Vanderbilt University
Department of Economics

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MACROECONOMIC THEORY PRELIMINARY EXAMINATION

Spring 2016

General Instructions: There are 7 questions, and you must answer all questions. Please read the questions very carefully and make each of your answers as clear and rigorous as possible. Write legibly and budget your time carefully.
Consider an overlapping generations economy in which the representative agent lives for two periods, works in the first and consumes in the second. The representative old agent at time $t$ holds $M_t$ dollars at the beginning of the period. He then gets a random money transfer in a lump sum form. The post-transfer money supply per old agent is $M_t(1 + \bar{x})$ where $\bar{x}$ is an iid random variable. Prices are determined at the beginning of the period before the realization of $\bar{x}$ is known. The price at time $t$ is:

$$P_t = pM_t$$

for all $t$, where here $p$ (the normalized price) is an exogenously given constant.

The representative agent’s utility function is: $c_{t+1} - v(L_t)$, where $v(L) = (\frac{1}{2})L^2$. Output is equal to labor input.

(a) The young agent computes his real wage ($w(x)$ = the amount of second period consumption per unit of labor) after the realization $x$ of $\bar{x}$ is observed. Write the real wage as a function of $x$.

(b) Define labor supply as the solution to: $w(x)L - v(L)$, where $w(x)$ is the real wage computed in (a). What is the labor supply as a function of the realization of $\bar{x}$?

(c) Assume now that the young agent is committed to satisfy the demand of $\frac{M_t(1 + x)}{P_t}$ units. Express the amount of work as a function of $x$ and the normalized price $p$.

(d) What is his utility when satisfying demand?

(e) Is this utility greater or lower than $\max_x w(x)L - v(L)$? As before, $w(x)$ is the real wage computed in (a).

(f) Write the problem of a planner who wants to maximize the welfare of the representative consumer in the steady state.
(g) Assume that the normalized price is $p = 1$ and consider the problem of a policy-maker that can choose the distribution of $\tilde{x}$. What is the optimal policy? (Assume that the policy-maker wants to maximize steady state welfare).

2. Consider an economy with two assets: bonds and money. The representative agent lives for two periods. The real rate of return on bonds is $r$ and the real rate of return on money is $r_m$. The representative agent chooses consumption ($C_t$), labor ($L_t$) and the real amounts of bonds ($b$) and money ($m$). The real value of his initial financial wealth is $A_0$. His labor income is equal to his labor input and he pays a fraction $\tau$ of his income to the government ($\tau$ is the proportional income tax rate). His first period consumption is given by:

$$C_1 = (1 - \tau)L_1 - b - m + A_0$$

His second period consumption is given by:

$$C_2 = (1 - \tau)L_2 + b(1 + r) + m(1 + r_m)$$

(a) Derive the budget line of the representative agent in a present value form.

(b) In equilibrium the budget line that you have derived in (a) must hold. In addition the following market conditions must hold:

$$C_t + G_t = L_t$$

where $G_t$ is real government spending.

Use the representative agent's budget constraint and the market clearing conditions to derive the so called "government budget constraint".

(c) What is the effect of an increase in the financial wealth ($A_0$) on the present value of government spending? Assume that ($r, r_m, \tau, L_1, L_2$) remain constant.

(d) What is the effect of an increase in the financial wealth ($A_0$) on $\tau$? Assume that ($r, r_m, L_1, L_2, G_1, G_2$) remains constant.
3. Consider the following UST model. There are two goods: $X, Y$ with lower case letters denoting quantities. There is one seller, $N$ definite buyers and $\Delta$ possible buyers. The seller can produce $x$ units of $X$ at the cost of $C(x)$ units of $Y$. The cost function $C(x)$ is standard ($C' > 0, C'' > 0$).

The $N$ definite buyers arrive first and buy in the first market. A second group of $\Delta$ possible buyers arrives only if $s = 2$ and if they arrive they buy in the second market. The probability that $s = 2$ and the second market opens is $\pi$.

The seller is a price-taker. The price in the first market is $P_1$ and the price in the second market (if it opens) is $P_2$. The seller chooses the supply to the first market $x_1$ and the supply to the second market $x_2$ by solving the following problem:

\begin{equation}
\max_{x_1,x_2} P_1 x_1 + \pi P_2 x_2 - C(x_1 + x_2)
\end{equation}

The first order conditions that an interior solution to (1) must satisfy are:

\begin{equation}
P_1 = \pi P_2 = C'(x_1 + x_2)
\end{equation}

(a) Write the market clearing conditions under the assumption that each buyer that arrives wants to buy 1 unit (assume inelastic demand for this unit).

(b) Assume now that storage is possible. There are agents who specialize in storing goods and these agents are willing to pay $\lambda$ per unit regardless of the state. Write the problem of the seller for this case.

(c) Under what condition the seller will supply a positive amount to both markets?

(d) Under what condition the seller will supply only to market 1 (and nothing to market 2)?
4. Consider the following UST model that allows for storage. The beginning of period inventories is $I$. The number of active buyers $\tilde{N}$ can take two possible realizations: $N$ and $N+\Delta$. A group of $N$ buyers arrive first with probability 1. A second group of $\Delta$ buyers arrive after the first group with probability $\frac{1}{2}$. Production $x$ is chosen prior to the information about the realization of $\tilde{N}$ and the cost of production is $C(x) = x^2$.

Consider the following two planner's problem that employ different assumptions about the information received by the planner.

(1) $V(I) = \max_{k_1, k_2, k_3, x} \left\{ NU \left( \frac{k_1}{N} \right) + \frac{1}{2} \Delta U \left( \frac{k_2}{\Delta} \right) + \bar{y} - x^2 + \frac{1}{2} \beta V(k_1) + \frac{1}{2} \beta V(k_2) \right\}$

s.t. $k_1 + k_2 + k_3 = I + x$

(2) $W(I) = \max_{k_{11}, k_{12}, k_2, k_3, x} \left\{ \frac{1}{2} NU \left( \frac{k_{11}}{N} \right) + \frac{1}{2} NU \left( \frac{k_{12}}{N} \right) + \frac{1}{2} \Delta U \left( \frac{k_2}{\Delta} \right) + \bar{y} - x^2 + \beta W(k_3) \right\}$

s.t.

$k_{11} + k_3 = I + x$

$k_{12} + k_2 + k_3 = I + x$

(a) When does the planner learn about the realization of $\tilde{N}$ in problem (1)?

(b) When does the planner learn about the realization of $\tilde{N}$ in problem (2)?

(c) Problem (1) can be stated by adding a constraint to problem (2). What is the constraint?

(d) Compare $V(I)$ to $W(I)$. Which one is larger? Explain.

(e) What is the level of next period's inventories if the planner solves problem (1)?

(f) What is the level of next period's inventories if the planner solves problem (2)?
5. Consider an economy populated by identical representative agents who have preferences of the following form

\[ \sum_{i=0}^{\infty} \beta^i \log(c_i). \]

Here, \( c_t \) represents consumption in period \( t \). Also, \( \beta \in (0,1) \) is the discount factor. However, each period there is a probability \( \theta \) that the individual would survive until the following period, and a probability \( 1 - \theta \) that they will not survive (i.e. die). When this latter event happens, the agent’s capital stock will be lost, and they will receive utility of zero forever after. That is, in any period the individual knows that he will consume in the present period, but does not know if he will consume in the following period.

The technology faced by every individual the economy is the following

\[ c_t + k_{t+1} = z(k_t^\alpha) \]

Here \( k_t \) represents the capital stock chosen in period \( t-1 \) which will produce output in the following period. The restrictions are that \( z > 0, 0 < \alpha < 1 \).

a) Set up the dynamic programming problem, and identify the state and control variables.

b) Obtain the optimal decision rules for investment and consumption. Use the method of undetermined coefficients to do this.

c) What parameter restrictions are necessary to insure that the value function is well-defined?

d) Calculate the savings rate, and explain how the parameters \( \alpha, \beta, \theta \) affect the savings rate. Explain why you think your answer makes sense.

e) Calculate a difference equation for \( \log(k_t) \) as a function of \( \log(k_{t-1}) \), and other variables or constants. Also, calculate the value of \( \log(k_t) \) as a function of \( \log(k_0) \).

f) Suppose that \( \alpha = 1 \). Develop a condition that would guarantee that aggregate output (i.e. not per-capita output) would grow at a positive, constant rate.
6. Consider an economy populated by identical, representative agents who derive utility from both consumption and leisure, and have preferences of the following form

$$\sum_{t=0}^{\infty} \beta^t [U(c_t) + V(1 - n_t)].$$

Here $c_t$ represents period-$t$ consumption, and $n_t$ is labor or employment. Also, $\beta \in (0,1)$ is the discount factor. The individual is endowed with one unit of time in each period. The technology for this economy uses both capital ($k_t$) and labor as inputs, and is written as: $y_t = A(k_t^\alpha)(n_t^{1-\alpha})$. Capital depreciates at the rate of $\delta$ per period. The resource constraint for this economy can then be written as follows:

$$c_t + i_t = A(k_t^\alpha)(n_t^{1-\alpha})$$

and

$$k_{t+1} = (1 - \delta)k_t + i_t$$

There is no uncertainty in this problem.

a) Set up the dynamic programming problem for a planner who seeks to maximize the discounted utility of the individuals, subject to the resource constraint. Identify the state and control variables.

b) Develop the optimization conditions for the optimal consumption/saving decision, as well as that of employment.

c) Show how the steady-state capital-labor ratio ($k_t/n_t$) is influenced by the parameter $A$. That is, you should be able to develop a formula for $\frac{d(k_t/n_t)}{dA}$.

d) In a competitive equilibrium, the wage of labor will equal the marginal product of labor $\left(\frac{dy_t}{dn_t}\right)$, while the net rate of return to capital will equal the marginal product of capital less the depreciation rate $\left(\frac{dy_t}{dk_t}\right) - \delta$. Show how the steady-state wage will be influenced by the parameter $A$.

e) Show how the steady-state net rate of return to capital will be influenced by the parameter $A$.

f) Suppose that these allocations were in existence in a competitive equilibrium, but that there also was a government that was taxing the net rate of return to capital, and consuming the proceeds itself. Suppose that this tax rate ($\tau$) was constant. Show how this tax rate would affect the steady-state capital-labor ratio.
7. An employer is searching for a single worker to work for him. Each period he is permitted to hire a worker. Workers have productivity $z_t$ which is drawn from a distribution with cdf denoted by $F(z)$. All workers are identical a priori. That is, successive workers chosen by the firm represent identical, independent draws from the distribution $F(z)$.

When a worker with productivity $z_t$ works for the employer, they share the output equally. That is, they each get $(z_t/2)$ in each period the worker is employed. If the employer chooses to hire the worker permanently, he will get that amount each period forever (since the employer has an infinite planning horizon). The employer has a discount factor of $\beta = .5$. Each period the employer must decide whether to keep the worker he currently has working for him (whose value of $z_t$ is known), or to fire him, and search again for another worker again. If the employer fires the existing worker, he must pay him “severance pay” $\theta$ units at the beginning of the period after he was hired (which has a discounted value of $\beta \theta$). If the employer fires his current worker, he pays the severance payment next period, and immediately gets to hire a new worker. The employer cannot recall workers who were previously employed.

a) Set up the dynamic programming problem.
b) Present an argument as to why the employer should use a “reservation productivity” strategy for choosing whom to hire.
c) Explain why the employer would never want to recall workers whom he had previously rejected.
d) Suppose that the distribution is uniformly distributed between zero and 100. Furthermore, let $\theta = 12$. Determine the optimal hiring strategy for the employer.
e) Explain why you think that your answer is a sensible strategy.