purchase for themselves \( \hat{x}_{22} \) and the transfers/gifts made among them \( \hat{x}_{21} \) (equation (14)),
which by symmetry should be identical to the quantities given to member 1 in (13).

\[
x_1 = \hat{x}_{11} + (n-1)\hat{x}_{21}
\]

(13)

\[
x_2 = \hat{x}_{12} + \hat{x}_{22} + (n-2)\hat{x}_{21}
\]

(14)

The purchases of sharable goods go to a common pool formed by member 1’s contribution \( \hat{s}_1 \) and the
\( n-1 \) contributions of others. Each member consumes an equal share.

\[
s_1 = s_2 = \frac{\hat{s}_1 + (n-1)\hat{s}_2}{n}
\]

(15)

\(^{11}\)In equilibrium \( x_2 = x_3 = \ldots = x_n \), thus \( \frac{1}{n-1} \sum_{j=2}^{n} U(x_j, s_j, Q) = U(x_2, s_2, Q) = \left( \frac{x_2^{1-\sigma}}{1-\sigma} + \frac{(s_2 - \gamma)^{1-\sigma}}{1-\sigma} + \frac{Q_1^{1-\sigma}}{1-\sigma} \right) \)
Finally, the consumption of public goods $Q$ equals the contribution of member 1 ($\tilde{Q}_1$) plus the contribution of the other $(n - 1)$ members in the household ($\tilde{Q}_2$), equation (16).

$$Q = \tilde{Q}_1 + (n - 1)\tilde{Q}_2$$  \hfill (16)

The budget constraint for the symmetric case is as follows.

$$p_x(\tilde{x}_{11} + (n - 1)\tilde{x}_{12}) + p_s\tilde{s}_1 + p_q\tilde{Q}_1 = y$$  \hfill (17)

In (17), increasing the quantity of private goods consumed by others in one unit requires member 1 to buy $(n - 1)$ goods (e.g. one pair of shoes for each child) for this reason $\tilde{x}_{12}$ is multiplied by this constant. The variable $y$ represents individual income and also per capita income considering that all members are identical. The Nash equilibrium after each member maximizes (12) subject to (13)-(17) and plays a simultaneous game of complete information gives the following system of equations for per capita purchases (see appendix I for derivation).

$$
\begin{align*}
\tilde{x} &\equiv \tilde{x}_{11} = \tilde{x}_{22} = \frac{y - p_s\gamma}{\mathbb{P} p_x^{1/\sigma} (1 + \tau)^{1/\sigma}} \\
\tilde{x}_{12} = \tilde{x}_{21} &= 0 \\
\tilde{s} &= \frac{y - p_s\gamma}{\mathbb{P} p_s^{1/\sigma} n^{1/\sigma}} + \gamma \\
\tilde{Q} &= \frac{y - p_s\gamma}{\mathbb{P} p_q^{1/\sigma} n}
\end{align*}
$$  \hfill (18)

where

$$\mathbb{P} \equiv \left( (1 + \tau)^{-1/\sigma} p_x^{-1/\sigma} + n^{-1/\sigma} p_s^{-1/\sigma} + n^{-1} p_q^{-1/\sigma} \right)$$  \hfill (19)

When altruism is imperfect (i.e. the consumption of other members is valued less than own consumption $\tau < (n - 1)$), each person will only purchase private goods for herself and makes no transfer/gift to others in the household. All members will purchase a non-zero quantity of food $\tilde{s}$ and public goods $\tilde{Q}$.

**Explaining regularity i) As household size increases, the share of food in total expenditures declines (Deaton-Paxson paradox)**

Contrary to what the standard economic theory predicts, the share of food expenditure tends to decrease with household size. As explained previously, this regularity is known as the Deaton-Paxson paradox. The sharing model presented in this section is capable of reconciling economic theory with empirical evidence. The expression for the share of food in total expenditures $\omega_s \equiv p_s\tilde{s}/y$ derived from (18) is given by

$$\omega_s = \frac{p_s^{1-1/\sigma}}{n^{1/\sigma} \mathbb{P}} \left( 1 - \frac{p_x\gamma}{y} \right) + \frac{p_s\gamma}{y}$$  \hfill (20)
Then, the condition for the share of food to decrease with household size as observed in the data is

\[
\frac{\partial \omega_s}{\partial n} < 0 \iff (1 + \tau)^{-1/\sigma} \left( \frac{p_x}{p_q} \right)^{1-1/\sigma} + n^{-1}(1 - \sigma) > 0
\]

\[
\iff \sigma < 1 + \left( \frac{\omega_x}{\omega_q} \right) \quad \text{s.t.} \quad \omega_x \equiv \frac{p_x \tilde{x}}{y} \quad \omega_q \equiv \frac{p_q \tilde{Q}}{y}
\]

Inequality (22) indicates that there is an upper limit in relation to the complementarity of goods for the model to be consistent with the observed consumption patterns. Intuitively, if the complementarity of a sharable good, say meat, and a private good, say wine, is too high, then the expenditure on meat will not decline (or may even increase) with household size to keep enjoying wine. One the other hand, if private goods and sharable goods are substitutes, the addition of a member to the household makes everyone reduce the expenditures on sharable goods because they have to be divided among more people, favoring the consumption of private goods which benefits are fully appropriable by the buyer. The elasticity of substitution between the good composites \( s, x \) and \( Q \) is expected to be relatively low (Deaton and Paxson (1998)). Inequality (22) indicates that the sharing model can explain the Deaton-Paxson paradox for the realistic case when good composites are gross complements.

**Explaining regularity ii) food as a share of food plus housing increases with household size (consistent with Barten model) despite a decline in per capita food expenditure**

A potential explanation for the Deaton-Paxson paradox is that there are other goods consumed by household members that are more rival than food.\(^{12}\) This idea is used by Gan and Vernon (2003) to argue that there is no contradiction between theory and empirical evidence. Using the same data as Deaton and Paxson (1998), Gan and Vernon estimate Engel’s curves replacing the dependent variable with the share of food in food plus housing expenditures only. Since housing is presumably less rival than food, the coefficient associated with household size should be positive according to the Barten model and not negative as shown by Deaton and Paxson. Results in Gan and Vernon show that the share of food in food plus housing increases with household size in accordance to economic theory.

The sharing model is consistent with the evidence presented by Gan and Vernon’s paper but provides a different explanation.

\[
\frac{p_s(\tilde{s} - \gamma)}{p_q \tilde{Q}/y} = \left( \frac{p_s n}{p_q} \right)^{1-1/\sigma} \quad \Rightarrow \quad \frac{p_s \tilde{s}/(p_q \tilde{Q} + p_s \tilde{s})}{\partial n} > 0, \quad \text{if} \quad \sigma > 1
\]

The first term in (23) is approximately the ratio of food share to housing share. This ratio is clearly increasing in household size \( n \) for a low elasticity of substitution (i.e. \( \sigma > 1 \)), a realistic assumption considering that the model is written for good composites (see Deaton and Paxson (1998) page 901). Since \( \tilde{Q} \) is

\(^{12}\)Both, Deaton and Paxson (1998) and Gan and Vernon (2003) mention goods that are less public than food rather than more rival. However, the model they use cannot deal with non-excludable goods (see section 2.2). Thus, rivalry is the only characteristic modeled in their papers.
decreasing in household size, the share of food in food plus housing increases with household size consistent with the evidence presented in Gan and Vernon (2003).\textsuperscript{13}

Deaton and Paxson (2003) point out that Gan and Vernon’s argument does not solve the paradox since it does not explain why per capita food consumption declines with household size. It only shows that food consumption decreases less rapidly than housing. In the sharing model, the condition for per capita food consumption to decline is given by

\[
\frac{\partial \tilde{s}}{\partial n} < 0 \quad \text{if} \quad \sigma < 1 + \left(\frac{\omega_x}{\omega_q}\right) 
\]

s.t. \( \omega_x \equiv p_x \tilde{x}/y \) \( \omega_q \equiv p_q \tilde{Q}/y \)

(24)

which is identical to inequality (22) and the core of the paradox. While the hypothesis that other goods are more rival than food is insufficient (at least empirically) to explain the decline in per capita food consumption, the sharing model gives the theoretical conditions in (24) for this regularity.\textsuperscript{14}

**Explaining regularity iii) food consumed away from home increases with household size**

A given explanation for the Deaton-Paxson paradox is the plausible existence of economies of scale in food preparation. However, Deaton and Paxson discard this hypothesis because the decline in per-capita time required to prepare food associated with an increase in household size should induce individuals to substitute food consumed away from home for food prepared at home. But, the data tend to show the opposite.\textsuperscript{15}

While this regularity may serve as evidence against the existence of strong economies of scale in food preparation, it is difficult to be explained by standard models of the household. However, the sharing model gives an intuitive explanation. Food at home is a common-pool good - rival but non-excludable. Then as the household becomes larger the share consumed by the person who purchases it becomes smaller. But, food away from home is a private good. If desired, it is consumed entirely by the person who purchases it. To illustrate this point, assume that food \( s \) in equation (12) is a composite of food at home \( f_h \) and food away from home \( f_a \) given by (25).

\[
s = f_h + f_a^\kappa, \quad 0 < \kappa < 1
\]

(25)

Since food at home is a non-appropriable good, it follows allocation rule (15). But food consumed away from home is a private good and the allocation is given by rules (13)-(14). It can be shown that per capita demand for food consumed away from home is given by equation (26).

\[
\tilde{f}_a = \left(\frac{np_h \kappa}{p_a (1 + \tau)}\right)^{1/(1-\kappa)}
\]

(26)

\textsuperscript{13}p_s (\tilde{s} - \gamma)/y = p_x \tilde{x}/y - p_q \gamma/y p_q Q/y, since \( \frac{\partial Q}{\partial n} < 0 \), then \( \frac{p_x \tilde{x}/y}{p_q Q/y} \) is necessarily increasing in \( n \).

\textsuperscript{14}As Deaton and Paxson (2003) indicate in relation to Gan and Vernon (2003) argument, “Gan and Vernon focus on the possibility that there are substantial economies of scale in food consumption, which, if true, would certainly help resolve the puzzle. But they generate no empirical evidence to support their contention that food has greater economies of scale than clothing or transportation” page 1362.

\textsuperscript{15}Deaton and Paxson (1998) main argument against this hypothesis is that the food shares contain only food purchases not the combination of food expenditure plus the time allocated to food preparation.
where \( p_h \) and \( p_a \) are the prices of food consumed at home and food consumed away from home. Consistent with the evidence presented in Deaton and Paxson (1998), food away from home is clearly increasing in household size even when food at home declines with \( n \) (see equation (24)).

The sign of \( \frac{\partial \hat{f}_a}{\partial n} \) depends of the relative income elasticities of food at home and food away from home. The quasi-linearity functional form imposed in (25) is an extreme case where every additional amount of money allocated to food is spent on food at home. The demand for food away from home responds only to the price effect induced by an extra member in the household.\textsuperscript{16}

*Explaining regularities iv) The Engel’s curve tends to be hump-shaped in poor countries and v) The Deaton-Paxson paradox is more evident in low-income countries*

Regularity v) is documented in Deaton and Paxson (1998). The decline in food share when household size increases is more pronounced in low-income countries. This fact exacerbates the paradox. Poor people are expected to have a higher income elasticity in food expenditure making (11) more likely to hold. Regularity iv) has received less attention but it is equally problematic to be explained by existing models of the households. The Engel’s curve in low-income countries tends to be increasing at the low end of the income distribution and declining for the rest of the population. If food is the most important necessity for survival, how is it possible that the extreme poor spend a lower proportion of their income in food than households with higher living standards?

The sharing model can explain, at least partially, regularities iv) and v) by making the altruistic parameter \( \tau \) to depend on income.\textsuperscript{17} The positive relationship between altruism and income has been documented by Chowdhury and Jeon (2014) in an experimental study.\textsuperscript{18} Assume that \( \tau \) in the utility function (12) is heterogeneous.

\[
\tau = \frac{(n-1)y^3}{1+y^\beta} \quad (27)
\]

Expression (27) allows the level of altruism \( \tau \) to depend positively on income and household size. This functional form restricts the level of altruism to be non-negative (i.e. no hate or envy among household members) and less than \((n-1)\) (i.e. utility derived from own consumption of goods is valued more than the

\textsuperscript{16}Gan and Vernon (2003) point out that regularity iii) in remark 1 does not hold for all of countries. A plausible explanation is that the income effect of food consumed away from home is relatively large.

\textsuperscript{17}There may be other reasons in addition to variable altruism to explain hump-shaped Engel’s curves. I am not aware of any other theory explaining this regularity. More research is needed in this area.

\textsuperscript{18}The same paper discusses alternative behavioral models. However, the psychological reasons linking altruism and income are beyond the scope of the sharing model. Here, the relationship between altruism and income is taken as an assumption.
utility obtained from the consumption of others).

\[ \omega_s = \frac{p_{s}^{1-1/\sigma}}{n^{1/\sigma} \mathbb{P}} \left(1 - \frac{p_s \gamma}{y}\right) + \frac{p_s \gamma}{y} \]  \hspace{1cm} (28)

where \( \mathbb{P} \equiv \left(1 + \frac{(n-1)y^\beta}{1 + y^\beta}\right)^{-1/\sigma} p_{x}^{1-1/\sigma} + n^{-1/\sigma} p_s^{1-1/\sigma} + n^{-1} p_q^{1-1/\sigma} \)

Equation (28) is the theoretical Engel’s curve obtained from equations (18) and (27). Consistent with evidence, it is non-monotonic in income. Figure 1 shows the Engel’s curve (28) for a two-person household. It has a hump-shaped profile.

Expression (28) is cumbersome but the intuition is simple. As income increases, household members become more altruistic creating incentives to contribute more to common-pool goods. But, simultaneously food share tends to decrease because it is a necessity good (1st Engel’s law). For poor people, the changes in altruism may dominate resulting in increasing levels of food consumption. Because altruism is assumed to be bounded from above (i.e. it cannot grow indefinitely), then there is an income threshold for which the first Engel’s law begins to dominate.

Allowing altruism to be non-fixed as in (27) can also explain why the Deaton-Paxson paradox is stronger in low-income countries. As \( \tau \) increases, the demand system (18) approaches that obtained from the Barten Model.

\[ \text{as } \tau \to (n-1), \quad \omega_s \to \frac{p_{s}^{1-1/\sigma}}{p_{x}^{1-1/\sigma} + p_{s}^{1-1/\sigma} + (p_q/n)^{1-1/\sigma}} \left(1 - \frac{p_s \gamma}{y}\right) + \frac{p_s \gamma}{y} \]  \hspace{1cm} (29)

Expression (29) is exactly the share of food derived from a Barten model with individual utility function (12).\textsuperscript{19}

In addition to higher income, another reason to expect relatively high levels of intra-household altruism in developed countries is that households are usually formed by nuclear families, in contrast to developing countries where households are more likely to be organized around extended families. Since altruistic behaviors are plausibly less likely to occur towards in-laws than towards spouses or children, (29) is expected to be closer to the Barten model in developed countries.

3.2 Economies of scale, diseconomies of scale and household division

The sharing model presented in this section generates both economies of scale and diseconomies of scale depending on the number of members, which naturally explains why households cannot grow indefinitely. In contrast, the Barten model can only generate economies of scale, implying that the larger the household

\textsuperscript{19}When \( \tau = (n-1) \), the collective model with identical individuals and weights \( \mu_i = 1/n \) in (2) reduces to maximizing the function \( \left(\frac{s^{1-\sigma}}{1-\sigma} + \frac{(s-\gamma)^{1-\sigma}}{1-\sigma} + \frac{Q^{1-\sigma}}{1-\sigma}\right) \) subject to \( p_s x + p_s s + \frac{p_x}{n} Q = y \)
is, the better their members are.

\[ V = \frac{1 + \tau}{1 - \sigma} (y - p_s \gamma)^{1-\sigma} \frac{\tilde{P}}{P^{1-\sigma}} \]  
((30))

where \( P \) is defined in (19) and

\[ \tilde{P} = (1 + \tau)^{1-1/\sigma} p_x^{1-1/\sigma} + n^{1-1/\sigma} p_s^{1-1/\sigma} + p_q^{1-1/\sigma} \]

The indirect utility function (30) resulting from converting quantities purchased into quantities consumed in (18) and replacing them in (12) is increasing in household size when this is small and decreasing when large. The reason for a non-monotonic indirect utility function is that an additional member in the household generates two opposite effects. On the one hand, it decreases the per capita cost of public goods. On the other hand, it reduces the individual consumption share of the marginal common-pool good. When the first effect dominates the second one, there are household economies of scale, otherwise there are diseconomies of scale. Figure 2 shows the theoretical indirect utility function for different levels of altruism \( \tau \). The optimal household size positively depends on the level of altruism. Above the maximum, members benefits by splitting the household.

As indicated in the conclusions, the sharing model can be used to study the economic incentives for household formation and household dissolution together with group behaviors in an integrated framework.

4 The limited power of a collective model test

Attanasio and Lechene (2014) develop and implement a clever test for the collective model. This test exploits that the distribution factors \( z \) introduced in section 2.2 affect the demand for goods only indirectly by modifying individual weights \( \mu_i \) in equation (2) (Bourguignon et al. (2009)). Then, under the null hypothesis that the collective model is true, for any pair of goods \((v, w)\) and any pair of distribution factors \((z_1, z_2)\), equality (31) should hold.\(^{20}\)

\[ H_0: \frac{\partial v}{\partial z_1} \frac{\partial s}{\partial z_2} = \frac{\partial w}{\partial z_1} \frac{\partial w}{\partial z_2} \]  
((31))

If the sharing model from section 2.3 is true (i.e. the collective model is false), the null hypothesis (31) should not hold for at least one pair \((v, w)\) of goods. If \( v \) and \( w \) are taken from different commodity composites (e.g. \( v \) is a sharable good and \( w \) is a private good), equality (31) certainly fails. However, Attanasio and Lechene use only the components of total food expenditures to test (31). In such a case, when only sharable goods are used, the collective model test is likely to have no power against the sharing model.

In equation (5), \( \delta_i \) is a parameter that indicates what proportion of the sharing composite is consumed by member \( i \). But, the model is silent about how the allocation of sharable goods is done to obtain such

\(^{20}\)Attanasio and Lechene (2014) do not test directly condition (31) but the equivalent \( z \)-conditional demands.
composite good. If the allocation within s is (conditionally) efficient in the sense that ratio of marginal utilities for any pair a sharable goods is the same for all household members, then the Attanasio and Lechene test (31) has zero power against the the sharing model when only food items are used for its computation. Appendix II shows formally such case.

5 Measuring economies of scale in the household

This section presents a method to estimate household economies of scale. The procedure is derived from the sharing model introduced in section 2.3. The method imposes no functional form for the utilities \( U_i(\cdot) \). However, it assumes that all household members are identical. This assumption can be relaxed in the implementation by including observable characteristics of the household and its members as regressors.

Consider the indirect utility function obtained from problem (3)-(7)

\[
V(p_x, p_s, p_q, y, n)
\]

The derivative of the indirect utility function with respect to household size \( n \) can be written as a function of marginal utilities evaluated at the optimum and changes in quantities demanded with respect to \( n \).

\[
\frac{\partial V}{\partial n} = \frac{\partial U^*}{\partial x} \frac{\partial x}{\partial n} + \frac{\partial U^*}{\partial s} \frac{\partial s}{\partial n} + \frac{\partial U^*}{\partial Q} \frac{\partial Q}{\partial n} \tag{32}
\]

Expression (32) is the increase in 'utils' as a result of having a extra member in the household. The first order conditions in the maximization problem and the symmetry obtained from identical individuals generate the following expressions for the Lagrange multiplier \( \lambda \) (see appendix I).

\[
\lambda = \frac{\partial U^*}{\partial x} \frac{1}{p_x} = \frac{\partial U^*}{\partial s} \frac{1 + \tau}{np_s} = \frac{\partial U^*}{\partial Q} \frac{1 + \tau}{p_q} \tag{33}
\]

Using (33) and the fact than in equilibrium the relationship between per-capita consumed and per-capita purchased goods are: \( s = \tilde{s} \), \( x = \tilde{x} \) and \( Q = n\tilde{Q} \), then (32) simplifies to the following expression:

\[
\frac{\partial V}{\partial n} / \lambda = p_x \frac{\partial \tilde{x}}{\partial n} + np_s \frac{\partial \tilde{s}}{(1 + \tau) \partial n} + np_q \frac{\partial \tilde{Q}}{(1 + \tau) \partial n} + \frac{p_q \tilde{Q}}{(1 + \tau)} \tag{34}
\]

\[
y \left( \frac{1 + \tau}{n} \frac{\partial \omega_x}{\partial \log(n)} + \frac{\partial \omega_s}{\partial \log(n)} + \frac{\partial \omega_q}{\partial \log(n)} + \omega_q \right) \tag{35}
\]

Equation (35) is obtaining using the fact that the change in the budget share \( \omega_h \) of good \( h \in \{s, x, Q\} \) with respect to household size \( n \) is \( \partial \omega_h / \partial n = (p_h/y)(\partial h / \partial n) \).

\[
\frac{\partial V}{\partial n} / \lambda C = \frac{y}{C(1 + \tau)} \left( \frac{1 + \tau}{n} \frac{\partial \omega_x}{\partial \log(n)} + \frac{\partial \omega_s}{\partial \log(n)} + \frac{\partial \omega_q}{\partial \log(n)} + \omega_q \right) \tag{36}
\]
where
\[ C = \left[ 1 + \tau \left( \frac{\partial \omega_x}{\partial \log(y)} + \omega_x \right) + (n - 1) \left( \frac{\partial \omega_s}{\partial \log(y)} + \omega_s \right) + (n - 1) \left( \frac{\partial \omega_q}{\partial \log(y)} + \omega_q \right) \right] \] (37)

The left hand side of (36) is the economies of scale coefficient. It is the amount of money that a person living in a household with \( n \) members should receive to have the same utility as a person living in an household with \( n + 1 \) members if both households have the same per capita income and preferences. The right hand side of (36) is formed by quantities that can be estimated with a standard Engel’s curve, with the exception of the altruistic parameter. Section 6.3 estimates economies of scale coefficients for different values of \( \tau \).

6 Empirical Analysis

6.1 Data

The dataset used in the analysis is the Mexican Family Life Survey (MxFLS). It is a multi-themed nationally representative longitudinal survey. Individuals are followed over time regardless of changes in residence and household division. This characteristic makes the data particularly suitable to study economies of scale and resource allocation within the household as explained below. There are currently three rounds available. The first round took place in 2002, the second round in 2005-2006 and the third round in 2009-2010. All the rounds contain information about consumption and income. Consumption expenditures are measured at the household level and include a detailed enumeration of food and non-food items. Labor income is reported at the individual level and non-labor income at both, the individual level and the household level. The sample used in this study includes all households with non-zero income and non-zero food expenditure. The last row of table 2 shows the total number of households with these characteristics disaggregated by year.

The family and the household

The analysis in this paper exploits the distinction between the family and the household. This is new in this literature. The definition of the household is taken from the survey. It consists of people who live under the same roof and share resources.\(^{22}\) The family is defined as a group of people that belonged to the same household in 2002 regardless of whether they moved and formed a new household in subsequent years. This definition is purely instrumental for the econometric analysis. It does not try to follow any sociological or anthropological definition. Figure 3 shows diagrammatically the relationship between households and families. For example, in the first round of the survey, two identical households are observed, say the father, the mother and one son. In both households, the son got married and had a baby between the first and the second rounds. However, in one case the son and his nuclear family moved to a new house and in the other case the son and his nuclear family stayed in the same house. Family

\(^{22}\)The survey defines the household as “A person or group of people, related or unrelated by biological bonds, who usually live together in a part of or in an entire building/dwelling and usually consume meals prepared with a common budget on the same stove/oven and even use the same tools for preparing the meals.”
A and family B are treated as identical in all dimensions except that A is considered to be split (i.e. living in two houses rather than in one) in the last two rounds.

The longitudinal structure of the data allows the empirical analysis to follow closely the ideal situation to study economies of scale. According to the Barten model, when a household splits their members are on average worse off because they have to duplicate the expenditure on public goods to obtain the same per capita consumption. For example, if all members in families A and B in figure 3 want a new refrigerator in 2005, then family A have to reduce more the consumption of other goods to release resources to buy two new refrigerators, one for each house, and obtain the same per capita fridge consumption as family B. Table 2 shows the total number of families defined as in figure 3. In 2002, the number of households and the number of families are the same by construction.

Consumption shares and rent imputation The largest component of household public goods is presumably housing. However, most of the household heads report paying no rent or mortgage. Ignoring housing in total consumption underestimates the share of public goods and overestimates the shares of food and other non-food items in total expenditures. A solution to this problem is imputing the rent. Since the 2005-2006 round, interviewees were asked about the hypothetical rent that they would pay for the house they live. Although this measure is expected to be noisy, on average it is assumed to be correct. The rent imputation procedure is based on the following hedonic regression.

\[ \log(h_{jls}) = x'_{jls} \alpha + d_s + u_l + \epsilon_{jl} \quad (38) \]

The log reported hypothetical rent \( h_{jls} \) for house \( j \) in administrative unit (state) \( s \) in locality \( l \) is regressed on characteristics of the house \( x_{jls} \), dummy variables for the state \( d_s \) where the house is located and dummy variables for the population size of the locality \( u_l \). The house characteristics included in the regression are the materials of the roof, walls and floors, the access to drinkable water inside the house, the number of rooms, sewage, whether the house has a kitchen and the fuel used to cook. Regression (38) is computed for round 2005-2006 when the hypothetical rent is available.\(^{23}\) Since the regressors are available in all periods, the imputed rent (39) for each household in each period \( t \) is computed taking the exponential of the predictions.

\[ \text{irent}_{jlst} = \exp(x'_{jlst} \hat{\alpha} + \hat{d}_s + \hat{u}_l) \quad t = 2002, 2005, 2009 \quad (39) \]

The total consumption is computed by adding the imputed rent and the reported consumption expenditures of other goods. All values are adjusted by inflation. Following the theory from previous section, goods are classified into: i) sharable goods (food), ii) public goods (housing and durables) and iii) private goods (the rest of the goods). The shares are reported in table 2 with and without imputed rent. A detailed list of the components of each commodity composite is in table 1.

\(^{23}\)The round 2009-2010 also contains information about hypothetical rent. Results are almost identical when this round is used in the hedonic regression. The correlation of imputed rents using the 2005-2006 and the 2009-2010 rounds is above 0.9.
6.2 Empirical strategy

As discussed in previous sections, the sharing model is able to explain the fact that food as share of total expenditures tends to decline with household size (Deaton-Paxson paradox). This apparent contradiction between economic theory (i.e. Barten model) and empirical evidence cannot be explained by the unitary model or the collective model. The first part of the empirical section revisits the Deaton-Paxson paradox enhancing the econometric method commonly used to measure household economies of scale. In addition to analyzing how food share correlates with household size, the specification exploits household splits in longitudinal data.

\[
\omega_{ft}^s = \beta_1^s \log(y_{ft}) + \beta_2^s \log(n_{ft}) + \beta_3^s \text{split}_{ft} + z_{jt}^f \beta_4^s + \zeta_f^s + \psi_t^s + \epsilon_{ft}^s \tag{40}
\]

The estimating equation (40) is an augmented Engel’s curve where the share of food in total expenditures \(\omega_{ft}^s\) for family \(f\) in period \(t\) is regressed on (log) family per capita expenditures \(y_{ft}\). As is common practice, the size of the family \(n_{ft}\) in the regression is intended to measure the existence of economies of scale. In the presence of public goods, \(\beta_2\) should be positive to agree with the prediction of the Barten model. Enhancing the standard model, regression (40) also includes the variable \(\text{split}\) to measure economies of scale. It is an indicator that takes the value one if the family lives in two houses and zero if it lives in one house (see figure 3). Conditional on family size, \(\beta_3\) is expected to be negative in the presence of economies of scale. The coefficients \(\beta_2\) and \(\beta_3\) measures the same underlying phenomenon. So, they have to agree on the conclusions.

If the Deaton-Paxson paradox is genuinely a behavioral fact contradicting the Barten model rather than the result of an econometric misspecification, not only should \(\beta_2\) be negative, but also \(\beta_3\) positive.

A potential econometric problem generally not addressed in the estimation of Engel’s curves is the endogeneity of family size. The desire for a relatively large family may be associated with the composition of goods consumed. Although heterogeneous preferences are partially controlled for with the inclusion of demographic variables \(z_{ft}\), age and sex may not be sufficient statistics. To the extent that preferences are stable over time, the inclusion of family fixed effects \(\zeta_f\) in longitudinal data solves the problem.\(^{24}\)

Previous studies on Engel’s curves focus on the consequences of a potential misspecified functional forms and the presence of measurement error. The evidence indicates that, even though the relationship between per capita household expenditure and the share of food consumed may not be log linear as in (40), allowing more flexibility in the functional form does not alter the conclusions in relation to economies of scale. Potentially more problematic is the measurement error in total family expenditures because it affects simultaneously the right hand side and the denominator of the left hand side of (40). Following previous studies, (log) family per capita expenditures is instrumented with (log) family per capita income. The measurement

\(^{24}\)There are no clear instruments in the literature for household size. Studies about fertility and child investment use twinning as a natural experiment. However, twins’s datasets containing detailed information about income and consumption are rare. Even when such data are obtained, changes household size cannot be disentangled from changes in household composition.
errors between the instrument and the instrumented variables are likely uncorrelated. Finally, \( \psi_t \) in (40) is a set of survey round fixed effects and \( \epsilon_{ft} \) is a disturbance term.

**Economies of scale**  Equation (40), which is estimated for the share of food, can also be computed for the share of public goods \( Q \) and private goods \( x \) as defined in the previous section. Then, the estimated coefficients can be used to construct a plug-in estimator of economies of scale by taking the sample counterpart of equations (36) and (37).

\[
\hat{\text{scale}}(n, y, \tau) = \frac{1}{\hat{C}(1 + \tau)} \left( \frac{(1 + \tau)}{n} \hat{\beta}_s + \hat{\beta}_q + \hat{\omega}_q(y) \right)
\]

where

\[
\hat{C} = \left[ 1 + \tau \left( \hat{\beta}_1 + \hat{\omega}_s(y) \right) + (n - 1) \left( \frac{1}{n} \right) \left( \hat{\beta}_1 + \hat{\omega}_s(y) \right) + (n - 1) \left( \hat{\beta}_q + \hat{\omega}_q(y) \right) \right]
\]

Equation (41) differs from (36) in that the right hand side is divided by family per capita expenditures. Then, the scale coefficient is now interpreted as the equivalent percentage change in total expenditures corresponding to an additional member in the family. The scale coefficient depends on family size, per capita expenditures and level of altruism. Two of these variables are observed, \( \tau \) has to be assumed.

The coefficient superscripts in (41)-(42) indicate the dependent variable used to estimate (40) (s: food, x: private goods and q: public goods). The linearity in (40) guarantees that the estimated elasticities sum zero, so there is no need to impose that \( \beta^s_m + \beta^x_m + \beta^q_m = 0 \) for \( m = 1, 2 \). The non-homotheticity of preferences implies that the shares depend on household per capita expenditures. The next section shows results for the mean and quartiles of the distribution.

### 6.3 Results

Table 3 shows results from estimating Engel’s curves. Panel A columns 1 to 3 use the standard method where the unit of observation is the household and the presence of economies of scale is identified with the coefficient associated with log number of members. Column 1 shows the result of estimating equation (40) by OLS after pooling the three rounds of the survey. Column 2 instruments household per capita expenditures with income. Column 3 exploits the panel structure of the data by including household fixed effects in addition to instrumenting household per capita income. In these first three regressions, the computation of household expenditures does not include imputed rent. The specifications used in columns 4 and 5 are identical to those used in columns 2 and 3 but including imputed rent in the denominator of food shares and in total log household per capita expenditures.

In all the regression in panel A, the coefficient associated with household size is negative and statistically different from zero. The results contradict the Barten model and are in line with the Deaton-Paxson paradox described for other countries.
In table 3 panel B, the unit of observation is the family instead of the household. Then, it is possible to measure the impact that living in two houses \((split = 1)\) rather than in one \((split = 0)\) has on the share of food. Other than this, the specifications in panel B are identical to those in panel A. Per capita expenditures on public goods, such as housing, appliances, furniture and house decoration, are expected to increase when households split. As a result, the share of food should decline according to the Barten model. However, all the regressions in panel B show the opposite. Holding family size and total family per capita consumption constant, the share of food increases when households split. These results lead to the same conclusions obtained in panel A and reinforces the Deaton-Paxson observation that the empirical evidence contradicts the standard theory.

The enhanced method to estimate Engel’s curve suggests that the lack of agreement between theory and empirics is not the result of econometric problems. Instead, it is the theory that should be revised to reconcile it with the facts. The sharing model introduced in section 2.3 is a plausible explanation of this old puzzle.

Gan and Vernon (2003) suggest that the Deaton-Paxson paradox is not such. They claim that there are goods consumed in the household that are more rival than food.\(^{25}\) When they consider a basket of goods containing only food and goods that are presumably less rival than food (e.g. housing), the predictions of the Barten model holds. That is, the evidence shows that the share of food on food plus housing increases with household size.

Table 4 shows the result of estimating (quasi) Engel’s curves where the dependent variable is the share of food in the consumption of food and housing. Columns 1 and 2 follow the standard method. As in Gan and Vernon (2003), the share of food in food plus rent increases with household size. This result is maintained in columns 3 and 4. As predicted by the Barten model, families that live in two houses rather than in one \((split = 1)\) consume a smaller share of food in food plus housing.

Despite that results in table 4 agree with the Barten model, Deaton and Paxson (2003) argue that the paradox is not solved because the fact that, at a given level of total per capita consumption, per capita food consumption (not the food share) tends to decline as households become larger remain unexplained. In the presence of public goods, the monetary contribution each member has to make to obtain a given level of consumption declines as household size increases. Then, the freed resources can be used to consume more. Being food a normal good, the Barten model predicts that its consumption should increase.

Table 5 shows the demand for food. The specifications are identical to those in table 3 but with log per capital food consumption as a dependent variable instead of the food share. Deaton and Paxson (2003) critique is evident. Panel A shows that per capita food consumption declines with household size. This

\(^{25}\)This paper as well as others in the the literature refer as goods that are more private rather than more rival. However, the only characteristic in these models that distinguishes private and public goods is rivalry. They cannot incorporate excludability (see discussion in section 2.2)
fact cannot be explained by arguing that other goods are less rival than food as in Gan and Vernon (2003). Panel B corroborates the results in panel A. Families that live in two houses rather that in one, consume on average more food per capita at a given level of total per capita expenditures.

The sharing model presented in section 2.3 is capable of explaining the consumption patterns in tables 3-5. If food is a common-pool good but still rival in consumption, then the larger the household is, the smallest the portion a member gets from her marginal contribution to the common pot. Thus, as households become larger, their members have more incentives to reallocate resources from sharable goods (e.g. food) to private goods (e.g. clothing).

**Economies of scale coefficients** The Engel’s curve specification used to compute scale coefficients in this section corresponds to the pooled instrumental variable as shown in column 4 table 3. This specification generates similar results to panel-fixed-effects but allows the computation of scale coefficients for different quartiles of the distribution.\(^{26}\)

Table 6 shows estimated household economies of scale using formula (41)-(42). In this case, the public goods, private goods, and food shares are evaluated at the sample mean. Each column is generated using a different assumption about the level of altruism in the family. The results indicate, for example, that a two-person household with no altruism \((\tau = 0)\) would need 12.4% more income to obtain the same utility level as adding an extra member and maintaining the same per capita income.

The last row of table 6 shows the single adjustment factor that best fits the results. Inequality and poverty studies that attempt to adjust per capita consumption/income by economies of scale usually rely on the following single parameter formula (Deaton (1997)).

\[
y^{adj} = \frac{Y}{n \xi}, \quad 0 \leq \xi \leq 1
\]  

(43)

Adjusted household per capita income/consumption \(y^{adj}\) is computed dividing total income/consumption \(Y\) by household size adjusted by \(\xi\). Depending on the level of altruism assumed, the estimated adjustment factor \(\hat{\xi}\) ranges from 0.7 to 0.84. Figure 4 plots two of the columns in table 6 (solid lines) and the implied coefficients computed with formula (43) and the estimated \(\xi\) at the bottom of the table (dotted lines). The estimated values and the fitted lines using (43) are remarkably similar.

For comparison reasons, the last column of table 6 reports similar statistics than previous columns but computed with scale parameters used by the OECD, see Cowell (2011). The OECD assumes significantly stronger economies of scale from a one-person household to a two-person households.\(^{27}\)

\(^{26}\) Some families change quartiles from one round to another creating problems in partitioning the sample by living standards.\(^{27}\) The formula used by the OECD to adjust family income is \(y^{adj} = \frac{Y}{\sum d_i},\) where \(Y\) is total family income and \(d_i\) is a value assigned to household member \(i = 1, \ldots, n\). The values are \(d_1 = 0.67\) for the first adult in the household \(d_i = 0.33\) for each of the other adults and \(d_{14} = 0.2\) for each child under 14 years old (Cowell (2011) page 105). Last column in table 6 assumes that all members are adults.
Table 7 shows the scale coefficients for different quartiles of the total family expenditures distribution. The adjustment for economies of scale is not monotonic along the income distribution because the slope of the Engel’s curve is highest (in absolute terms) for the second quartile.

The method used in this section to measure economies of scale in the family is easily implemented and does not require panel data. It can be computed with any cross-section data containing information about household consumption.

6.4 Public goods with congestion

For the computation of economies of scale, goods are considered extreme cases in relation to rivalry and excludability (e.g. public goods are considered fully non-rival and non-excludable). This section presents a sensitivity analysis changing these assumptions.

When congestion affects public goods, the sharing rule (16) should be modified:

\[ Q = \frac{\hat{Q}_1 + (n-1)\hat{Q}_2}{n^\phi}, \quad 0 \leq \phi \leq 1 \] (44)

The parameter \( \phi \) controls the degree of congestion. When \( \phi = 0 \), the good is completely non-rival, when \( \phi = 1 \) the good is fully rival and the allocation rule becomes identical to that for sharing goods in equation (15). It can be shown that congestion affects the measurement of economies of scale by adjusting the last term of (36). This expression becomes (see appendix IV)

\[ \frac{\partial V}{\partial n}/\lambda C = \frac{y}{C(1+\tau)} \left( \frac{(1+\tau)}{n} \frac{\partial \omega_x}{\partial \log(n)} + \frac{\partial \omega_q}{\partial \log(n)} + \frac{\partial \omega_s}{\partial \log(n)} + \omega_q(1-\phi) \right) \] (45)

Table 8 shows the economies of scale coefficients for congestion levels \( \phi = 0.1 \) and \( \phi = 0.3 \) (low and medium levels of altruism). The values in this table are lower in relation to those in table 6. For relatively high congestion \( \phi = 3 \) the scale coefficients decline approximately one third.

7 Summary and conclusions

This paper presents a new model for the allocation of resources in the household where individuals are assumed to control their own income and voluntarily share purchased goods with other members of the family. The allocation rule for each good depends on the intrinsic characteristic of the commodity and on the norms governing the interaction of household members. In this respect, in addition to public goods and private goods commonly included in other models, this paper recognizes the existence of common-pool goods. Common-pool goods are rival in consumption, but contrary to private goods, they are non-excludable. These goods cannot be fully appropriated by the buyer because they are supposed to be shared with other members in the household. Food is argued to be a common-pool good. The ‘common provision of food’ is considered a distinctive characteristic in many definitions of the household.
The model in the paper challenges the assumption made by the collective model that the allocation of resources in the household is efficient. A Pareto-efficient allocation is incompatible with the consumption patterns observed in the data, but not with the Nash equilibrium described in the paper. More specifically, the model is able to explain why per capita food consumption tends to decline as household size increases. This fact is considered an unresolved puzzle (Deaton-Paxson paradox). Previous models of the household are not capable of explaining this consumption patterns.

The first part of the empirical analysis revisits the Deaton-Paxson paradox exploiting household divisions in longitudinal data. Results using the new approach reinforces the conclusions found in Deaton and Paxson (1998). The second part of the empirical section derives and implements a method to compute household economies of scale. The estimation of economies of scale coefficients has been elusive because of the unresolved Deaton-Paxson paradox. The correct computation of economies of scale has enormous policy implications since it affects how to measure inequality and poverty.

Empirical papers in the literature have rejected intra-household Pareto-optimality but have not modeled households’ behavior to explain their findings. Udry (1996) suggests that more theoretical research is needed to explain why households do not reach the utility-possibility frontier as described in his paper. The sharing model is a simple and tractable theory that breaks the tradition of imposing intrahousehold efficiency. It is grounded on the reasonable idea that allocation rules differ by good type. The ability of the sharing model to explain consumption regularities that other theories cannot describe suggests that the model deserves more examination.

The sharing model presented in this paper can be used to study a variety of topics. Ongoing research includes the unification of theories about household formation, household dissolution and intrahousehold allocation of resources, as well as the study of joint labor supply.
References


8 Figures and tables

Figure 1: Hump-shaped Engel’s curve (n=2):
Food share as a function of log per capita income

The parameter values are: $\sigma = 2$, $\beta = 3$, $\gamma = 0.043$. The prices of all goods are set to 1.

Figure 2: Indirect utility as a function of household size:
Different levels of altruism

The parameter values are: $\sigma = 2$, $\gamma = 0.043$. Per capita expenditures are set to 10. The three indirect utilities are rescaled to equal one for $n = 1$. 
Figure 3: ‘Empirical’ definition of households and families

Note: Each diamond represents a person-round observation. The arrows follow persons across the longitudinal dataset rounds.

Figure 4: Percentage increase in per capita income equiv. to adding a household member (by family size)

Table 1: Consumption components

<table>
<thead>
<tr>
<th>Commodity composite</th>
<th>Goods included</th>
<th>List</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharable goods</td>
<td>food except meals consumed away from home</td>
<td>vegetables, fruits, cereals, grains, meats, industrially-processed food</td>
</tr>
<tr>
<td>Public goods</td>
<td>housing, durables and school fees</td>
<td>imputed rent, TV sets, radios, cameras, washing machines, refrigerators, furniture, school tuition and fees, etc.</td>
</tr>
<tr>
<td>Private goods</td>
<td>clothing, tobacco, transportation, hygiene, etc.</td>
<td>clothing, toys, medicines, doctor’s visits lotions, deodorants, magazines, etc.</td>
</tr>
</tbody>
</table>

Note: Some studies consider children as household public goods, this is why school tuitions and fees are included in this category. Results are very similar when these items are considered private goods.
Table 2: Summary statistics

<table>
<thead>
<tr>
<th></th>
<th>Survey year</th>
<th></th>
<th></th>
<th></th>
</tr>
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<tr>
<td></td>
<td></td>
<td>2002</td>
<td>2005-2006</td>
<td>2009-2010</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>s.d.</td>
<td>mean</td>
<td>s.d.</td>
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<td>1.982</td>
<td>4.939</td>
<td>2.343</td>
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<td>0.000</td>
<td>0.082</td>
<td>0.275</td>
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<td></td>
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<tr>
<td>share food</td>
<td>0.427</td>
<td>0.163</td>
<td>0.456</td>
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<td>share private</td>
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<td>0.155</td>
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<tr>
<td>share public</td>
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<td>0.127</td>
<td>0.237</td>
<td>0.120</td>
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<tr>
<td>shares without imp. rent</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>share food</td>
<td>0.520</td>
<td>0.196</td>
<td>0.564</td>
<td>0.194</td>
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<td>share private</td>
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<td>(log) per capita expend.</td>
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<td>(log) per capita income</td>
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<td>1.232</td>
<td>7.239</td>
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<tr>
<td>households in sample</td>
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<td>5,205</td>
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Table 3: Engel’s curve

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<tr>
<th></th>
<th>Food share (excl. rent)</th>
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<th>Food share (w/ imp. rent)</th>
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<td>IV-FE</td>
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<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
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<tr>
<td>Panel A: Households</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>log household per capita exp. ( + )</td>
<td>-0.132***</td>
<td>-0.175***</td>
<td>-0.174***</td>
<td>-0.137***</td>
</tr>
<tr>
<td></td>
<td>(0.00169)</td>
<td>(0.00468)</td>
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<td>-0.0999***</td>
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<td>(0.00341)</td>
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<td></td>
<td>(0.0155)</td>
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<td>15,639</td>
<td>15,639</td>
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<td>Panel B: Families</td>
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<td>-0.173***</td>
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</table>

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

All regressions include survey round fixed effects and dummies for the share of household members with ages 0-5, 6-11, 12-17, 18-64 and 65+ by gender

+ instrumented with log household per capita income in columns 2 to 5

++ instrumented with log family per capita income in columns 2 to 5
Table 4: (quasi) Engel’s curve:
Dep. var.: share of food in food plus imputed rent

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
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<tr>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0366)</td>
<td>(0.169)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.797***</td>
<td>0.0283</td>
<td>0.797***</td>
<td>0.0283</td>
</tr>
<tr>
<td></td>
<td>(0.0367)</td>
<td>(0.157)</td>
<td>(0.0367)</td>
<td>(0.157)</td>
</tr>
<tr>
<td>Observations</td>
<td>15,633</td>
<td>15,633</td>
<td>14,609</td>
<td>14,609</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
All regressions include survey round fixed effects and dummies for the share of household members with ages 0-5, 6-11, 12-17, 18-64 and 65+ by gender
⁺ instrumented with log household per capita income
++] instrumented with log family per capita income
Table 5: Demand for food

Dep. var. log per capita food consumption

<table>
<thead>
<tr>
<th></th>
<th>Panel A: Households</th>
<th>Panel B: Families</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS (1)</td>
<td>IV-GMM (2)</td>
</tr>
<tr>
<td>log household per capita exp. +</td>
<td>0.688***</td>
<td>0.590***</td>
</tr>
<tr>
<td></td>
<td>0.00431</td>
<td>0.0123</td>
</tr>
<tr>
<td>log household size</td>
<td>0.121***</td>
<td>0.197***</td>
</tr>
<tr>
<td></td>
<td>0.00868</td>
<td>0.0133</td>
</tr>
<tr>
<td>Constant</td>
<td>1.633***</td>
<td>2.445***</td>
</tr>
<tr>
<td></td>
<td>0.0394</td>
<td>0.104</td>
</tr>
<tr>
<td>Observations</td>
<td>15,630</td>
<td>15,630</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>w/imputed rent</th>
</tr>
</thead>
<tbody>
<tr>
<td>log household per capita exp. +</td>
<td>0.687***</td>
</tr>
<tr>
<td></td>
<td>0.00442</td>
</tr>
<tr>
<td>log family size</td>
<td>0.123***</td>
</tr>
<tr>
<td></td>
<td>0.00924</td>
</tr>
<tr>
<td>split</td>
<td>0.0960***</td>
</tr>
<tr>
<td></td>
<td>0.0139</td>
</tr>
<tr>
<td>Constant</td>
<td>1.645***</td>
</tr>
<tr>
<td></td>
<td>0.0405</td>
</tr>
<tr>
<td>Observations</td>
<td>14,609</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

All regressions include survey round fixed effects and dummies for the share of household members with ages 0-5, 6-11, 12-17, 18-64 and 65+ by gender

+ instrumented with log household per capita income in columns 2 to 5

++ instrumented with log family per capita income in columns 2 to 5
Table 6: Perc. increase in per capita income equiv. to adding a household member

<table>
<thead>
<tr>
<th>family size</th>
<th>altruism ( \tau = 0 )</th>
<th>altruism ( \tau = 0.2 )</th>
<th>altruism ( \tau = 0.5 )</th>
<th>altruism ( \tau = 0.8 )</th>
<th>OECD( ^\dagger )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.241</td>
<td>0.200</td>
<td>0.159</td>
<td>0.132</td>
<td>0.340</td>
</tr>
<tr>
<td>2</td>
<td>0.124</td>
<td>0.103</td>
<td>0.081</td>
<td>0.067</td>
<td>0.128</td>
</tr>
<tr>
<td>3</td>
<td>0.086</td>
<td>0.071</td>
<td>0.056</td>
<td>0.046</td>
<td>0.068</td>
</tr>
<tr>
<td>4</td>
<td>0.067</td>
<td>0.055</td>
<td>0.043</td>
<td>0.036</td>
<td>0.037</td>
</tr>
<tr>
<td>5</td>
<td>0.055</td>
<td>0.045</td>
<td>0.036</td>
<td>0.029</td>
<td>0.030</td>
</tr>
<tr>
<td>6</td>
<td>0.047</td>
<td>0.039</td>
<td>0.030</td>
<td>0.025</td>
<td>0.022</td>
</tr>
<tr>
<td>7</td>
<td>0.041</td>
<td>0.034</td>
<td>0.027</td>
<td>0.022</td>
<td>0.013</td>
</tr>
<tr>
<td>8</td>
<td>0.036</td>
<td>0.030</td>
<td>0.024</td>
<td>0.019</td>
<td>0.014</td>
</tr>
<tr>
<td>9</td>
<td>0.033</td>
<td>0.027</td>
<td>0.021</td>
<td>0.018</td>
<td>0.011</td>
</tr>
</tbody>
</table>

\( \hat{\xi} \quad 0.704 \quad 0.753 \quad 0.803 \quad 0.837 \quad 0.752 \)

In all cases shares are evaluated at sample mean

\( \dagger \) Computed using modified-OECD equivalence scales

Table 7: Economies of scale by income quartile, altruism and family size

<table>
<thead>
<tr>
<th>family size</th>
<th>low altruism (( \tau = 0 ))</th>
<th>medium altruism (( \tau = 0.5 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q1</td>
<td>q2</td>
</tr>
<tr>
<td>1</td>
<td>0.295</td>
<td>0.257</td>
</tr>
<tr>
<td>2</td>
<td>0.135</td>
<td>0.089</td>
</tr>
<tr>
<td>3</td>
<td>0.087</td>
<td>0.052</td>
</tr>
<tr>
<td>4</td>
<td>0.064</td>
<td>0.036</td>
</tr>
<tr>
<td>5</td>
<td>0.051</td>
<td>0.028</td>
</tr>
<tr>
<td>6</td>
<td>0.042</td>
<td>0.022</td>
</tr>
<tr>
<td>7</td>
<td>0.036</td>
<td>0.019</td>
</tr>
<tr>
<td>8</td>
<td>0.031</td>
<td>0.016</td>
</tr>
<tr>
<td>9</td>
<td>0.027</td>
<td>0.014</td>
</tr>
</tbody>
</table>

\( \hat{\xi} \quad 0.696 \quad 0.796 \quad 0.614 \quad 0.638 \quad 0.791 \quad 0.833 \quad 0.768 \quad 0.774 \)
Table 8: Economies of scale in the presence of congestion

<table>
<thead>
<tr>
<th>family</th>
<th>$\tau = 0$</th>
<th>$\tau = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>size</td>
<td>$\phi = 0.1$</td>
<td>$\phi = 0.3$</td>
</tr>
<tr>
<td>1</td>
<td>0.217</td>
<td>0.169</td>
</tr>
<tr>
<td>2</td>
<td>0.110</td>
<td>0.080</td>
</tr>
<tr>
<td>3</td>
<td>0.075</td>
<td>0.054</td>
</tr>
<tr>
<td>4</td>
<td>0.058</td>
<td>0.041</td>
</tr>
<tr>
<td>5</td>
<td>0.048</td>
<td>0.033</td>
</tr>
<tr>
<td>6</td>
<td>0.040</td>
<td>0.028</td>
</tr>
<tr>
<td>7</td>
<td>0.035</td>
<td>0.024</td>
</tr>
<tr>
<td>8</td>
<td>0.031</td>
<td>0.021</td>
</tr>
<tr>
<td>9</td>
<td>0.028</td>
<td>0.019</td>
</tr>
</tbody>
</table>

$\hat{\xi}$  0.739  0.808  0.825  0.869
Appendix I: Derivation of demand system

The Kuhn-Tucker conditions in problem (12)-(17) obtained from increasing in one unit the quantity purchased of each good are:

\[ x_1^{-\sigma} \leq \lambda p_x , \quad \tilde{x}_{11} \geq 0 , \quad \tilde{x}_{11} (x_1^{-\sigma} - \lambda p_x) = 0 \]
\[ \tau x_2^{-\sigma} \leq \lambda (n-1) p_x , \quad \tilde{x}_{12} \geq 0 , \quad \tilde{x}_{12} (\tau x_2^{-\sigma} - \lambda (n-1) p_x) = 0 \]
\[ (s_1 - \gamma)^{-\sigma} + \tau (s_2 - \gamma)^{-\sigma} \leq \lambda n p_s , \quad \tilde{s}_1 \geq 0 , \quad \tilde{s}_1 ((s_1 - \gamma)^{-\sigma} + \tau (s_2 - \gamma)^{-\sigma} - \lambda n p_s) = 0 \]
\[ Q^{-\sigma} + \tau Q^{-\sigma} \leq \lambda p_q , \quad \tilde{Q}_1 \geq 0 , \quad \tilde{Q}_1 (Q^{-\sigma} + \tau Q^{-\sigma} - \lambda p_q) = 0 \]

In the symmetric case \( x_1 = x_2 \), which implies that the second equation holds with strict inequality when the first one holds with equality for \( \tau < (n-1) \). This means that there will be zero gift/transfer of private goods to other members \( (\tilde{x}_{12} = 0) \). The symmetry of the game implies that \( \tilde{x}_{11} = \tilde{x}_{22} , \tilde{x}_{12} = \tilde{x}_{21} = 0 , \tilde{s}_1 = \tilde{s}_2 \) and \( \tilde{Q}_1 = \tilde{Q}_2 \). Then, using conditions (13)-(16), the Kuhn-Tucker conditions simplifies to

\[ \tilde{x}^{-\sigma} = \lambda p_x \quad (46) \]
\[ (1 + \tau)(\tilde{s} - \gamma)^{-\sigma} = \lambda n p_s \quad (47) \]
\[ (1 + \tau)(n\tilde{Q})^{-\sigma} = \lambda p_q \quad (48) \]

where \( \tilde{x} , \tilde{s} \) and \( \tilde{Q} \) are per capita expenditures on each of the good composites. These equalities together with the budget constraint \( p_x \tilde{x} + p_x \tilde{s} + p_q \tilde{Q} = y \) determine the per capita demand system (18).

Appendix II: The allocation of sharable goods

The goals of this appendix are i) to show that \( \delta_i \) in the allocation rule (5) can be obtained from a nested collective model, and ii) to show that the Attanasio and Lechene (2014) test implemented with sharable goods has no power against the sharing model.

When there are multiple shareable goods, the model has to specify a mechanism to allocate them. It is reasonable to assume that household members consume a relatively higher proportion of the sharable goods they prefer and leave other sharable goods to members who value them most, generating a (conditionally) efficient allocation of common-pool goods.

Consider a household with two non-identical members. Let there be two sharable goods \( v \) and \( w \) in the sharable composite \( s_1 \) for member 1. The functional form for the sharable composite is as follows.

\[ s_1 = (\theta_1 v_1^{1-\pi} + (1 - \theta_1) w_1^{1-\pi})^{1/(1-\pi)} \quad (49) \]

Household member 1 and household member 2 may differ in the relative preferences of sharable goods, i.e. \( \theta_1 \) and \( \theta_2 \). The efficient allocation of sharable goods can be obtained maximizing a household 'social
planner’ objective function that consists of a weighted sum of sharable composites for member 1 and member 2.

\[
\max \beta s_1 + (1 - \beta)s_2
\]  

(50)

Instead of using this standard linear function for the social planner, for convenience I uses a CES type function, which also results in an efficient allocation of resources.

\[
\max \left( \beta \pi_1 s_1^{1-\pi} + (1 - \beta) \pi_2 s_2^{1-\pi} \right)^{1/(1-\pi)}
\]  

(51)

Replacing (49) into (51) the problem becomes

\[
\max \left( \beta \pi_1 v_1^{1-\pi} + (1 - \beta) \pi_2 v_2^{1-\pi} + (1 - \beta_2)\pi_2 w_2^{1-\pi} \right)^{1/(1-\pi)}
\]  

(52)

\[
st \quad p_v(v_1 + v_2) + p_w(w_1 + w_2) = Y_t
\]  

(53)

where \(Y_t\) is the total income of all members use to purchase sharable goods. The efficient quantities are:

\[
v_1 = \frac{Y_t \beta \theta_1}{P_1^{(1-\pi)/\pi} p_v^{1/\pi}}
\]  

(54)

\[
w_1 = \frac{Y_t \beta(1 - \theta_1)}{P_1^{(1-\pi)/\pi} p_w^{1/\pi}}
\]  

(55)

\[
v_2 = \frac{Y_t (1 - \beta)\theta_2}{P_2^{(1-\pi)/\pi} p_v^{1/\pi}}
\]  

(56)

\[
w_2 = \frac{Y_t (1 - \beta)(1 - \theta_1)}{P_2^{(1-\pi)/\pi} p_w^{1/\pi}}
\]  

(57)

where

\[
P = \left( \beta P_1^{(1-\pi)/\pi} + (1 - \beta)P_2^{(1-\pi)/\pi} \right)^{\pi/(\pi-1)}
\]  

(58)

\[
P_1 = \left( \theta_1 p_v^{(1-\pi)/\pi} + (1 - \theta_1)p_w^{(1-\pi)/\pi} \right)^{\pi/(\pi-1)}
\]  

(59)

\[
P_2 = \left( \theta_2 p_w^{(1-\pi)/\pi} + (1 - \theta_2)p_v^{(1-\pi)/\pi} \right)^{\pi/(\pi-1)}
\]  

(60)

Replacing demands (54)-(57) in (49) obtains the sharable good composites for each member and for the household.

\[
s_1 = \frac{Y_t \beta}{P_1^{(1-\pi)/\pi} P_1^{1/\pi}} = \frac{Y_t}{P} \left( \frac{P}{P_1} \right)^{1/\pi} \beta
\]  

(61)

\[
s_2 = \frac{Y_t (1 - \beta)}{P_2^{(1-\pi)/\pi} P_2^{1/\pi}} = \frac{Y_t}{P} \left( \frac{P}{P_2} \right)^{1/\pi} (1 - \beta)
\]  

(62)

\[
s = \frac{Y_t}{P}
\]  

(63)

where

\[
\left( \frac{P_1}{P} \right)^{1/\pi} s_1 + \left( \frac{P_2}{P} \right)^{1/\pi} s_2 = s
\]  

(64)
\( P \) is the price of increasing in one unit of the composite good \( s \). From (61), (62) and (49)

\[
\begin{align*}
    s_1 &= s \left( \frac{P}{P_1} \right)^{1/\pi} \beta \\
    s_2 &= s \left( \frac{P}{P_2} \right)^{1/\pi} (1 - \beta)
\end{align*}
\] (65) (66)

Notice that (65) and (66) relate the total consumption (and purchases) of sharable goods in the household with individual consumptions. These expressions are identical to (5) with \( \delta_1 = \beta \left( \frac{P}{P_1} \right)^{1/\pi} \) and \( \delta_2 = (1 - \beta) \left( \frac{P}{P_2} \right)^{1/\pi} \). It is evident the relationship between the allocation parameters \( \delta_i \) and the individual weight \( \beta \).

If \( \beta \) is assumed to depend on ‘distribution factors’ as the collective model, equality (31) holds for any pair of sharable goods.

**Appendix III: Marginal utility of income**

The Lagrange function evaluated at the optimum is

\[
\mathcal{L}^* = (1 + \tau)U(\tilde{x}^*, \tilde{s}^*, n\tilde{Q}^*) + \lambda \left[y - p_x \tilde{x} - p_s \tilde{s} - p_q \tilde{Q}\right]
\] (67)

The marginal utility of per capita income (after arriving to a new Nash equilibrium) is

\[
\frac{\partial \mathcal{L}^*}{\partial y} = (1 + \tau) \left[U'_x \frac{\partial \tilde{x}^*}{\partial y} + U'_s \frac{\partial \tilde{s}^*}{\partial y} + nU'_q \frac{\partial \tilde{Q}^*}{\partial y}\right] + \lambda \left[1 - p_x \frac{\partial \tilde{x}^*}{\partial y} - p_s \frac{\partial \tilde{s}^*}{\partial y} - p_q \frac{\partial \tilde{Q}^*}{\partial y}\right]
\] (68)

\[
= \lambda \left[1 + \tau p_x \frac{\partial \tilde{x}^*}{\partial y} + (n - 1)p_s \frac{\partial \tilde{s}^*}{\partial y} + (n - 1)p_q \frac{\partial \tilde{Q}^*}{\partial y}\right]
\] (69)

\[
= \lambda \left[1 + \tau \left(\frac{\partial \omega_x}{\partial \log(y)} + \omega_x\right) + (n - 1) \left(\frac{\partial \omega_s}{\partial \log(y)} + \omega_s\right) + (n - 1) \left(\frac{\partial \omega_q}{\partial \log(y)} + \omega_q\right)\right]
\] (70)

Using the FOC (46)-(48) expression (68) becomes (69). Then, the fact that

\[
\frac{\partial \omega_x}{\partial \log(y)} = p_x \frac{\partial \tilde{x}^*}{\partial y} - \frac{p_x \tilde{x}^*}{y}
\]

is used to obtain (70)

**Appendix IV: Measuring economies of scale in the presence of public goods with congestion**

Consider the sharing rule (44). The three expressions for the Lagrange multiplier in (33) become:

\[
\lambda = \frac{\partial U^*}{\partial x} \frac{1}{p_x} = \frac{\partial U^*}{\partial s} \frac{(1 + \tau)}{np_s} = \frac{\partial U^*}{\partial Q} \frac{(1 + \tau)}{n^p p_q}
\] (71)
Replacing (71) in (32) and using the fact than in equilibrium the relationship between per-capita consumed and per-capita purchased goods are: \( s = \hat{s}, \ x = \hat{x} \) and \( Q = n^{(1-\phi)}\hat{Q} \), then (36) simplifies to the following expression:

\[
\frac{\partial V}{\partial n}/\lambda C = p_s \frac{\partial \hat{x}}{\partial n} + \frac{np_s}{(1 + \tau)} \frac{\partial \hat{s}}{\partial n} + \frac{np_q}{(1 + \tau)} \frac{\partial \hat{Q}}{\partial n} + \frac{p_q\hat{Q}(1 - \phi)}{(1 + \tau)}
\]

\[
= \frac{y}{C(1 + \tau)} \left( \frac{1 + \tau}{n} \frac{\partial \omega_x}{\partial \log(n)} + \frac{\partial \omega_s}{\partial \log(n)} + \frac{\partial \omega_q}{\partial \log(n)} + \omega_q(1 - \phi) \right)
\]